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Analysis of a Pseudo-Decentralized Discrete-Time Algorithm for Estimating Algebraic Connectivity of Multiagent Networks

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Abstract—The algebraic connectivity is an important measure of a network because it represents how well the network is connected. Recently, Yang *et al.* proposed a pseudo-decentralized continuous-time algorithm for all agents in a multiagent network to estimate the algebraic connectivity. In this paper, we consider a discrete-time version of this algorithm, and examine the validity through theoretical analysis and numerical experiments.

1. Introduction

Multiagent networks have attracted a great deal of attention recently [1]. In some applications of multiagent networks such as the formation flight [2], it is desired that each agent can estimate the connectivity of the network in a decentralized manner. Yang *et al.* [3] proposed a continuous-time algorithm for agents to estimate the algebraic connectivity of the network. The algebraic connectivity, which is defined as the second smallest eigenvalue of the Laplacian matrix [4], is widely used as a measure representing how well the network is connected. The method of Yang *et al.* was proved to work properly under a certain condition on the parameters [3]. However, it relies on the strong assumption that every agent can compute the average of the state values of all agents instantaneously.

A truly decentralized algorithm for the algebraic connectivity estimation based on the algorithm of Yang *et al.* was recently proposed by Yamane and Takahashi [5]. Furthermore, a discrete-time version of this algorithm was proposed by Endo and Takahashi [6]. Although the validity of these algorithms have been confirmed through numerical experiments, their dynamical behavior is not well understood theoretically because these algorithms are much more complicated than the algorithm of Yang *et al.*

In this paper, we focus our attention on a discrete-time version of the algorithm of Yang *et al.* and analyze the dynamical behavior. Although this is not a truly decentralized algorithm, clarifying its properties is an important preliminary step to complete understanding of the algorithm of Endo and Takahashi. We first describe the discrete-time algorithm obtained from the algorithm of Yang *et al.* We next perform a theoretical analysis of the algorithm. We finally conduct numerical experiments to confirm the results of theoretical analysis.

2. Continuous-Time Algorithm Proposed by Yang et al.

2.1. Algebraic Connectivity of Multiagent Networks

Let us consider a network of n agents labeled by integers from 1 to n communicating with each other. We assume that the communication between agents is symmetric, i.e., if agent i can send information to agent j then agent j can send information to agent i. Under this setting, the communication in the multiagent network is expressed by a simple undirected graph $G = (V, E)$ where $V = \{1, 2, \ldots, n\}$ is the vertex set and E is the edge set which contains unordered pairs of distinct vertices. The edge $\{i, j\}$ is a member of E if and only if agents i and j can communicate with each other. Let $\mathcal{N}_i$ denote the set of agents with which agent i can directly communicate. In other words, let $\mathcal{N}_i = \{j \mid \{i, j\} \in E\}$. Let the adjacent and degree matrices of the graph G be denoted by $\boldsymbol{A} = (a_{ij})$ and $\boldsymbol{D} = \mathrm{diag}(d_1, d_2, \ldots, d_n)$, respectively. Then $a_{ij} = 1$ if $\{i, j\} \in E$ and $a_{ij} = 0$ otherwise. Also, $d_i = \sum_{j=1}^{n} a_{ij}$ represents the number of agents with which agent i can directly communicate. The Laplacian matrix $\boldsymbol{L}$ of G is defined by $\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{A}$. Because $\boldsymbol{L}$ is positive semi-definite, all eigenvalues of $\boldsymbol{L}$ are nonnegative real numbers. Furthermore, because $\boldsymbol{L}\mathbf{1} = \mathbf{0} (= 0 \cdot \mathbf{1})$ where $\mathbf{1}$ ($\mathbf{0}$, resp.) is the vector of all ones (zeros, resp.), the smallest eigenvalue is 0 and the corresponding eigenvector is $\mathbf{1}$. In what follows, we denote the eigenvalues of $\boldsymbol{L}$ as $\lambda_1 (= 0) \leq \lambda_2 \leq \cdots \leq \lambda_n$. The second smallest eigenvalue λ_2, which is known as the algebraic connectivity, is an important measure that represents how well G is connected. In particular, λ_2 is positive if and only if G is connected.

