

Design of Balanced Code Sequences for DS/CDMA Using Genetic Algorithm

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Abstract– In this paper, a multi-objective Genetic Algorithm (MOGA) is applied for the design of balanced code sequences in DS/CDMA system. By rearranging the column of the Hadamard matrix according to order-based chromosomes, the non-dominated set of solutions for different cross-correlation and auto-correlation performance can be obtained using MOGA. Such scheme can provide the designer with feasibility under different system environments.

1. Introduction

In mobile communication, Direct-Sequence Code Division Multiple Access (DS/CDMA) technology [1] is commonly used. DS/CDMA can support multiple accesses, for which the same frequency band is shared by a group of users, each identified by his unique code sequence.

The major design criterion of such a DS/CDMA system is the multiple access interference (MAI). As a measurement depended on the periodic cross-correlation properties of the code sequences, the MAI will affect the system capacity performance. However, under some other situations, periodic cross-correlation may not be the sole consideration. For example, the performance of the correlator is also affected by the odd cross-correlation under the condition that information bits are changed over one integration interval [2, 3]; a good auto-correlation is important for reducing the interference and allowing fast synchronization for a multi-path environment, and so on.

Designing sequences with good odd cross-correlation and/or good auto-correlation is a difficult task [4, 5], and a tradeoff relationship between these two correlation factors is well-known [6]. In this paper, by taking the correlation values as our objectives, a multi-objective genetic algorithm (MOGA) [7,8] is proposed and designed so that the design problem can be solved.

In order to formulate our code generation in the framework of GA, the Sylvester-type Hadamard matrix is adopted. In [9], a random process has been introduced to modulate the regular structure of the Hadamard matrix for the generation of code sequences, and some improvements in auto-correlation and odd cross-correlation have been observed. However, since a random process is used, the obtained result is always sub-optimal.

In our approach, an order-based GA is proposed and designed for modifying the Hadamard matrix so that balanced code sequences can be generated. The

organization of the paper is as follows. The Hadamard coding method is revisited in Sect. 2 and the correlation properties are briefly explained. In Sect. 3, the proposed MOGA is explained in details with the results given in Sect. 4. Finally, conclusion remarks are given in Sect. 5.

2. Walsh-Hadamard Code and Correlation Properties

Walsh-Hadamard code, well-known for its ability in eliminating the multi-access interference, is one of the most commonly used sequences in DS/CDMA system. It is generated by Hadamard function $H^{(k)}$ of order k which can be recursively defined by

$$H^{(k)} = \begin{bmatrix} H^{(k-1)} & H^{(k-1)} \\ H^{(k-1)} & -H^{(k-1)} \end{bmatrix} \quad (1)$$

with $k \geq 1$ $H^{(0)} = 1$.

A set of orthogonal codes of length $n = 2^k$ (the set is denoted as A), can be obtained by taking the rows (except the first row) of the matrix in (1), or

$$a_i = H_{i+1}^{(k)} \quad (2)$$

where $a_i \in A$, $i = 1, 2, \dots, 2^k - 1$; $H_i^{(k)}$ is the i -th row vector; and $a_{i,j} = H_{i+1,j}^{(k)}$ is the element at the i -th row and j -th column in $H^{(k)}$ or the j -th element in the sequence a_i .

The cross-correlation function (CCF) and auto-correlation function (ACF) of the sequences in A are then defined as follows [10]:

Even periodic CCF:

$$\theta_{x,y}(l) = C_{x,y}(l) + C_{x,y}(l-n) \quad (3)$$

Odd periodic CCF:

$$\hat{\theta}_{x,y}(l) = C_{x,y}(l) - C_{x,y}(l-n) \quad (4)$$

where $C_{x,y}(l)$ is the aperiodic CCF defined by

$$C_{x,y}(l) = \begin{cases} \sum_{j=0}^{n-1-l} a_{x,j} a_{y,j-l}^* & \text{if } 0 \leq l \leq n-1 \\ \sum_{j=0}^{n-1-l} a_{x,j-l} a_{y,j}^* & \text{if } 1-n \leq l < 0 \\ 0 & \text{if } |l| \geq n \end{cases} \quad (5)$$

with $a_x, a_y \in A$.

The even and odd periodic ACFs, $\theta_{x,x}(l)$ and $\hat{\theta}_{x,x}(l)$, are defined similarly as that in (3) and (4), respectively, but with $x = y$.

