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Numerical analysis on dynamics-dependent synchronization in mutually-coupled semiconductor lasers

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Abstract—We numerically investigate chaos synchronization property with delayed feedback and coupling. We focus on the different synchronization states for high and low frequency components by applying a low-pass filter on the change of each laser dynamics between chaotic oscillations and low-frequency fluctuations (LFF). We demonstrate dynamics-dependent synchronization, where the occurrence of in-phase and anti-phase synchronization at different frequency components is determined by the chaotic and LFF dynamics. These numerical results agree well with our previous experimental results in a photonic integrated circuit with mutually-coupled semiconductor lasers.

1. Introduction

Coupled nonlinear systems show a variety of dynamics. Recently, chaos synchronization in coupled semiconductor lasers with delayed optical feedback has been widely studied for understanding fundamental physical phenomenon and information security application. For example, chaos-based optical communication schemes [1], secure key distribution [2, 3], and neuromorphic information processing [4] have been demonstrated. When lasers are mutually coupled, the laser output can be synchronized with the time lag corresponds to the coupling delay time, which is known as the leader-laggard relationship [5]. In addition, semiconductor lasers with optical self-feedback show low-frequency fluctuations (LFF) dynamics [6]. Synchronization phenomenon of LFF dynamics in mutually-coupled semiconductor lasers has been reported, such as anti-phase synchronization [7] and episodic synchronization [8].

Recently, photonic integrated circuits (PICs) have been demonstrated and attractive as suitable photonic devices for random number generation [9, 10] and chaos communications [11]. The relationship between synchronization state and laser dynamics in a PIC with mutually-coupled semiconductor lasers has been investigated experimentally and this phenomenon is termed dynamics-dependent synchronization [12]. However, the mechanism of this synchronization phenomenon has not been analyzed yet in detail.

In this study, we numerically investigate dynamics-dependent synchronization in two mutually-coupled semiconductor lasers with asymmetric feedback strengths. We focus on the synchronization state of differential frequency components by applying a low-pass filter to analyze the influence of the laser dynamics on synchronization quality. In particular, we study the relationship between synchronization state and asymmetric configuration when the feedback strength for one laser and the coupling strength are changed simultaneously.

2. Numerical model

Our numerical model is depicted in Fig. 1. This scheme consists of two semiconductor lasers that are mutually coupled (we call laser 1 and laser 2, respectively) via a common external mirror. Each laser has an external cavity and their cavity lengths are 11.0 mm and 10.3 mm for laser 1 and 2, respectively. The coupling length is 21.3 mm, which correspond to the lengths of the PIC used in the previous experiment [12]. We assume the refractive index of 3.9.

We execute numerical simulations with the rate equations known as the Lang-Kobayashi equations [13] in order to reproduce the experimental results and analyze the mechanism of dynamics-dependent synchronization. The Lang-Kobayashi equations are written as follows:

Laser 1:

$$\begin{aligned}\frac{dE_1(t)}{dt} &= \frac{1+i\alpha}{2}\left[\frac{G_N(N_1(t)-N_0)}{1+\epsilon|E_1(t)|^2}-\frac{1}{\tau_p}\right]E_1(t)\\ &\quad+\kappa_1 E_1(t-\tau_1)\exp(-i\omega_1\tau_1)\\ &\quad+r_{SOA}\kappa_{inj}[E_2(t-\tau_{inj})\exp[i(\Delta\omega t-\omega\tau_{inj})]\end{aligned} \tag{1}$$

$$\frac{dN_1(t)}{dt} = J_1 - \frac{N_1(t)}{\tau_s} - \frac{G_N(N_1(t)-N_0)}{1+\epsilon|E_1(t)|^2}|E_1(t)|^2 \tag{2}$$

Laser 2:

$$\begin{aligned}\frac{dE_2(t)}{dt} &= \frac{1+i\alpha}{2}\left[\frac{G_N(N_2(t)-N_0)}{1+\epsilon|E_2(t)|^2}-\frac{1}{\tau_p}\right]E_2(t)\\ &\quad+{r_{SOA}}^2\kappa_2 E_2(t-\tau_2)\exp(-i\omega_2\tau_2)\\ &\quad+r_{SOA}\kappa_{inj}[E_1(t-\tau_{inj})\exp[i(\Delta\omega t-\omega\tau_{inj})]\end{aligned} \tag{3}$$

$$\frac{dN_2(t)}{dt} = J_2 - \frac{N_2(t)}{\tau_s} - \frac{G_N(N_2(t)-N_0)}{1+\epsilon|E_2(t)|^2}|E_2(t)|^2 \tag{4}$$