2.2. Algorithm of Yang et al.

The algorithm proposed by Yang *et al.* [3] is described by the following differential equations:

$$\dot{x}_i(t) = -k_1 \left(\frac{1}{n} \sum_{j=1}^{n} x_j(t) \right) - k_2 \sum_{j \in \mathcal{N}_i} (x_i(t) - x_j(t)) - k_3 \left(\frac{1}{n} \sum_{j=1}^{n} x_j(t)^2 - 1 \right) x_i(t), \quad i = 1, 2, \ldots, n \qquad (1)$$

where $x_i(t)$ is the state value of agent i at time t, and k_1, k_2, k_3 are positive constants. They proved that if $k_1 > k_2\lambda_2$ and $k_3 > k_2\lambda_2$ then the solution $\boldsymbol{x}(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}}$

of (1) converges to an eigenvector of $\boldsymbol{L}$ corresponding to λ_2 for almost all initial solutions [3]. It is easy to see that $a_i(t) = \sum_{j\in\mathcal{N}_i}(x_i(t) - x_j(t))/x_i(t)$ converges to λ_2 if $\boldsymbol{x}(t)$ converges to an eigenvector corresponding to λ_2 and $\lim_{t\to\infty} x_i(t) \neq 0$. Note that this algorithm is not truly decentralized because it is assumed that every agent can compute $\frac{1}{n}\sum_{j=1}^n x_j(t)$ and $\frac{1}{n}\sum_{j=1}^n x_j(t)^2$ instantaneously.

3. Theoretical Analysis of Discrete-Time Algorithm

3.1. Discrete-Time Version of Yang et al.'s Algorithm

We consider a discrete-time version of (1) described by

$$\begin{aligned} & x_i(k+1) \\ &= x_i(k) + \epsilon\left[-k_1\left(\frac{1}{n}\sum_{j=1}^n x_j(k)\right) - k_2\sum_{j\in\mathcal{N}_i}(x_i(k) - x_j(k)) \right.\\ &\quad \left. - k_3\left(\frac{1}{n}\sum_{j=1}^n x_j(k)^2 - 1\right)x_i(k)\right], \quad i = 1, 2, \ldots, n \end{aligned}$$

where ϵ is a positive constant. These difference equations can be rewritten in a vector form as:

$$\begin{aligned} \boldsymbol{x}(k+1) = \boldsymbol{x}(k) + \epsilon\Bigg[&-k_1\left(\frac{1}{n}\boldsymbol{1}^{\mathrm{T}}\boldsymbol{x}(k)\right)\boldsymbol{1} - k_2\boldsymbol{L}\boldsymbol{x}(k) \\ &- k_3\left(\frac{1}{n}\|\boldsymbol{x}(k)\|^2 - 1\right)\boldsymbol{x}(k)\Bigg] \end{aligned} \quad (2)$$

where $\boldsymbol{x}(k) = (x_1(k), x_2(k), \ldots, x_n(k))^{\mathrm{T}}$ and $\boldsymbol{L}$ is the Laplacian matrix of the network. In the following discussion, we impose for simplicity the following assumption.

Assumption 1 All eigenvalues of $\boldsymbol{L}$ are simple.