As found in [6], there is a tradeoff relationship between CCF and ACF, which can be expressed as:

$$\sum_{l=0}^{n-1} \theta_{x,y}^2(l) \leq n^2 + \left(\sum_{l=1}^{n-1} \theta_{x,x}^2(l) \right)^{\frac{1}{2}} \left(\sum_{l=1}^{n-1} \theta_{y,y}^2(l) \right)^{\frac{1}{2}} \quad (6)$$

It should be emphasized that the odd CCF and the ACF will also play an important roles on the performance of DS/CDMA system under the situations such as transmitting consecutive data bits with a polarity transition [2] or multipath environments.

3. GA-Optimization of Balance Code Sequences

GA [11] is an optimization technique that inspired by the mechanism of natural selection, where potential solutions are formulated as chromosomes. Initially, a group of chromosomes, or called population, is randomly created. The chromosomes are evaluating based on the objective functions, and their goodness, represented by fitness values, can be reflected. One of the major advantages in using GA [12] is that multiple objectives can be easily transformed into a fitness value, for example, the rank of chromosome in the population.

In this paper, an order-based multiobjective GA (MOGA) is designed to obtain the optimal set of sequences that can achieve both good ACF and CCF performance. Guided by their fitness values, chromosomes are evolved and new solutions are generated from the genetic operations. The newly-formed chromosomes will replace some or all of the chromosomes in the original population to form a new population, and the performance of the population will then be improved. The cycle is repeated until some criteria are fulfilled. The flow diagram of GA is depicted in Fig. 1.

3.1. Representation of Gene

The order based chromosome in our design, comprises a string of distinct integers, where each integer indicates a particular column of the Hadamard matrix H_k . Given that the total number of columns in H_k is 2^k , the order genes are in the range of $[1, 2^k]$.

The sequence of the genes reflects the order of columns in the Hadamard matrix, so that a new matrix can be formed. Two examples are depicted in Fig. 2, where 8-bit length sequences (i.e. $k=3$) are assumed.

The obtained sequences in Fig. 2(a) are
 (1, -1, 1, -1, 1, -1, 1, -1), (1, 1, -1, -1, 1, 1, -1, -1),
 (1, -1, -1, 1, 1, -1, -1, 1), (1, 1, 1, 1, -1, -1, -1, -1),
 (1, -1, 1, -1, -1, 1, -1, 1), (1, 1, -1, -1, -1, -1, 1, 1),
 (1, -1, -1, 1, -1, 1, 1, -1)

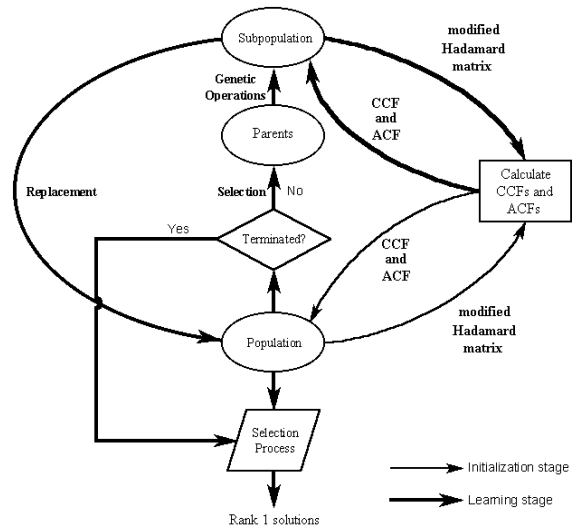


Fig. 1. Flow diagram of GA

1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1
1	-1	1	-1	1	-1	1	-1
1	1	-1	-1	1	1	-1	-1
1	-1	-1	1	1	-1	-1	1
1	1	1	1	-1	-1	-1	-1
1	-1	1	-1	-1	1	-1	1
1	1	-1	-1	-1	-1	1	1
1	-1	-1	1	-1	1	1	-1

(a)

2	3	1	7	4	5	8	6
1	1	1	1	1	1	1	1
-1	1	1	1	-1	1	-1	-1
1	-1	1	-1	-1	1	-1	1
-1	-1	1	-1	1	1	1	-1
1	1	1	-1	1	-1	-1	-1
-1	1	1	-1	-1	-1	1	1
1	-1	1	1	-1	-1	1	-1
-1	-1	1	1	1	-1	-1	1

(b)

Fig. 2. Chromosome and resultant matrices

while the sequences from Fig. 2(b) are

(-1, 1, 1, 1, -1, 1, -1, -1), (1, -1, 1, -1, -1, 1, -1, 1),
 (-1, -1, 1, -1, 1, 1, 1, -1), (1, 1, 1, -1, 1, -1, -1, -1),
 (-1, 1, 1, -1, -1, -1, 1, 1), (1, -1, 1, 1, -1, -1, 1, -1),
 (-1, -1, 1, 1, 1, -1, -1, 1).

