

## A Stochastic Resonance Receiver for 4-PAM signals

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**Abstract**—Stochastic Resonance (SR) is considered as a noise enhanced phenomenon, that a response of a nonlinear system is improved by noise. In previous studies, an application of SR for wireless communication has been discussed and an SR receiver, the receiver that demodulate a received signal by SR, was proposed. However, these studies have used a basic modulation signal and have not focused on a signal that transmit information by multi-level amplitude. In this paper, we consider an SR receiver for 4-pulse amplitude modulation (4-PAM) signals. By applying SR, it can demodulate 4-PAM signals by a 1-bit resolution device. We show the system model of the receiver, evaluate its performance by simulation, and show its availability.

### 1. Introduction

Stochastic Resonance (SR) is a nonlinear phenomenon that can enhance the response of a system by adding noise under certain condition[1]. In contrast to many other systems that deal with noise negatively, SR utilizes noise positively. By integrating noise as a positive factor of a system, the energy-saving and high performance system can be realized. Particular advantages of SR are that it can detect weak signals buried in noise, detect weak signals that is sub-threshold of detectable level, and linearize a nonlinear system by adding noise[2]. The characteristic of this interesting phenomenon has been discussed in the context of nonlinear physics[3, 4], and some applications of SR to wireless communication have been proposed. SR is expected to be utilized for spectrum sensing in cognitive radio[5], signal detection[6, 7], and improvement of receiver sensitivity[8].

We focus on applying SR to a receiver for communication. In previous studies, a SR receiver for communication is proposed and have been discussed[8, 9, 10, 11]. These studies focused on detecting sub-threshold signals by SR and the transmitted signal was a basic modulation signal.

However, these studies have not been focused on a utility of a signal that transmit information by multi-level amplitude, like a 4-pulse amplitude modulation (4-PAM) signal. Such a signal can use a frequency bandwidth more efficiently and a modulated signal progressed from this signal is practically used. Applying SR to demodulation of these signals is essential for an application of SR to wireless communication.

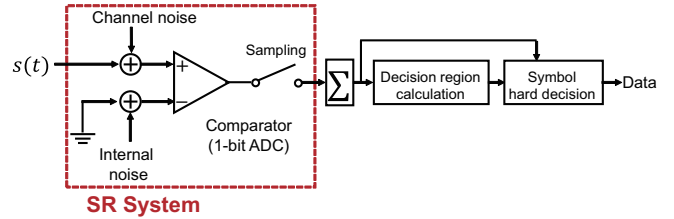


Figure 1: System model of SR receiver

Herein, we consider a SR receiver for the 4-PAM signal. Contributed by SR characteristics that it can linearize a nonlinear system by using noise, this receiver can demodulate the 4-PAM signal, that have 2-bit amplitude, by using a 1-bit resolution device. we show a system model of the receiver, evaluate the symbol error rate (SER) performance of this receiver by simulation and show its availability. This receiver is useful for the situation that we can only use a low-resolution device, e.g. a communication by weak signals, or a wide-band communication because the wide-band communication requires a high speed device but the device has a trade-off between a sampling speed and a resolution.

This paper is organized as follows. First, we show the system model of an SR receiver for 4-PAM signals in Sec. 2. In Sec. 3, we evaluate the SER performance of the SR receiver by a simulation. Conclusions are given in Sec. 4 and future work is given in Sec. 5.

### 2. System model

Figure 1 shows the system model of the SR receiver for a 4PAM signal. The transmitted 4-PAM signal  $s(t)$  is given by

$$s(t) = \sum_i d_i g(t - iT). \quad (1)$$

where  $d_i$  is the  $i$ th symbol of a data sequence, that takes  $\{-3, -1, +1, +3\}$ . And  $g(t)$  is a rectangular pulse that has duration  $T$  and its amplitude is unity. Through a wireless communication channel, a channel noise  $n_c(t)$  is add to  $s(t)$ . The received signal  $r(t)$  is given by