Let $\boldsymbol{q}_i$ be a unit eigenvector of $\boldsymbol{L}$ corresponding to the eigenvalue λ_i for $i = 1, 2, \ldots, n$. Then we have $\boldsymbol{L} = \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}^{\mathrm{T}}$ where $\boldsymbol{Q} = (\boldsymbol{q}_1\boldsymbol{q}_2\cdots\boldsymbol{q}_n)$ is an orthonormal matrix and $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$. Multiplying the both sides of (2) on the left by $\boldsymbol{Q}^{\mathrm{T}}$, letting $\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{x}(k) = \boldsymbol{y}(k)$, and noting $\frac{1}{n}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{1}\boldsymbol{1}^{\mathrm{T}}\boldsymbol{Q} = \mathrm{diag}(1, 0, 0, \ldots, 0)$, we have

$$\boldsymbol{y}(k+1) = \boldsymbol{y}(k) - \epsilon\left[\tilde{\boldsymbol{\Lambda}} + k_3\left(\frac{1}{n}\|\boldsymbol{y}(k)\|^2 - 1\right)\boldsymbol{I}\right]\boldsymbol{y}(k) \quad (3)$$

where

$$\tilde{\boldsymbol{\Lambda}} = \mathrm{diag}(\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_n) = \mathrm{diag}(k_1, k_2\lambda_2, k_2\lambda_3, \ldots, k_2\lambda_n).$$

In the next subsection, we study the behavior of the solution of (3) instead of (2).

3.2. Equilibrium Point Analysis

We first specify all equilibrium points of (3) under some condition.

Theorem 1 Suppose that the constants k_1, k_2, k_3 satisfy

$$k_1 > k_2\lambda_n, \quad (4)$$

$$k_3 > k_2\lambda_n. \quad (5)$$

If $k_3 > k_1$ then the set of all equilibrium points of (3) is given by $\{\boldsymbol{y}^{(0)}\} \cup \{\pm\boldsymbol{y}^{(j)}\}_{j=1}^n$ where

$$\begin{aligned} \boldsymbol{y}^{(0)} &= \boldsymbol{0}, \\ \pm\boldsymbol{y}^{(1)} &= \pm\left(\sqrt{n\left(1 - \frac{k_1}{k_3}\right)}, 0, 0, \ldots, 0\right)^{\mathrm{T}}, \\ \pm\boldsymbol{y}^{(j)} &= \pm\left(\overbrace{0, 0, \ldots, 0}^{j-1}, \sqrt{n\left(1 - \frac{k_2\lambda_j}{k_3}\right)}, 0, 0, \ldots, 0\right)^{\mathrm{T}}, \\ & \quad j = 2, 3, \ldots, n. \end{aligned}$$

If $k_3 \leq k_1$ then the set of all equilibrium points of (3) is given by $\{\boldsymbol{y}^{(0)}\} \cup \{\pm\boldsymbol{y}^{(j)}\}_{j=2}^n$.

Proof: Proof is omitted because it is similar to [3]. □

We next show that under a certain condition only $\pm\boldsymbol{y}^{(2)}$ are stable and all other equilibrium points are unstable.

Theorem 2 If the positive constants k_1, k_2, k_3, ϵ satisfy (4), (5) and

$$\frac{1}{\epsilon} > \max\left\{\frac{1}{2}(k_1 - k_2\lambda_2), k_3 - k_2\lambda_2, \frac{k_2}{2}(\lambda_n - \lambda_2)\right\} \quad (6)$$

then $\pm\boldsymbol{y}^{(2)}$ are stable equilibrium points of (3) and all other equilibrium points of (3) are unstable.

Proof: Let $\boldsymbol{y}^*$ be any equilibrium point of (3). Let $\boldsymbol{d}(k) = (d_1(k), d_2(k), \ldots, d_n(k))^{\mathrm{T}} = \boldsymbol{y}(k) - \boldsymbol{y}^*$. Then the linearized system at $\boldsymbol{y}^*$ is given by $\boldsymbol{d}(k+1) = \boldsymbol{A}(\boldsymbol{y}^*)\boldsymbol{d}(k)$ where

$$\boldsymbol{A}(\boldsymbol{y}^*) = \boldsymbol{I} - \epsilon\tilde{\boldsymbol{\Lambda}} + \epsilon k_3\left(\boldsymbol{I} - \frac{2}{n}\boldsymbol{y}^*\boldsymbol{y}^{*\mathrm{T}} - \frac{1}{n}\|\boldsymbol{y}^*\|^2\boldsymbol{I}\right). \quad (7)$$