3.2. Genetic Operations

After selecting two parents based on the fitness proportionate scheme, order crossover [11] is used to generate new valid chromosomes (also known as

offspring and grouped as a sub-population). By randomly choosing two crossover points, the parents are divided into three portions. The genes in the middle portion are to be swapped and the rests are filled up by the remaining genes based on the original relative order in the parents, starting from the first crossover point.

Inversion [11] is also adopted in our design. The order of genes between two randomly chosen positions in a chromosome will be inverted accounting to some operational probability. In our design, it is set to be 0.05.

For the exploration of the genetic search, mutation operation is used to swap two randomly chosen genes, also with a small operational rate of 0.05.

3.3. Fitness Evaluation

In our design, both the ACF and CCF are to be optimized simultaneously. Let $\theta_{x,x}(l)$ be the ACF of sequence x at time l and $\theta_{x,y}(l)$ be the CCF between sequences x and y at time l , the objectives are:

- 1) to minimize the sum of average ACFs at all time delays

$$S_{ACF} = \sum_l \left(\frac{1}{n} \sum_{x=1}^n \theta_{x,x}(l) \right) \quad (7)$$

- 2) to minimize the sum of average CCFs at all time delays

$$S_{CCF} = \sum_l \left(\frac{1}{C_2^n} \sum_{x,y \in [1,n], x \neq y} \theta_{x,y}(l) \right) \quad (8)$$

Instead of aggregating the objectives with a weighting function, the multi-objective approach suggested in [12] is applied. The fitness of the chromosome is specified by its rank in the population, which in turns, defined as

$$rank(I) = 1 + p \quad (9)$$

if chromosome I is dominated by other p chromosomes in the population, and the definition of domination [16] for an n -objective minimization problem is given by:

Chromosome u is dominated by chromosome v if
 $\forall i = 1, \dots, n, \quad f_i(u) \geq f_i(v)$

and

$$\exists j = 1, \dots, n, \quad s.t. \quad f_j(u) > f_j(v).$$

where f is the objective function of the problem.

To further enhance the searching, fitness sharing [12] is adopted to prevent the finite population from converging to only one or few of the points in Pareto optimal set.

4. Results

Assuming that 32-bit code sequences are to be generated, the MOGA described in the previous section is designed and the Pareto set obtained is depicted in Fig. 3. The tradeoff relationship between ACF and CCF is clearly observed and each point is a possible solution. To compare their performance, four sets of code sequences are chosen with an equi-distance along the ACF-axis and their ACFs and CCFs are tabulated in Table 1.

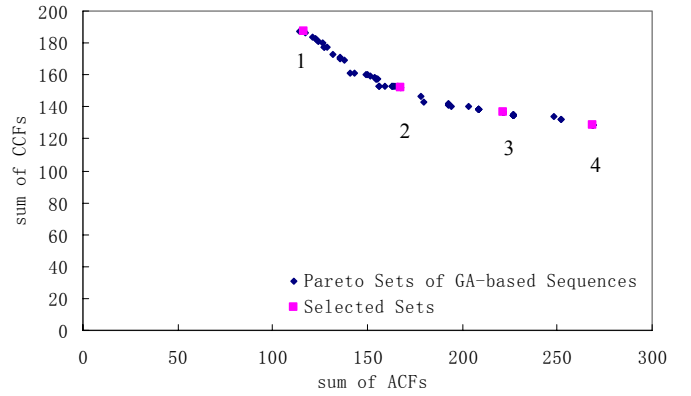


Fig. 3. Pareto optimal set of GA-based Sequences

Table 1: Comparison of Four Selected Solution Sets

Solution Set	Sum of average ACFs (7)	Sum of average CCFs (8)
1	116.71 (-56.54%) [†]	187.041 (+45.30%)
2	167.161 (-37.75%)	152.348 (+18.35%)
3	221.484 (-17.52%)	136.555 (+6.08%)
4	268.516 (0%)	128.731 (0%)