where, E and N are the complex electric field and the carrier density, respectively. $\tau_{1,2}$ and $\kappa_{1,2}$ represent the feedback delay times and the feedback strengths. τ_{inj} and κ_{inj} represent the coupling delay time and the coupling strength. α is the linewidth enhancement factor, J is the laser injection current. G_N is the gain coefficient. N_0 is the carrier density at transparency. τ_p and τ_s are the photon and carrier lifetimes. ϵ is the gain saturation coefficient. $\Delta\omega$ (= $2\pi\Delta$f)is the detuning of the optical angular frequencies between the two lasers, where Δf is set to 3.0 GHz. The parameter values are set as follows: J_1 = 1.02 $J_{1,th}$, J_2 = 1.10 $J_{2,th}$, κ_1 = 0.05, κ_2 = 0.05, κ_{inj} = 0.05, τ_1 = 0.29 ns, τ_2 = 0.27 ns, and τ_{inj} = 0.28 ns. In addition, we set a semiconductor optical amplifier (SOA) between the laser 2 and the external mirror to change the feedback strength for laser 2 and the coupling strength simultaneously, for asymmetric coupling configuration [12]. We multiply the amplifier coefficient of SOA (defined as r_{SOA}) and the initial value of κ_2, and κ_{inj}. The coupling strength is written as r_{SOA} κ_{inj}. The feedback light for laser 2 passes the SOA twice, and the feedback strength for laser 2 is written as ${r_{SOA}}^2$ κ_{inj}. We investigate the synchronization states for differential frequency components by changing r_{SOA}.

We calculate the cross-correlation value between the temporal waveforms of the laser 1 and 2 to quantitatively evaluate the synchronization quality. The cross-correlation function is defined as follows.

$$C = \frac{<(I_1(t-\tau)-\bar{I}_1)(I_2(t)-\bar{I}_2)>}{\sigma_1 \cdot \sigma_2} \tag{5}$$

where, $I_1(t)$ and $I_2(t)$ are the output intensities of the laser 1 and 2, respectively. $\bar{I}_1$ and $\bar{I}_2$ are the mean values of $I_1(t)$ and $I_2(t)$. σ_1 and σ_2 are the standard deviations of $I_1(t)$ and $I_2(t)$. The bracket <> represents time averaging. We calculate the cross-correlation function and obtain the peak value of the cross correlation C by changing the delay time τ continuously.

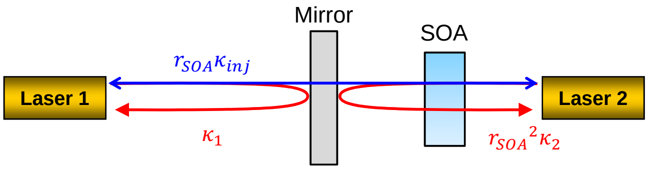

Figure 1: Schematics of mutually-coupled semiconductor lasers with semiconductor optical amplifier (SOA).

3. Chaos synchronization for different amplifier values

The LFF dynamics consists of high-frequency chaotic oscillations and low-frequency intensity dropouts [6]. We apply a low-pass filter to the laser output signals to separate these two dynamics. We calculate the cross correlation between the temporal waveforms of the laser 1 and 2 to evaluate the synchronization quality for different frequency components, which are the original and low-pass filtered signals at the cut-off frequency of f_c = 1.0 GHz. We show the numerical results of chaos synchronization when the value of amplifier coefficient of SOA is changed (r_{SOA} = 1.3 and 2.5) in Figs. 2 and 3, respectively. The temporal waveforms of Fig. 2 show chaotic dynamics for both lasers, and in-phase synchronization (the peak of the cross correlation shows a high positive value) is observed for both high and low frequency components, as shown in Fig. 2(c). On the other hand, anti-phase synchronization is observed for the low-pass filtered signals, while in-phase synchronization is observed for the original signals, as shown in Fig. 3. Both lasers show LFF dynamics in Fig. 3(a)(b). We consider that anti-phase synchronization is achieved due to the occurence of LFF dynamics for large r_{SOA}.

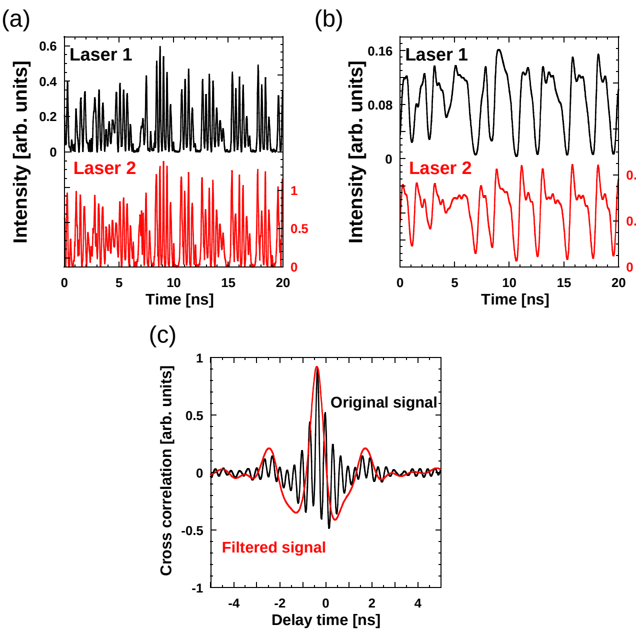

Figure 2: Temporal waveforms of (a) original signals and (b) low-pass filtered signals (f_c = 1 GHz), and (c) cross-correlations function of original signal (black line) and filtered signal (red line) for r_{SOA} = 1.3.

