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Pull-in Characteristics of Self-Synchronizing Linkage Inverter System between Distributed Power Source and Power System

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Abstract—DC/AC inverters are indispensable for the interconnection of distributed power sources to power systems. The high confidential characteristics of inverter operation is strongly requested in order to keep the fault tolerant power suppliance. In this paper, the self synchronizing characteristics of our proposed inverter system based on a pull-in characteristic of PLL is discussed. As results, it is shown that the inverter system is able to behave as a synchronous generator depending on the power flow between the distributed power source and power system.

1. Introduction

Recent increase of distributed power sources requests us to develop the method for flexible interconnections to conventional power systems. DC power sources, for example solar cells, fuel cells etc., are inevitable to be connected with inverters for changing DC into AC power. The linkage inverters between distributed power sources and power system are requested to adjust the amplitude and the frequency of AC output voltage to power system. Moreover, the phase of AC voltage must be controlled according to the power flow between them. Generally speaking, the inverters do not have any inertia as synchronous generators and the ability to synchronize themselves to the linked power system. Therefore, the inverters have been controlled to fixed their voltage and current phases just at the same phase of the voltage at the linkage point.

The conventional linkage inverter is controlled with using the phase signal obtained through phase looked loop (PLL) or zero cross detecting circuits [1, 2]. In the static operation, they can work well enough to keep their output constant even under the fluctuation of input power. On a standpoint of nonlinear dynamics, we cannot recognize that they are the synchronization under which synchronous generators operate in the power system. They will keep their revolution speed with regulating their power flow in and out with slightly shifting their phase difference between themselves and power system. They show the autonomous synchronization with restoring force.

In this paper, we will discuss the self synchronizing characteristics of our proposed single phase inverter [5], which can be applied to the linkage system between distributed power sources and power systems. The linkage method is based on PLL pull-in characteristics and has self-synchronizing characteristics. The method is based on the principles reported by Harada [3] and Ohnishi [4] for the static operation. The discussions in this paper support our experimental results obtained in [5] and give a possibility of contribution by distributed power sources in power system.

2. Principle of Self Synchronizing Inverter

2.1. Original Work by Harada (Methid-1)

The inverter system is described in Fig.1 with connection to the infinite bus. Here, the distributed dc power source consists of a dc voltage source E_s and a resistor R_s. The assumption does not loose the generality. The original control method for the linkage system was proposed by Harada [3]. The block diagram is shown in Fig.2. It shows the principle of inverter operation which keeps DC voltage E_I constant. If DC voltage E_I is higher than commanded value E_{OPT}, inverter output angular frequency is higher than power system one. Then phase angle difference between inverter and power system, and inverter output current I_{inv} increases. Thus distributed power source output current I_s increases and E_I decreases. If E_I is lower than E_{OPT}, E_I increases. It is shown that DC voltage is kept constant in this method if distributed power sources and loads vary.

Here let us consider the operation of the inverter. The output angular frequency ω_{inv} is represented by

$$\omega_{inv} = KE_I. \tag{1}$$

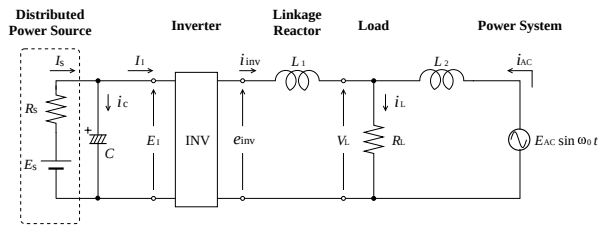

Figure 1: Configuration of linkage circuit

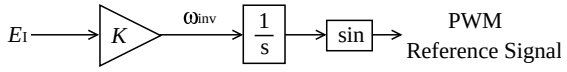

Figure 2: Block diagram by Harada (Method-1)

Then the differential equation is given as follows:

$$\frac{\mathrm{d}\Delta\omega}{\mathrm{d}t} = K\frac{\mathrm{d}E_{\mathrm{I}}}{\mathrm{d}t}, \tag{2}$$

where $\Delta\omega = \omega_{\mathrm{inv}} - \omega_0$. The current of conductance C is represented by

$$i_{\mathrm{C}} = C\frac{\mathrm{d}E_{\mathrm{I}}}{\mathrm{d}t} = \frac{E_{\mathrm{s}} - E_{\mathrm{I}}}{R_{\mathrm{s}}} - I_{\mathrm{I}} \tag{3}$$