$$r(t) = s(t) + n_c(t). \quad (2)$$

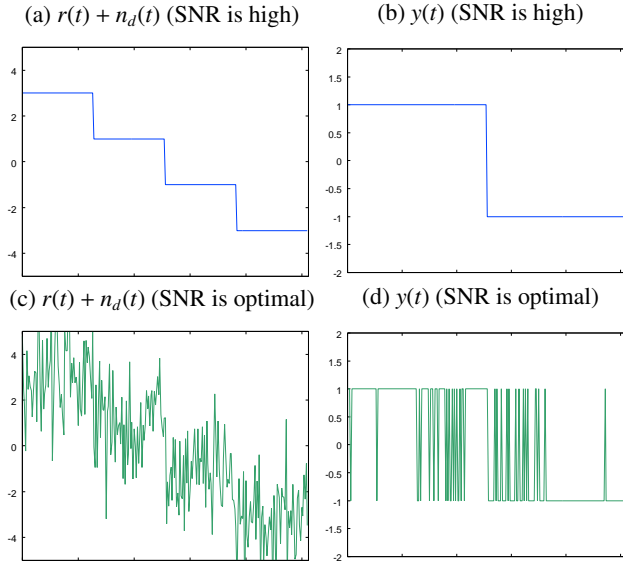


Figure 2: Examples of the input and the output signals when (a)(b)SNR is high and (c)(d)SNR is optimal.

We assume demodulating such a signal by a 1-bit device. we assume a threshold of this device is set to 0, and the threshold has internal noise  $n_d(t)$ . This noise also affect a output of the device. The output  $y(t)$  is given as follows.

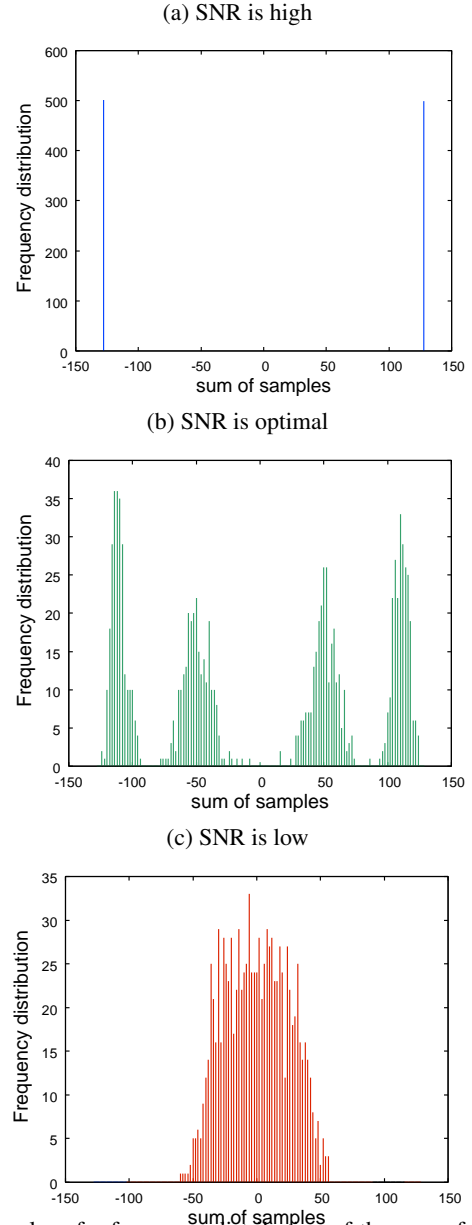
$$y(t) = \begin{cases} +V & r(t) + n_d(t) > 0 \\ -V & \text{otherwise} \end{cases} \quad (3)$$

After that, the output  $y(t)$  is sampled  $N$  times during the symbol duration  $T$ .

Figure 2 shows examples of waveforms of  $r(t) + n_d(t)$  and  $y(t)$ . As Fig. 2 (a) and (b) show, when SNR of  $r(t) + n_d(t)$  is high,  $y(t)$  is constant between each symbol and we cannot decide the transmitted symbol is +1 or +3, and  $-1$  or  $-3$ . However, as Fig. 2 (c) and (d) show, when SNR of  $r(t) + n_d(t)$  is optimal,  $y(t)$  changes in each symbols. The output probability of  $y(t)$  depends on original amplitude of  $s(t)$ . Therefore, we can decide the transmitted symbol from a sum of the sampled value in each symbol.

