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Collective Dynamics in Networks of Bistable Time-Delayed Feedback Oscillators Coupled via the Mean Field

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Abstract– We investigate the collective dynamics of oscillators in a network of identical bistable time-delayed feedback systems globally coupled via the mean field. The variety of dynamical regimes in the network results from the presence of bistable states with substantially different frequencies in coupled oscillators. The existence of chimera states is shown, in which some part of oscillators in the network exhibits synchronous oscillations, while all other oscillators remain asynchronous.

1. Introduction

Networks of coupled oscillators have been studied by many authors for several decades. These investigations have revealed many nonlinear phenomena, including the formation of various structures, clusterization, and synchronization of oscillators in the network [1]. It was believed for a long time that the regions of synchronous behavior of network elements can coexist with the regions of asynchronous behavior only in heterogeneous networks, in which oscillators with close frequencies become synchronized, while oscillators with appreciably different frequencies exhibit asynchronous dynamics. Afterwards, it was found out that coexistence of synchronous and asynchronous groups of oscillators is possible also in networks of coupled identical oscillators [2]. Such state was named in [3] as chimera state.

Chimera states have been identified in networks of identical oscillators with nonlocal [3], local [4], and global coupling [5, 6]. Chimera states have also been observed in various experiments [7–10]. In the present paper, we study experimentally the collective dynamics of oscillators, including chimera states, in a network of identical bistable oscillators with time-delayed feedback globally coupled via the mean field. In contrast to experiments dealing with a single electronic time-delay system [9], we examine a network composed of eight experimental electronic delayed-feedback oscillators globally coupled via the mean field.

The paper is organized as follows. In Section 2, we describe a network under study. In Section 3, the results of the network experimental investigation are presented. In Section 4, we summarize our results.

2. Network of Time-Delay Systems Globally Coupled via the Mean Field

We consider a network consisting of coupled identical time-delay systems, with each system described in the absence of coupling by the following delay-differential equation:

$$\varepsilon \dot{x}(t) = -x(t) + f\left(x(t-\tau)\right), \qquad (1)$$

where τ is the delay time, ε is the parameter of inertia, and f is a nonlinear function. Equation (1) is a mathematical model of electronic self-sustained oscillator composed of a ring with three elements: a delay line, nonlinear device, and low-pass first-order RC filter. In Fig. 1, such oscillator is enclosed by a dashed line. For this oscillator, $x(t)$ and $x(t-\tau)$ in Eq. (1) are the delay line input and output voltages, respectively, and $\varepsilon = RC$.

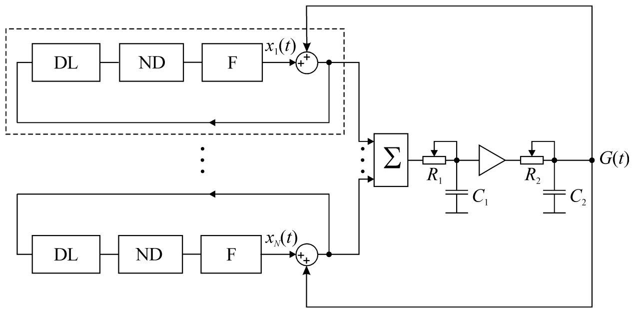

Fig. 1. Block diagram of a network of identical time-delayed feedback oscillators coupled via the mean field. The first and Nth oscillators are depicted. The delay lines, nonlinear devices, and filters are denoted as DL, ND, and F, respectively. The summary amplifier is denoted as Σ.

Let the nonlinear device in oscillator provide a transformation described by cubic function

$$f(x) = a + b(x-d) - c(x-d)^3 . \qquad (2)$$

This function is plotted in Fig. 2 for $a = 1.5$, $b = 2.3$, $c = 1.78$, and $d = 1.57$. With this nonlinearity, the system (1) shows bistability. Depending on the initial conditions, it can exhibit two regimes of oscillations, which occur in the vicinity of unstable fixed points A and B (Fig. 2). Nearby the fixed point A, periodic oscillations in the fundamental mode take place at a frequency close to $\nu_1 = 1/(2\tau)$. Nearby the fixed point B, chaotic oscillations at the third harmonic of the fundamental mode take place at a basic frequency close to $\nu_2 = 3/(2\tau)$.