From Eq.(1), (2) and (3), the state equation of inverter operation is induced as follows:

$$\begin{cases} \dfrac{\mathrm{d}\delta}{\mathrm{d}t} &= \Delta\omega \\ \dfrac{\mathrm{d}\Delta\omega}{\mathrm{d}t} &= \dfrac{K}{C}\left(\dfrac{E_{\mathrm{s}}}{R_{\mathrm{s}}} - I_{\mathrm{I}}\right) - \dfrac{1}{CR_{\mathrm{s}}}(\Delta\omega + \omega_0) \\ &= \dfrac{K}{C}\left(\dfrac{E_{\mathrm{s}} - E_{\mathrm{OPT}}}{R_{\mathrm{s}}} - I_{\mathrm{I}}\right) - \dfrac{1}{CR_{\mathrm{s}}}\Delta\omega \end{cases} \tag{4}$$

where δ is phase difference of inverter output voltage from system one.

2.2. Improved Method based on PLL (Method 2)

When LPF is ideal and harmonic component is completely eliminated, PLL output angular frequency deviation $\Delta\omega_{\mathrm{pll}}$ is represented by

$$\Delta\omega_{\mathrm{pll}} = -K_{\mathrm{p}}\sin(\delta_{\mathrm{pll}} - \delta_{\mathrm{ref}}), \tag{5}$$

where δ_{pll} and δ_{ref} are phase difference of PLL output voltage and PLL reference voltage from system ones. PLL reference voltage is load voltage V_{L}. $\Delta\omega_{\mathrm{avr}}$, which implies the angular frequency deviation in DC Automatic Voltage Regulator (DCAVR) output, is represented by

$$\Delta\omega_{\mathrm{avr}} = K(E_{\mathrm{I}} - E_{\mathrm{OPT}}) \tag{6}$$

From Eq.(5) and (6), the differential equation of inverter is formulated as follows:

$$\begin{cases} \dfrac{\mathrm{d}\delta_{\mathrm{avr}}}{\mathrm{d}t} &= \Delta\omega_{\mathrm{avr}} \\ \dfrac{\mathrm{d}\Delta\omega_{\mathrm{avr}}}{\mathrm{d}t} &= \dfrac{K}{C}\left(\dfrac{E_{\mathrm{s}} - E_{\mathrm{OPT}}}{R_{\mathrm{s}}} - I_{\mathrm{I}}\right) - \dfrac{1}{CR_{\mathrm{s}}}\Delta\omega_{\mathrm{avr}} \\ \dfrac{\mathrm{d}\delta_{\mathrm{pll}}}{\mathrm{d}t} &= \Delta\omega_{\mathrm{pll}} \\ &= -K_{\mathrm{p}}\sin(\delta_{\mathrm{pll}} - \delta_{\mathrm{ref}}) \end{cases} \tag{7}$$

where δ_{avr} is phase difference of DCAVR output voltage. δ and $\Delta\omega$ are given as $\delta_{\mathrm{avr}} + \delta_{\mathrm{pll}}$ and $\Delta\omega_{\mathrm{avr}} + \Delta\omega_{\mathrm{pll}}$.

The Method-2 has a feature that the inverter system can refer the change of angular velocity caused by the fluctuation in power system.

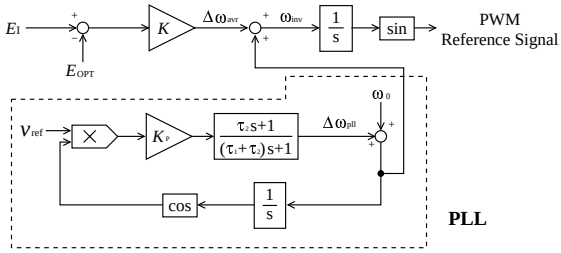

Figure 3: Block diagram of Method-2

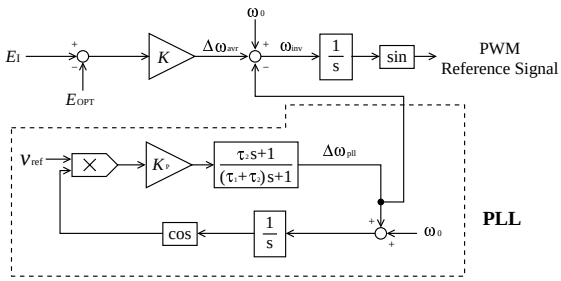

Figure 4: Block diagram of Method-3

2.3. Mehtod-3

Here LPF is ideal. The reference signal from the linked power system is useful to adjust the inverter output synchronize to the signal. However, under the transient state, the power flow will not depend on the angular velocity but the angular acceleration. Then the method owing to the change of angular velocity can be considered. The differential equation of inverter is formulated as follows:

$$\begin{cases} \dfrac{\mathrm{d}\delta_{\mathrm{avr}}}{\mathrm{d}t} &= \Delta\omega_{\mathrm{avr}} \\ \dfrac{\mathrm{d}\Delta\omega_{\mathrm{avr}}}{\mathrm{d}t} &= \dfrac{K}{C}\left(\dfrac{E_{\mathrm{s}} - E_{\mathrm{OPT}}}{R_{\mathrm{s}}} - I_{\mathrm{I}}\right) - \dfrac{1}{CR_{\mathrm{s}}}\Delta\omega_{\mathrm{avr}} \\ \dfrac{\mathrm{d}\delta^*_{\mathrm{pll}}}{\mathrm{d}t} &= \Delta\omega_{\mathrm{pll}} \\ &= -K_{\mathrm{p}}\sin(\delta^*_{\mathrm{pll}} - \delta_{\mathrm{ref}}) \\ \dfrac{\mathrm{d}\delta_{\mathrm{pll}}}{\mathrm{d}t} &= -\Delta\omega_{\mathrm{pll}} \end{cases} \tag{8}$$

3. Transient Dynamics of Inverter System

3.1. Feature of Method-1

3.1.1. Basin Portrait of Each Method

The inverter systems, which are controlled by three methods shown in block diagrams, show self-synchronizing characteristics. We can estimate the

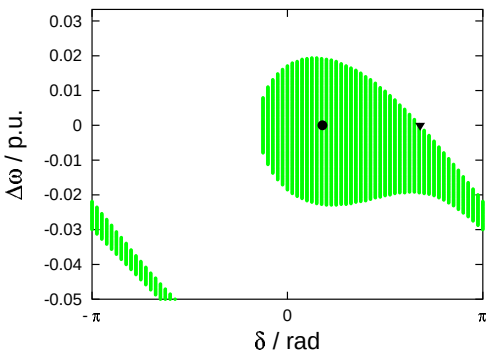

Figure 5: Basin by Method-1

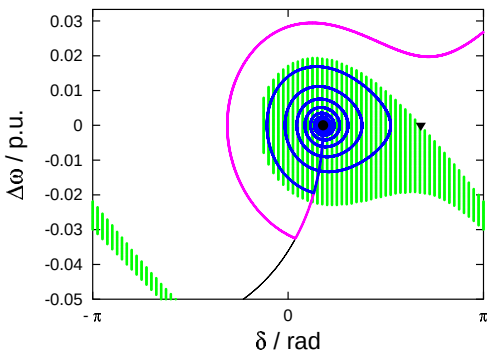

Figure 6: Pull-in characteristics of Method-1

improvement of the characteristics through the enlargement of domain of attraction in initial values of parameter space. Based on the estimated domain, the transient behavior governed by the methods can easily be shown as the pull-in characteristics after the onset of control.

In the following simulations, the parameters are set as follows: C = 60 mF, $K = 2.4\pi$, E_s = 60 V, E_{OPT} = 50 V, $R_s = 20\Omega$, L_1 = 60 mH, and E_{AC} = 50 V.

At first, Fig.5 shows the domain of attraction of the inverter system by Method-1. On the (δ, ω)–plane, the solution obtained from every initial value in the hatched domain converges to the equilibrium point at the center of the domain. The domain is surrounded by the stable manifold converged to a saddle marked by ▼ in the figure. It is not difficult to recognize that the controlled system has the similar characteristics to synchronous generators. The domain decreases by the increase of DC voltage E_s.

3.1.2. Pull-in Characteristics

On the standpoint of nonlinear dynamics, the control method by Method-1 has inherently a self-synchronizing parameter area. However, the transient characteristics is not enough. The damping is low to obtain the quick convergence. The feature is shown in Fig.6. The trajectory diverges from the equilibrium point because of the sudden decrease of E_s from 60 V to 30 V. The recovering time of E_s to 60 V decides the results of the stability of operation. If the time is long, the state cannot be pulled in to the equilibrium point.

3.2. Improved Methods

The improved methods have the higher dimension than the Method-1. Therefore, it is difficult to compare their characteristics in the same (δ, ω)–plane. Here we would like to project them on the plane without respect to the change of the control variables.