We first consider the stability of the equilibrium point $\boldsymbol{y}^{(0)}$ $(= \boldsymbol{0})$. Substituting $\boldsymbol{y}^* = \boldsymbol{0}$ into (7), we have $\boldsymbol{A}(\boldsymbol{0}) = \boldsymbol{I} - \epsilon\tilde{\boldsymbol{\Lambda}} + \epsilon k_3\boldsymbol{I}$. This is a diagonal matrix and the i-th diagonal entry $a_{ii}(\boldsymbol{0})$ is given by

$$a_{ii}(\boldsymbol{0}) = \begin{cases} 1 + \epsilon(k_3 - k_1), & \text{if } i = 1, \\ 1 + \epsilon(k_3 - k_2\lambda_i), & \text{otherwise.} \end{cases}$$

It follows from (5) that $a_{ii}(\boldsymbol{0}) > 1$ for $i = 2, 3, \ldots, n$. Therefore $\boldsymbol{y}^* = \boldsymbol{0}$ is unstable.

We next consider the stability of the equilibrium point $\pm\boldsymbol{y}^{(1)}$. Note that we are assuming implicitly that $k_3 > k_1$. Substituting $\boldsymbol{y}^* = \pm\boldsymbol{y}^{(1)}$ into (7), we have $\boldsymbol{A}(\pm\boldsymbol{y}^{(1)}) = \boldsymbol{I} - \epsilon\tilde{\boldsymbol{\Lambda}} + \epsilon k_3\boldsymbol{I} - 2\epsilon\,\mathrm{diag}\,(k_3 - k_1, 0, 0, \ldots, 0) - \epsilon(k_3 - k_1)\boldsymbol{I}$. This is a diagonal matrix and the i-th diagonal entry $a_{ii}(\pm\boldsymbol{y}^{(1)})$ is given by

$$a_{ii}(\pm\boldsymbol{y}^{(1)}) = \begin{cases} 1 - 2\epsilon(k_3 - k_1), & \text{if } i = 1, \\ 1 + \epsilon(k_1 - k_2\lambda_i), & \text{otherwise.} \end{cases}$$

It follows from (4) that $a_{ii}(\boldsymbol{y}^{(1)}) > 1$ for $i = 2, 3, \ldots, n$. Therefore $\boldsymbol{y}^{(1)}$ is unstable.

We finally consider the stability of the equilibrium points $\pm\boldsymbol{y}^{(j)}$ ($j = 2, 3, \ldots, n$). Substituting $\boldsymbol{y}^* = \pm\boldsymbol{y}^{(j)}$ into (7), we have $\boldsymbol{A}(\pm\boldsymbol{y}^{(j)}) = \boldsymbol{I} - \epsilon\tilde{\boldsymbol{\Lambda}} + \epsilon k_3\boldsymbol{I} - 2\epsilon\mathrm{diag}(\overbrace{0, 0, \ldots, 0}^{j-1}, k_3 - k_2\lambda_j, 0, 0, \ldots, 0) - \epsilon(k_3 - k_2\lambda_j)\boldsymbol{I}$. This is a diagonal matrix and the i-th diagonal entry $a_{ii}(\pm\boldsymbol{y}^{(j)})$ is given by

$$a_{ii}(\pm\boldsymbol{y}^{(j)}) = \begin{cases} 1 - \epsilon(k_1 - k_2\lambda_j), & \text{if } i = 1, \\ 1 - 2\epsilon(k_3 - k_2\lambda_i), & \text{if } i = j, \\ 1 - \epsilon k_2(\lambda_i - \lambda_j), & \text{otherwise.} \end{cases}$$