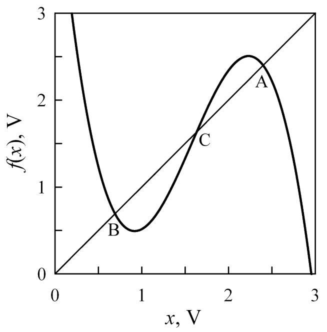

Fig. 2. Transfer function $f(x)$ of the experimental electronic oscillator. A, B, and C are unstable fixed points.

We couple the oscillators (1) via the mean field $G(t)$ in such a way that the dynamics of each oscillator in the network is described by the following equation:

$$\varepsilon \dot{x}_i(t) = -x_i(t) + f\big(x_i(t-\tau) + G(t-\tau)\big), \qquad (3)$$

where $i = 1, \ldots, N$ and N is the number of oscillators. A block diagram of the network of coupled oscillators under investigation is depicted in Fig. 1.

The mean field is formed by the summation of signals $x_i(t)$ from all oscillators using the summing amplifier with the transfer coefficient k and normalization of the summary signal to N. The resulting signal passes through a linear phase-shifting chain comprising two series-connected low-pass first-order RC filters and is fed into each oscillator as an external driving. The mean field is described by the following equation:

$$\gamma_1\gamma_2\ddot{G}(t) + (\gamma_1 + \gamma_2)\dot{G}(t) + G(t) = \frac{k}{N}\sum_{i=1}^{N} x_i(t), \qquad (4)$$

where $\gamma_1 = R_1C_1$ and $\gamma_2 = R_2C_2$. It should be noted that the signal $G(t)$ can be fed into the ring delayed-feedback oscillator at various points. For instance, it can be fed into oscillators between the nonlinear device and filter. In this case, the oscillators will be governed by the following equation:

$$\varepsilon \dot{x}_i(t) = -x_i(t) + f\big(x_i(t-\tau)\big) + G(t). \qquad (5)$$

In the present paper, we restrict our consideration to the case of oscillators described by Eq. (3).

3. Results of the Network Experimental Investigation

We study experimentally a network composed of eight self-sustained electronic oscillators described by Eq. (3) with parameters $\tau = 1$ ms and $\varepsilon = 0.084$ ms and cubic transfer function $f(x)$ depicted in Fig. 2. These oscillators contain analog RC filter and digital delay line and nonlinear element implemented on programmable microcontrollers. The analog and digital elements of oscillators are connected via analog-to-digital converters and digital-to-analog converters that are not shown in Fig. 1. The oscillators are coupled via the mean field $G(t)$ at $k = 0.01$.

Using programmable microcontrollers, we specify the initial conditions programmatically as a constant on the time interval equal to the delay time of oscillators. For four oscillators, the initial conditions were set equal to 2 V, while for the four other oscillators, the initial conditions were set equal to 0.5 V. These initial conditions belong to the basin of attraction of periodic attractor and chaotic attractor, respectively. As a result, four oscillators perform periodic oscillations in the fundamental mode (first harmonic), while the four other oscillators perform chaotic oscillations at the third harmonic of the fundamental mode. In this case, the oscillators in the network are separated into two clusters. One of these clusters contains oscillators with periodic behavior at a frequency close to ν_1 and another cluster contains oscillators with chaotic dynamics at a basic frequency close to ν_2.