The Method-2 has an additional control to the change of reference angular frequency. The reference angular frequency is given by the output of PLL circuit. The PLL substantially synchronizes to the input signal. But, it is only the signal level. The inverter system controlled by Method-1 has a self-synchronizing characteristics as synchronous generators through the power flow. The basin of the equilibrium point is narrowed down by Method-2 as shown in Fig.7. The tracking of power system frequency makes the system unstable under the transient state. It is the neck of Mehthod-2. However, the system shows the better damping characteristics to the disturbance as discussed later.

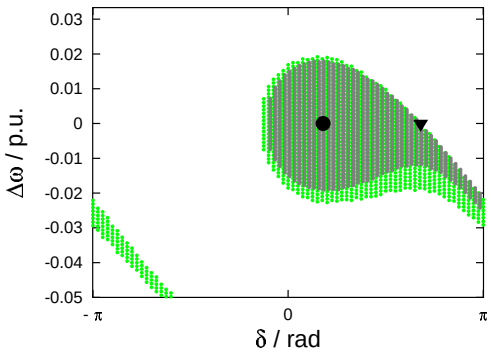

Figure 7: Basin by Method-2

The Method-3 shows the remarkable enlargement of the basin under control as shown in Fig.8. Especially, in the region of negative high angular frequency, the basin becomes wide. It implies the Method-3 has a potential to stabilize the huge frequency change. The enlargement into the low frequency area seems a remarkable benefit in the application to the distributed power sources.

Their pull-in characteristics are shown in Figs.9 and 10. In Fig. 9, the trajectories are obtained under different synchronizing method setting the same recovering time in their convergence from equilibrium. The Method-2, achieves the stable motion in spite of the divergence by Method-1.

Figure 10 shows the trajectories, which are obtained for the Method-1 and Method-3 with keeping the recovering time constant after the sudden decrease of E_s. The trajectories towards to the equilibrium point has large amplitude

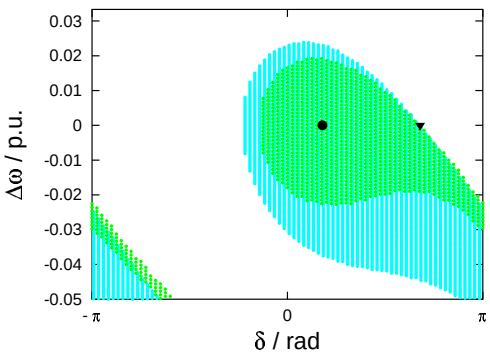

Figure 8: Enlargement of basin by Method-3

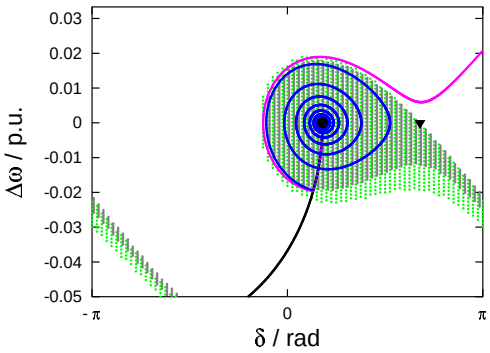

Figure 9: Pull-in characteristics of Method-2. Blue line is the trajectory under Method-1 and pink line under Method-2.

than in Fig.6 but shows a quick convergence.

4. Concluding Remarks

In this paper, we have confirmed the possibility of self-synchronization of the inverter systems. The self-synchronizing characteristics can be achieved by the method by Harada. It is remarkably improved by the proposed method with PLL signal. The feature of the synchronization is similar to synchronous generators.

The improved methods show the good pull-in characteristics even in the huge initial disturbance. Based on the obtained results, we can give the inverter system the substantial synchronizing characteristics, which have been managed by the timing of switching control. It implies that the distributed power sources have the potential to contribute the stable operation of power system through gathering numbers of spread systems.

Moreover, we can insist that the nonlinear characteristics should be added to the conventional linear system to establish the functionality in the highly connected network system.

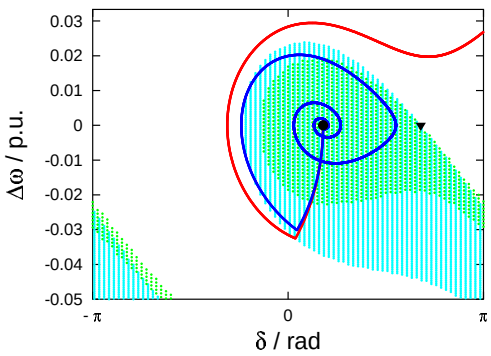

Figure 10: Pull-in characteristics of Method-3. Blue line is the trajectory under Method-3 and red line under Method-1.

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