Figure 3d shows frequency distributions of the sum of the sampled value in each symbol when 1000 symbols are transmitted and  $N=128$ ,  $V=1$ . When SNR is high, as Fig. 3d (a) shows, the frequency distribution has only 2 peaks and a correct decision of 4 types of the transmitted symbols is impossible. When SNR is optimal, as Fig. 3d (b) shows, the frequency distribution has 4 peaks and we can decide 4 types of the transmitted symbols correctly by hard decision. When SNR is low, as Fig. 3d (c) shows, the frequency distribution mostly depends on noise and we cannot decide the transmitted symbols.

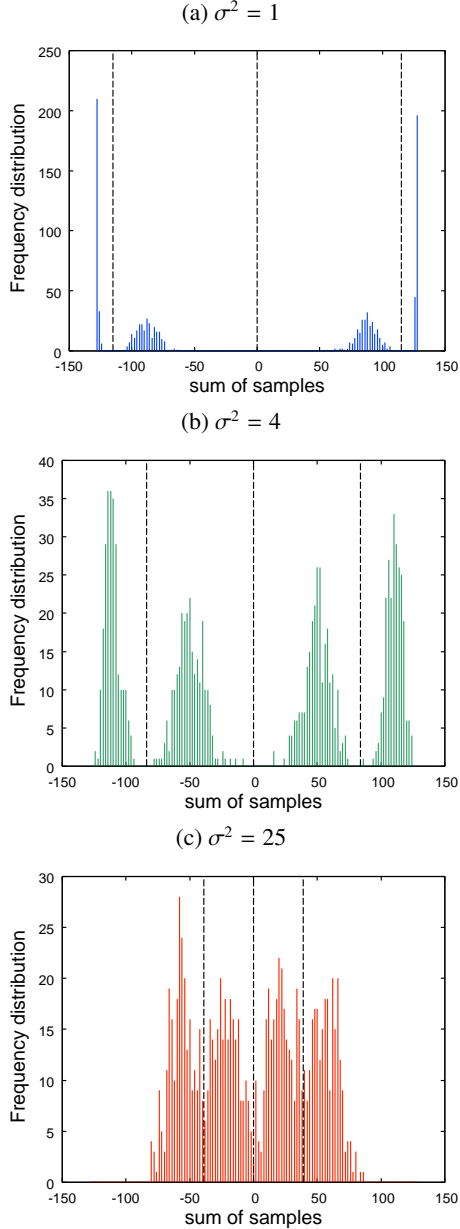
As explained, the performance of the SR receiver is improved under certain SNR. On the SR receiver, high SNR does not always lead to good performance. By tuning SNR, the SR receiver can demodulate the 4-PAM signal, that have 2-bit amplitude, by using a 1-bit resolution device.



(d) Examples of a frequency distribution of the sum of samples when (a)SNR is high, (b)SNR is optimal and (c)SNR is low.

The number of samples per symbol  $N$  is an important parameter for SR. In the SR system, the large  $N$  leads to improve the SR performance. We can enlarge  $N$  by extending the symbol duration  $T$ , shortening a sampling interval, or connecting statistically independent SR systems parallelly[12]. Especially, the characteristics that the performance of the receiver can be improved by parallel connecting is one of the advantages of this receiver.

A decision region for 4-PAM demodulation by the SR receiver depends on SNR, not the signal power. Figure 4d shows the frequency distribution with each variance of noise  $\sigma^2 = \sigma_c^2 + \sigma_d^2$  and expected thresholds for symbol decision. In Fig. 4d, the signal power of each distribu-



(d) Examples of a frequency distribution of the sum of samples in each variance of noise  $\sigma^2 = \sigma_c^2 + \sigma_d^2$  and expected thresholds for symbol decision

tion is the same. However, the thresholds are different in each SNR. Therefore the decision region must be decided dynamically. The method how to calculate the decision region is one of problems of this SR receiver.