4. Observation of dynamics-dependent synchronization

Figure 4 shows the evolution of cross correlation value for the original and low-pass filtered signals of the laser outputs when r_{SOA} is changed continuously. We calculate the maximum of the absolute value of the cross-correlation function for each signal because the delay time corresponds to the peak value changes when r_{SOA} is changed. We dis-

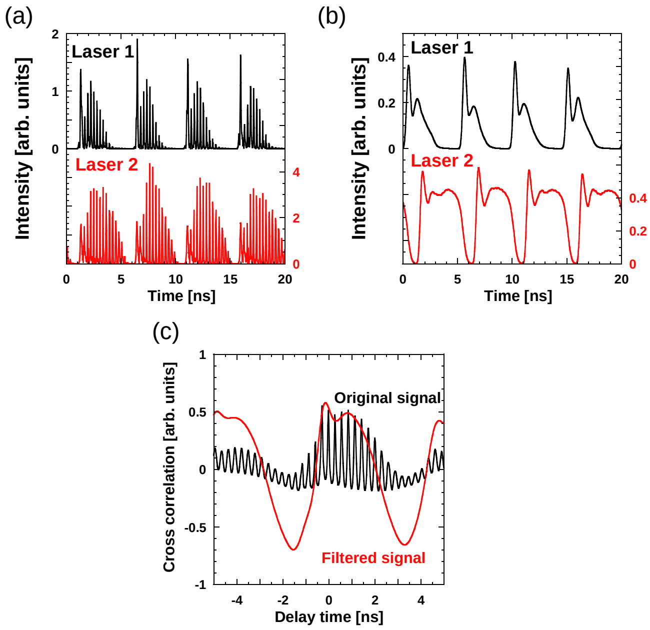

Figure 3: Temporal waveforms of (a) original signals and (b) low-pass filtered signals (f_c = 1 GHz), and (c) cross-correlations function of original signal (black line) and filtered signal (red line) for r_{SOA} = 2.5.

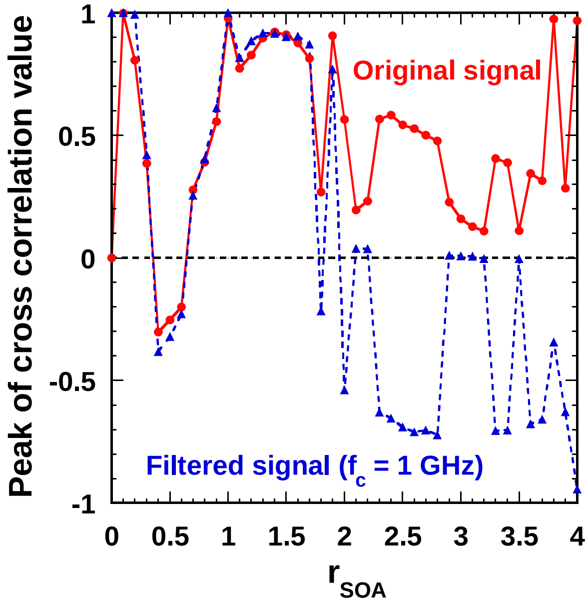

Figure 4: Evolution of cross-correlation value for the original and low-pass filtered signals when the value of the amplifier coefficient of SOA (r_{SOA}) is changed continuously.

cuss the dependence of the synchronization of LFF dynamics between the two lasers on the different frequency components. First, in-phase synchronization is observed for both original and filtered signals at near r_{SOA} = 1.5 with positive cross-correlation values. However, when r_{SOA} is increased to 2.5, in-phase synchronization is observed for the original signals and anti-phase synchronization is observed for the low-pass filtered signals. The dynamics changes from chaos to LFF as r_{SOA} is increased. For example, chaotic dynamics is observed for both lasers in the range of $0.4 < r_{SOA} < 1.5$, and LFF dynamics is observed in the range of $r_{SOA} > 2.3$. Thus, we numerically observe dynamics-dependent synchronization for asymmetric coupling configuration by changing r_{SOA}, where the two types of synchronization states appear, depending on the laser dynamics of chaos and LFF.

We speculate that dynamics-dependent synchronization results from the asymmetry of the feedback strengths between the two lasers. The feedback strength for the laser 2 increases with increase of r_{SOA} and LFF dynamics appear, while the feedback strength for the laser 1 is constant. This asymmetric changes of the feedback strength result in the different dynamics of chaos and LFF, and different synchronization states can be obtained based on these dynamics.

5. Conclusions

We numerically investigated dynamics-dependent synchronization in two mutually-coupled semiconductor lasers with asymmetric feedback strengths. We applied a low pass filter with the cut-off frequency of 1 GHz to the laser output signals to separate these two dynamics and to analyze the dependence of chaos synchronization state when laser dynamics is chaged from chaos to LFF. We observed in-phase synchronization at high-frequency components and anti-phase synchronization at low-frequency components by increasing the amplifier coefficient of the SOA. This result suggests that the occurrence of LFF dynamics results in anti-phase synchronization, and we numerically observed dynamics-dependent synchronization. This phenomenon results from the asymmetry of the feedback strengths between the laser 1 and 2.

Acknowledgments

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