Since all oscillators of the network take part in the formation of the mean field, the signal $G(t)$ has two main components with the frequencies close to ν_1 and ν_2. Each of these components, as it passes through a linear two-section RC filter, undergoes a phase shift

$$\Delta\varphi = -\arctan(2\pi\nu\gamma_1) - \arctan(2\pi\nu\gamma_2), \qquad (6)$$

which value depends on the frequency ν of the component. In Eq. (6), the first and second terms represent contribution of the first and second filter sections, respectively. For the low-frequency component of $G(t)$, $\nu = \nu_1$ and $\Delta\varphi = \Delta\varphi_1$ in Eq. (6), while for the high-frequency component of $G(t)$, $\nu = \nu_2$ and $\Delta\varphi = \Delta\varphi_2$ in Eq. (6).

The value of phase shift $\Delta\varphi$ determines the collective behavior of oscillators in the network. For $|\Delta\varphi| < \pi/2$, the coupling via the mean field is attractive and oscillators synchronize between themselves after a transient process, while for $|\Delta\varphi| \geq \pi/2$, the coupling is repulsive and oscillators remain asynchronous [5]. In our example, the phase shift $\Delta\varphi_1$ is less by the absolute value than the phase shift $\Delta\varphi_2$ because $\nu_1 < \nu_2$. By varying the values of R_1 and R_2 in the filter (Fig. 1), it is possible to control the phase shifts $\Delta\varphi_1$ and $\Delta\varphi_2$ so as to ensure three qualitatively different situations: i) $|\Delta\varphi_1| < \pi/2$ and $|\Delta\varphi_2| < \pi/2$, ii) $|\Delta\varphi_1| < \pi/2$ and $|\Delta\varphi_2| \geq \pi/2$, and iii) $|\Delta\varphi_1| \geq \pi/2$ and $|\Delta\varphi_2| \geq \pi/2$.

In the first case, synchronization takes place between periodic oscillators and between chaotic oscillators. Fig. 3(a) shows parts of the experimental time series of voltage in eight coupled oscillators for $|\Delta\varphi_1| = 0.002\pi$ and $|\Delta\varphi_2| = 0.006\pi$. As seen in Fig. 3(a), the time series of periodic oscillators are slightly different. It is explained by the fact that it is practically impossible to ensure the absolute identity of analog *RC* filters in experimental electronic oscillators. In the ideal case of identical oscillators, one would observe complete synchronization of periodic oscillators. The chaotic oscillators in Fig. 3(a) exhibit phase synchronization, but the amplitude of oscillations can be different.

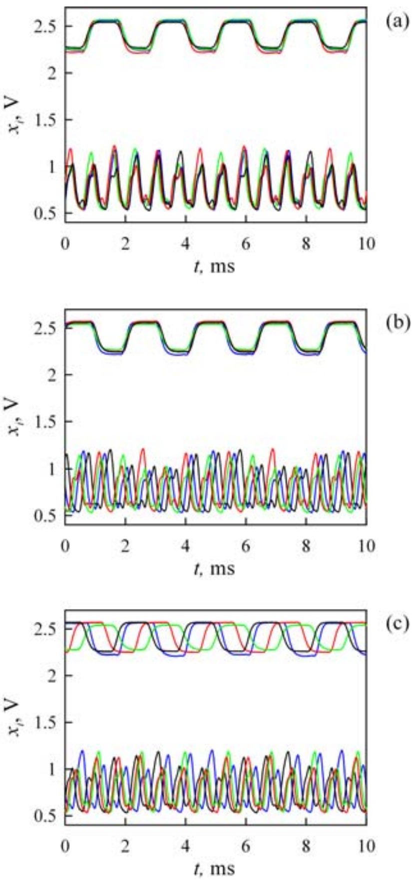

Fig. 3. Experimental time series of voltage in eight coupled electronic oscillators for $|\Delta\varphi_1| = 0.002\pi$ and $|\Delta\varphi_2| = 0.006\pi$ (a), $|\Delta\varphi_1| = 0.47\pi$ and $|\Delta\varphi_2| = 0.51\pi$ (b), and $|\Delta\varphi_1| = 0.8\pi$ and $|\Delta\varphi_2| = 0.99\pi$ (c). The time series of periodic and chaotic oscillators are shown at the top and at the bottom of the figures, respectively. The same set of colors is used for both periodic and chaotic time series.