[†] % difference as compared with 4th set (original Walsh code)

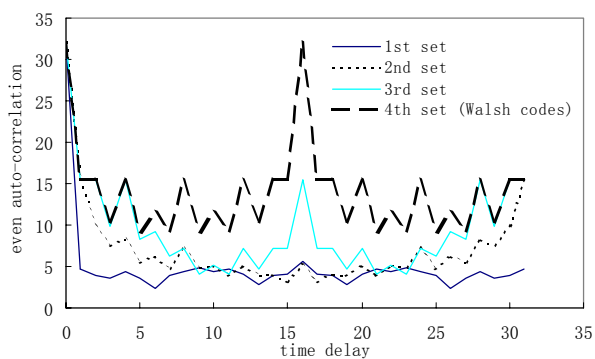
Due to the limitation of space, only the average values of periodic autocorrelation and cross-correlation are shown in Figs. 4 and 5.

From Fig. 4 (a) and (b), it can be observed that both the average values of the even and odd periodic autocorrelation of the 1st – 3rd solution sets are better than those of the Walsh code (the 4th set). In particular, a significant improvement is observed in the 1st set. Similar improvements are also found in comparing the maximum values of the periodic even and odd autocorrelations.

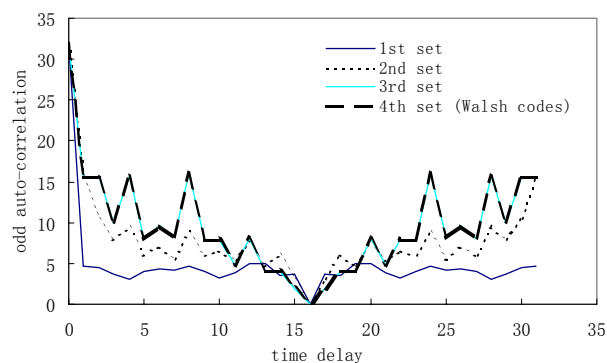
As expected, the periodic CCFs of the 1st – 3rd solution sets become worse than of the original Walsh codes (the 4th set), which is also illustrated in Fig. 5 (a) and (b). However, the maximum values, yet not shown here, are found to be even better than those of the Walsh codes in most of the delay intervals.

5. Conclusion

In this paper, a multi-objective GA is adopted for the design of balanced code sequences for DS/CDMA system. Using an order-based chromosome to rearrange the column of a Hadamard matrix, the Pareto optimal set of solutions with different cross-correlation and auto-correlation performance is obtained. From simulation, it is shown that the proposed approach can obtain those solutions with improved correlation properties as compared with the original Walsh code generated from the Hadamard matrix.

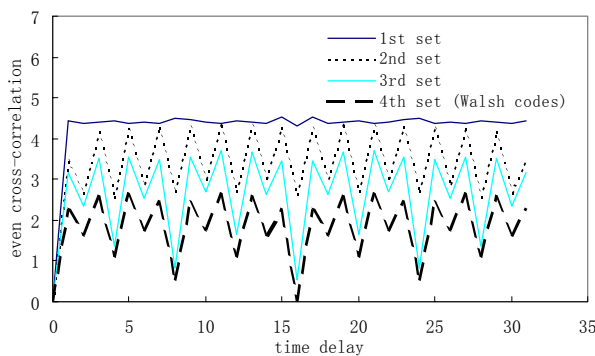


(a)

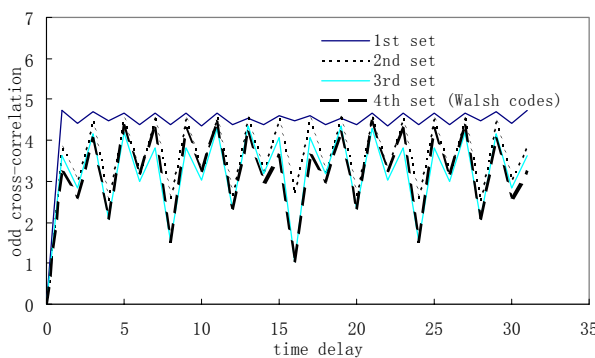


(b)

Fig. 4. (a) average values of even auto-correlation
(b) average values of odd auto-correlation



(a)



(b)

Fig. 5. (a) average values of even cross-correlation
(b) average values of odd cross-correlation

Acknowledgements

The work described in this paper was fully supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. CityU 1157/03E).

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