It follows from (4) and (6) that $|a_{11}(\pm\boldsymbol{y}^{(j)})| < 1$, and it follows from (5) and (6) that $|a_{jj}(\pm\boldsymbol{y}^{(j)})| < 1$. As for other diagonal entries, we have to consider two cases: $j = 2$ and $j \geq 3$. In the former case, it follows from Assumption 1 and (6) that $|a_{ii}(\pm\boldsymbol{y}^{(2)})| < 1$ for $i = 3, 4, \ldots, n$. In the latter case, it follows from Assumption 1 that $a_{ii}(\pm\boldsymbol{y}^{(j)}) = 1 + \epsilon k_2(\lambda_j - \lambda_i) > 1$ for $i = 2, 3, \ldots, j-1$. Therefore, $\pm\boldsymbol{y}^{(2)}$ is stable and $\pm\boldsymbol{y}^{(3)}, \pm\boldsymbol{y}^{(4)}, \ldots, \pm\boldsymbol{y}^{(n)}$ are unstable. □

3.3. Boundedness of Solutions

We finally give a sufficient condition for the solution of (3) to be bounded.

Theorem 3 Let $\tilde{\lambda}_{\min} = \min_{1\leq i\leq n}\{\tilde{\lambda}_i\} = \min\{k_1, k_2\lambda_2\}$ and $\tilde{\lambda}_{\max} = \max_{1\leq i\leq n}\{\tilde{\lambda}_i\} = \max\{k_1, k_2\lambda_n\}$. If the positive constants k_1, k_2, k_3, ϵ and the initial solution $\boldsymbol{y}(0)$ satisfy

$$k_3 \geq \tilde{\lambda}_{\max}, \tag{8}$$

$$\frac{1}{\epsilon} \geq 2(k_3 - \tilde{\lambda}_{\min}), \tag{9}$$

$$\|\boldsymbol{y}(0)\|^2 \leq \frac{n}{k_3}\left(k_3 - \tilde{\lambda}_{\max} + \frac{2}{\epsilon}\right) \tag{10}$$

then the solution $\boldsymbol{y}(k)$ of the difference equation (3) satisfies

$$\|\boldsymbol{y}(k)\|^2 \leq \frac{n}{k_3}\left(k_3 - \tilde{\lambda}_{\max} + \frac{2}{\epsilon}\right)$$

for all $k \geq 1$.

Proof: For any solution $\boldsymbol{y}(k)$ of the difference equation (3), we define $\gamma(k)$ as $\gamma(k) = k_3(\frac{1}{n}\|\boldsymbol{y}(k)\|^2 - 1)$. Then we have

$$\begin{aligned} &\|\boldsymbol{y}(k+1)\|^2 - \|\boldsymbol{y}(k)\|^2 \\ &= \left\|\boldsymbol{y}(k) - \epsilon\tilde{\boldsymbol{\Lambda}}\boldsymbol{y}(k) - \epsilon\gamma(k)\boldsymbol{y}(k)\right\|^2 - \|\boldsymbol{y}(k)\|^2 \\ &= \epsilon\boldsymbol{y}(k)^{\mathrm{T}}\left[\epsilon\gamma(k)^2\boldsymbol{I} + 2\gamma(k)(\epsilon\tilde{\boldsymbol{\Lambda}} - \boldsymbol{I}) + \epsilon\tilde{\boldsymbol{\Lambda}}^2 - 2\tilde{\boldsymbol{\Lambda}}\right]\boldsymbol{y}(k). \end{aligned}$$