The second situation is illustrated in Fig. 3(b) showing parts of the time series of voltage in all coupled oscillators for $|\Delta\varphi_1| = 0.47\pi$ and $|\Delta\varphi_2| = 0.51\pi$. The periodic oscillators exhibit synchronization similar to the case depicted in Fig. 3(a). The chaotic oscillators exhibit asynchronous behavior. This situation corresponds to a chimera state, since clusters with synchronized and desynchronized oscillators coexist in the network. It should be noted that chimera states can be observed in the network under study in spite of rather small number of oscillators. As it was shown recently in [10], chimera states can be identified even in small networks of coupled oscillators. Only four identical coupled oscillators are sufficient for observation of chimera states [10].

The last situation is illustrated in Fig. 3(c), which shows parts of the time series of voltage in eight coupled oscillators for $|\Delta\varphi_1| = 0.8\pi$ and $|\Delta\varphi_2| = 0.99\pi$. In this case, the oscillators in both clusters exhibit asynchronous behavior.

The snapshots of voltages $x_i(t)$ are presented in Fig. 4 for each of the three situations depicted in Fig. 3. The oscillators performing periodic oscillations are denoted by the numbers from 1 to 4, while the oscillators performing chaotic oscillations are denoted by the numbers from 5 to 8. The synchronized oscillators belonging to the same cluster have close x_i values [Fig. 4(a)]. In the regimes of asynchronous oscillations, the instantaneous voltages x_i are noticeably different both in periodic and chaotic oscillators [Fig. 4(c)]. Fig. 4(b) illustrates a chimera state, in which the periodic oscillators are synchronized, while the chaotic oscillators are desynchronized.

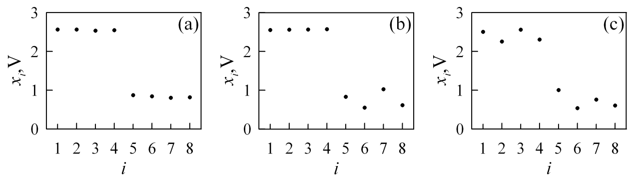

Fig. 4. Snapshots of voltages $x_i(t)$ in eight coupled experimental oscillators for $|\Delta\varphi_1| = 0.002\pi$ and $|\Delta\varphi_2| = 0.006\pi$ (a), $|\Delta\varphi_1| = 0.47\pi$ and $|\Delta\varphi_2| = 0.51\pi$ (b), and $|\Delta\varphi_1| = 0.8\pi$ and $|\Delta\varphi_2| = 0.99\pi$ (c).

Fig. 5 shows the space-time plots of the network of eight coupled experimental electronic oscillators. Since the analog *RC* filters in the real oscillators cannot be absolutely identical, the periodic oscillators 1–4 exhibit slightly different oscillations even in the synchronous regimes [Figs. 5(a) and (b)]. In Fig. 5(a), the chaotic oscillators 5–8 exhibit phase synchronization, but the amplitude of oscillations can be different. In Figs. 5(b) and (c), the chaotic oscillators exhibit asynchronous behavior. In this case, the difference between the amplitudes of their oscillations is more pronounced than in Fig. 5(a).

4. Conclusion

We have experimentally studied the collective dynamics of oscillators in the network of identical bistable time-delay systems globally coupled via the mean field. The variety of dynamical regimes in the considered network results from the presence of bistable states with substantially different frequencies in coupled oscillators.

One of the bistable regimes takes place in the fundamental mode of the time-delay system oscillations, while another regime takes place at the third harmonic of the fundamental mode. This feature of the bistable system under investigation allows us to ensure different phase shifts of the signal of the mean field for oscillators performing oscillations at different harmonics.