### 3. Simulation

In this section, we evaluate the SER performance of the SR receiver for 4-PAM signals by simulation. The parameter settings for this simulation are listed in Table 1. In this simulation, both the channel noise  $n_c(t)$  and the inter-

Table 1: Parameter settings for simulation

| parameter                       | value            |
|---------------------------------|------------------|
| transmitted symbols             | $-3, -1, +1, +3$ |
| Noise distribution              | AWGN             |
| Algorithm for decision region   | Otsu's method    |
| Number of symbols for algorithm | 1000             |
| Number of trials for simulation | 1000             |

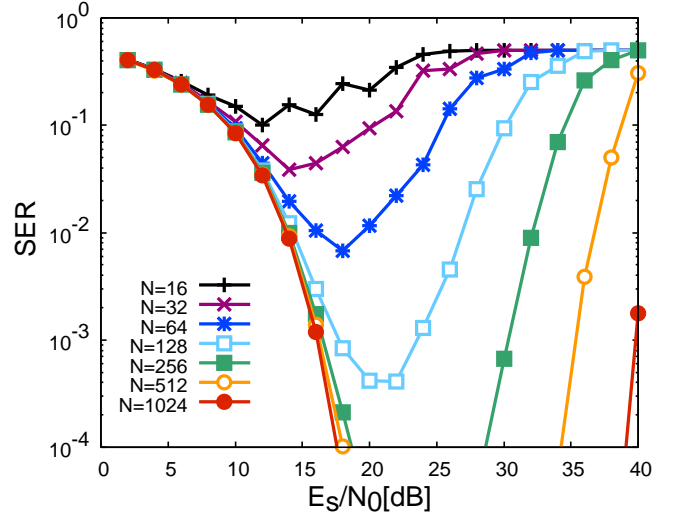


Figure 5: Simulation results of SER performance of the SR receiver with each number of samples per symbol  $N$

nal noise of device  $n_d(t)$  are a zero-mean white Gaussian noise and they are treated as identical. In this situation, because of positive/negative symmetry of noise and transmitted symbols, a threshold of the decision region between the symbols  $\{-1, +1\}$  is zero. Also we use Otsu's method for calculation of the thresholds of the decision region between the symbols  $\{+1, +3\}$  and between the symbols  $\{-1, -3\}$ . Note that Otsu's method is a popular method for clustering-based image thresholding and this method does not minimize SER, but easy to implement.

We evaluate  $N$  as a variable parameter. In this simulation, the sampling interval is constant and  $N$  is enlarged by extending the symbol duration  $T$ . In this case, an energy of the transmitted symbol is enlarged by increasing of  $N$ , therefore we compare the SER of the SR receiver on each  $N$  by SNR. Note that there are the three methods to enlarge  $N$  and this method leads to the worst performance.

Figure 5 shows the result of the simulation. The horizontal axis is a ratio of average energy per symbol  $E_s$  to noise power spectral density  $N_0$  of  $n_c(t) + n_d(t)$ , and the vertical axis is the SER performance of the SR receiver for 4-PAM signals. As the result shows, the SER performance is improved in a specific  $E_s/N_0$ , which is typical of SR.

The optimal SNR is that which achieves the required

SER. For example, as shown in Fig. 5, when the required SER is  $10^{-3}$  and  $N=256$ , the range of optimal SNR is  $E_s/N_0 = 17-30$ . In practice, we assume that the intentional use of an inexpensive and noisy device or adjusting the signal power for SR, with the SNR roughly adjusted to bring it within the optimal range.

Also, the SER performance of the SR receiver is improved with a increasing of  $N$ . When  $N$  is doubled, the optimal SNR range of the SR receiver is expand more than 3dB. This shows a expandability of the SR receiver.

#### 4. Conclusion

In this paper, we considered the stochastic resonance receiver for 4-PAM signals. Contributed by the SR characteristics that it can linearize a nonlinear system by using noise, this receiver can demodulate 4-PAM signal, that have 2-bit amplitude, with a 1-bit resolution device. The simulation result shows the availability of the receiver, and give a criterion of the number of samples per symbol  $N$  and optimal SNR.

#### 5. Future work

We show the system model of the SR receiver for 4-PAM signal in this paper. Next, we will extend this method to demodulation of a RF sampled signal. Demodulation of a RF sampled signal is important for a software defined radio (SDR). RF sampling requires a high speed devise but the device has a trade-off between a sampling speed and a resolution. Therefore, the demodulation method with a low-resolution device is required for the high frequency SDR and the development of the proposed method in this paper will be its resolution. Also, the RF sampled signal has a synergy with SR in that a transmitted symbol is oversampled.

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