Here $\epsilon\gamma(k)^2\boldsymbol{I} + 2\gamma(k)(\epsilon\tilde{\boldsymbol{\Lambda}} - \boldsymbol{I}) + \epsilon\tilde{\boldsymbol{\Lambda}}^2 - 2\tilde{\boldsymbol{\Lambda}}$ is a diagonal matrix and the i-th diagonal entry is given by

$$\begin{aligned} &\epsilon\gamma(k)^2 + 2\gamma(k)(\epsilon\tilde{\lambda}_i - 1) + \epsilon\tilde{\lambda}_i^2 - 2\tilde{\lambda}_i \\ &\qquad = \left(\gamma(k) + \tilde{\lambda}_i\right)\left(\epsilon\gamma(k) + \epsilon\tilde{\lambda}_i - 2\right) \end{aligned} \tag{11}$$

which takes a nonpositive value if and only if $-\tilde{\lambda}_i \leq \gamma(k) \leq -\tilde{\lambda}_i + \frac{2}{\epsilon}$ which can be rewritten as $\frac{n}{k_3}(k_3 - \tilde{\lambda}_i) \leq \|\boldsymbol{y}(k)\|^2 \leq \frac{n}{k_3}\left(k_3 - \tilde{\lambda}_i + \frac{2}{\epsilon}\right)$. Therefore, as far as $\boldsymbol{y}(k)$ satisfies

$$\frac{n}{k_3}(k_3 - \tilde{\lambda}_{\min}) \leq \|\boldsymbol{y}(k)\|^2 \leq \frac{n}{k_3}\left(k_3 - \tilde{\lambda}_{\max} + \frac{2}{\epsilon}\right), \tag{12}$$

$\|\boldsymbol{y}(k)\|^2$ does not increase because all diagonal entries of $\epsilon\gamma(k)^2\boldsymbol{I} + 2\gamma(k)(\epsilon\tilde{\boldsymbol{\Lambda}} - \boldsymbol{I}) + \epsilon\tilde{\boldsymbol{\Lambda}}^2 - 2\tilde{\boldsymbol{\Lambda}}$ are nonpositive. Note here that in (12) the right-hand side of the second inequality is greater than the left-hand side of the first inequality because it follows from (8) and (9) that

$$\frac{n}{k_3}\left(k_3 - \tilde{\lambda}_{\max} + \frac{2}{\epsilon}\right) - \frac{n}{k_3}(k_3 - \tilde{\lambda}_{\min}) \geq \frac{3n}{k_3}(\tilde{\lambda}_{\max} - \tilde{\lambda}_{\min}) > 0.$$

In order to complete the proof, it suffices for us to show that if $\|\boldsymbol{y}(k)\|^2$ is less than the left-hand side of the first inequality in (12) then $\|\boldsymbol{y}(k+1)\|^2$ is less than the right-hand side of the second inequality in (12). This can be done by using (8) and (9), but we omit it due to space limitation. □

From Theorems 1–3, we can conclude that if k_1, k_2, k_3, ϵ and the initial solution $\boldsymbol{y}(0)$ satisfy

$$k_1 > k_2\lambda_n, \tag{13}$$

$$k_3 > k_1, \tag{14}$$

$$\frac{1}{\epsilon} > \max\left\{\frac{1}{2}(k_1 - k_2\lambda_2), 2(k_3 - k_2\lambda_2)\right\}, \tag{15}$$

$$\|\boldsymbol{y}(0)\|^2 \leq \frac{n}{k_3}\left(k_3 - k_1 + \frac{2}{\epsilon}\right) \tag{16}$$

then the solution of the difference equation (3) always belongs to the bounded set

$$\left\{\boldsymbol{y} \in \mathbb{R}^n \;\middle|\; \|\boldsymbol{y}\|^2 \leq \frac{n}{k_3}\left(k_3 - k_1 + \frac{2}{\epsilon}\right)\right\}$$

and only $\pm\boldsymbol{y}^{(2)}$ are stable equilibrium points in this set. Therefore, it is expected that the solution $\boldsymbol{y}(k)$ of the difference equation (3) converges to either $\boldsymbol{y}^{(2)}$ or $-\boldsymbol{y}^{(2)}$ though this has not been rigorously proved.