The specific type of dynamical regime in the considered network is fully determined by the choice of initial conditions in coupled oscillators. We could observe chimera states in the network when the number of oscillators performing periodic oscillations was not less than two and the number of oscillators performing chaotic oscillators at the same time was also not less than two.

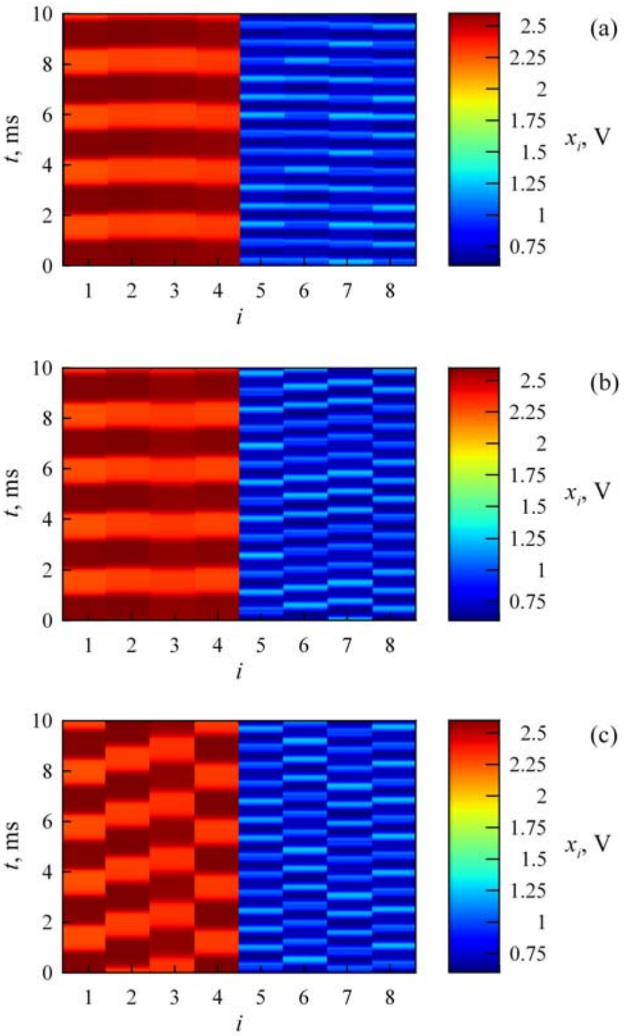

Fig. 5. Space-time plots of the network of eight coupled experimental oscillators for $|\Delta\varphi_1| = 0.002\pi$ and $|\Delta\varphi_2| = 0.006\pi$ (a), $|\Delta\varphi_1| = 0.47\pi$ and $|\Delta\varphi_2| = 0.51\pi$ (b), and $|\Delta\varphi_1| = 0.8\pi$ and $|\Delta\varphi_2| = 0.99\pi$ (c).

It is shown that two clusters coexist in the network. Depending on the phase shift of the mean field, each of these clusters can exhibit either synchronous or asynchronous behavior of oscillators in the cluster. In the case, where the coupling via the mean field is attractive for oscillators in one cluster and repulsive for oscillators in another cluster, a chimera state occurs in the network. In this state, clusters with synchronized and desynchronized oscillators coexist in the network.

We have considered the situation in which oscillators perform periodic oscillations in one of the bistable states and chaotic oscillations in another bistable state. However, qualitatively similar results can be obtained in the cases, where both bistable states are periodic or both bistable states are chaotic. It should be noted that in the case of attractive coupling, the identical periodic oscillators exhibit complete synchronization, while the chaotic oscillators exhibit phase synchronization.

The considered features of the collective dynamics in the network of identical bistable time-delayed feedback oscillators stand for different types of injection of the signal of the mean field into the ring time-delay systems.

Acknowledgments

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