4. Numerical Experiments

In order to understand how the algorithm (2) works, we conduct a numerical experiment using a network with 12 agents, which is shown in Fig. 1. The algebraic connectivity of the network is about 0.297. We set $k_1 = 12.0$, $k_2 = 1.0$ and $k_3 = 13.0$ so that both (13) and (14) are satisfied (recall that the largest eigenvalue λ_n of $\boldsymbol{L}$ is less than or equal to $n - 1$). As for the constant ϵ, we use three values: 0.001, 0.1 and 0.155. The first value satisfies (15) but the remaining two values do not. We choose the initial solution $\boldsymbol{x}(0)$ so that $\|\boldsymbol{x}(0)\| \leq \frac{n}{k_3}\left(k_3 - k_1 + \frac{2}{\epsilon}\right)$. Then (16) is satisfied (recall that $\|\boldsymbol{y}(k)\| = \|\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{x}(k)\| = \|\boldsymbol{x}(k)\|$).

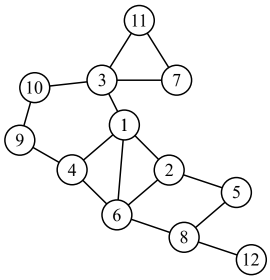
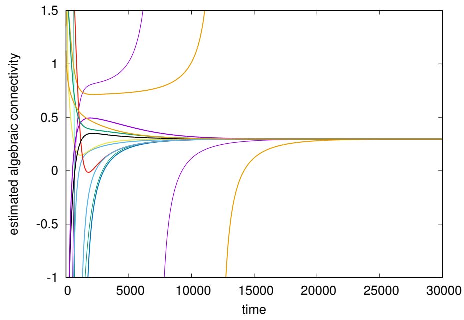
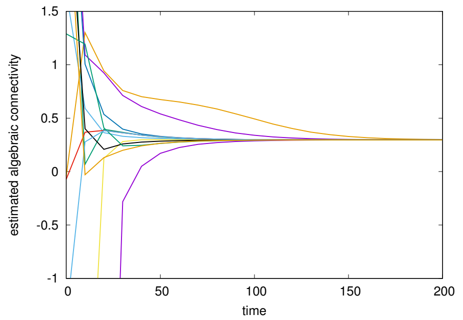


Figure 1: A network with 12 agents.

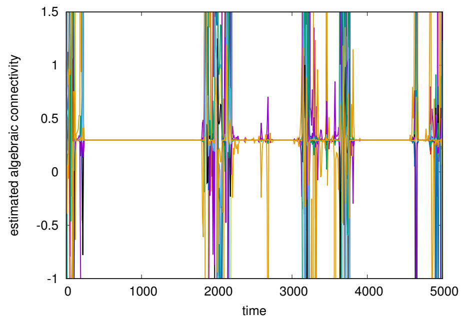
Figure 2 shows the results of the algebraic connectivity estimation with the algorithm (2). The horizontal axis represents discrete-time time k and the vertical axis represents the estimated value of the algebraic connectivity. In case of Fig. 2(a), the estimated values of all agents successfully converge to the true value before k reaches $25,000$. In case of Fig. 2(b), the estimated values of all agents converge to the true value much faster than the case of Fig. 2(a) while (15) is not satisfied. In case of Fig. 2(c), the period in which the estimated values stay close to the true value and the period in which they oscillate very rapidly appear alternately.



(a)



(b)



(c)

Figure 2: Waveforms of the estimated values of the algebraic connectivity. (a) $\epsilon = 0.001$. (b) $\epsilon = 0.1$. (c) $\epsilon = 0.155$.

5. Conclusions

We have studied the dynamical behavior of a discrete-time version of the algorithm proposed by Yang *et al.* for estimating the algebraic connectivity of multiagent networks. We have shown under a certain condition on the parameters that only a pair of equilibrium points corresponding to the algebraic connectivity are stable. We have also derived a sufficient condition for solutions to be bounded. We then have verified experimentally that the algebraic connectivity can be successfully estimated by the algorithm. A future problem is to prove the convergence of the solution to the stable equilibrium points.

Acknowledgments

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