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Design of Extremum Seeking Control with a Continuous-Time Accelerator

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Abstract—In this paper we are concerned with designing a continuous-time extremum seeking control law for nonlinear systems. This is a modification of a standard extremum seeking contoller. It is equipped with a continuous-time accelerator to the original one to be aimed at achieving the maximum operating point more rapidly. This accelerator is designed by making use of the Chebyshev polynomial identification of an uncertain output map. Numerical experiments show how this modified approach can be well in control of the Monod model of bioreactors.

1. Introduction

An extremum seeking control problem is classified in a category of adaptive control problems. Mainstream methods of adaptive control deal only with regulation to known set points or reference trajectories. However, extremum seeking controls are designed so as to operate at unknown set points that maximize the value of a performance function. Investigation of this problem dates back to 1922[1] and was very popular during 1950s and 1960s[2–4]. During the last two decades there has been a revival of interest in this problem.

An extremum seeking control, which is an old adaptive nonlinear control method from the 50-60s, is one of many interesting approaches. This type of approach is easy to implement to practical systems, but needs a longer time to reach the best operating point. The stability and applications of this type have been actively studied by Krstić et al. [5–7], so from now on we will simply refer this approach as Krstić type in this paper.

In order to improve the reaction time toward the optimal operating point, Takata et al.[8] developed an extremum seeking control which is added an accelerator to Krstić type. The design of this accelerator is based on the Chebyshev polynomial identification of discrete-time using a sampled-data technique, so it needs analog-digital converters, and besides, Butterworth filter with bilinear transformation.

In this paper we consider a modification of Krstić type approach which is equipped with a continuous-time accelerator for the extremum seeking control problems. It is aimed at shortening a period until the optimal operating point without the knowledge of the plant dynamic equation. This accelerator is designed by making use of the Chebyshev polynomial identification of continuous-time to estimate the uncertain output map. It does not need such additive equipments as analog-digital converters, because it uses continuous-time data directly without sampling.

The proposed approach is applied to the Monod model of bioreactors. Simulation results show that this enables to regulate the object around the best operating point speedily.

2. Problem Statement

We consider single-input-single-output systems of the form:

$$\dot{x}(t) = f(x(t), \alpha, u(t)) \tag{1}$$

$$y(t) = h(x(t), u(t)) \tag{2}$$

where $\bullet = d/dt$, $x \in R^n$ is the state, $u \in R$ is the control, $y \in R$ is the output, $\alpha \in R^L$ is the unknown parameter, and $f : R^{n+L+1} \longrightarrow R^n$ and $h : R^{n+1} \longrightarrow R$ are the unknown nonlinear smooth functions.

The performance function J is assumed to be the output equilibrium map such that

$$J(u) = h(z, u) \tag{3}$$

where $\dot{z} = f(z, \alpha, u) = 0, z \in R^n$.

The aim of this problem is to develop a feedback mechanism, which enables the given system to operate around the maximum point of the performance swiftly, without requiring the knowledge of the functions of f and h, and the parameter α.

3. Extremum Seeking Control of Krstić Type

Krstić type approach could be designed from the following basic idea and its feedback scheme is shown in Fig.1 (see[5–7]).

It is impossible to conclude that a certain point is a maximum without visiting the neighborhood on both sides of

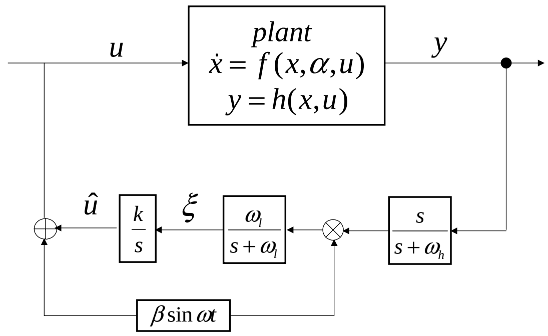

Figure 1: Krstić type control scheme.

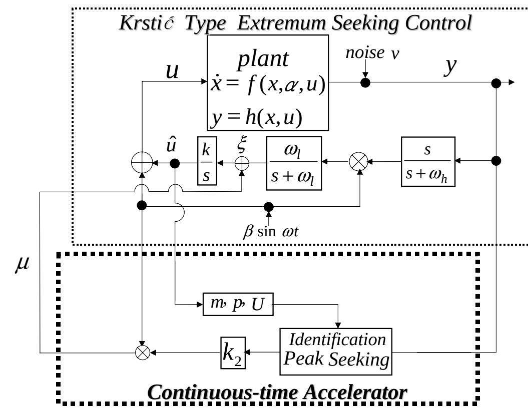

Figure 2: Extremum seeking control scheme.

the maximum. For this reason, this scheme employs a slow perturbation $\beta \sin \omega t$ which is added to the control signal $\hat{u}$. The persistent nature of $\beta \sin \omega t$ may be undesirable but is necessary to maintain a maximum in the face of changes in functions f and h.

The perturbation $\beta \sin \omega t$ will create a periodic response of y. The high-pass filter $s/(s+\omega_h)$ would eliminate the DC component of y. Then, the product of the sinusoids $\beta \sin \omega t$ produces $\beta^2/2(1+\cos 2\omega t)$, and its DC component $\xi \propto \beta^2/2$ is extracted by the low-pass filter $\omega_l/(s+\omega_l)$. The sign of this ξ provides the direction to the integrator $\hat{u} = K\xi/s$ moving $\hat{u}$ towards the optimal operating point u^*. Due to this, the output y gradually approaches the maximum output value $y^* = J(u^*)$.

Although it has the merit of easy implementation to practical systems, this Krstić type approach usually needs a longer time to reach the optimal point u^*, namely, the maximum output y^*. We will consider a modification of this controller to shorten a reaching time in the next section.

4. Control with Accelerator

In our feedback scheme, a continuous-time accelerator is added to the original structure and is shown in Fig.2.

Note that the state x approaches the stable equilibrium point z as the control progresses:

$$y = h(x,u) \simeq h(z,u) = J(u),$$

so we assume that

$$y = J(u) + w_1$$

where w_1 is error.

4.1. The Chebyshev Polynomial Identification

We interpolate the performance function curve via the Chebyshev polynomials [9] up to the N-th order using the data $\{y(\tau), \hat{u}(\tau) : t - T < \tau \le t\}$ at time t, where T is an accumulation period of data. The perturbation $\beta \sin \omega t$ is undesirable to estimate the performance function, so the principal control $\hat{u}$ instead of u is used to make the $J(u)$. At $u = \hat{u} + \beta \sin \omega t$, the $\beta \sin \omega t$ will be treated as free in the sections 4.1 and 4.2.

Let the domain of the principal control u be $D = [u_{min}, u_{max}]$. To transform into a standard domain $D_0 = [-1, 1]$, introduce a normalizing function:

$$U(u) = U(\hat{u} + \beta \sin \omega t) = \frac{(\hat{u} - m)}{p} \qquad (4)$$

where $U : D \to D_0$, $m = (u_{max} + u_{min})/2$, $p = (u_{max} - u_{min})/2$. The Chebyshev polynomials are then defined by

$$\Phi_r(u) = \cos(r \cdot \cos^{-1} U(u)) \quad (r = 0, 1, 2, \cdots)$$

or

$$\begin{aligned} \Phi_0(u) &= 1 \\ \Phi_1(u) &= U(u) \\ \Phi_2(u) &= 2U^2(u) - 1 \\ \Phi_3(u) &= 4U^3(u) - 3U(u) \\ \Phi_4(u) &= 8U^4(u) - 8U^2(u) + 1 \\ \Phi_5(u) &= 16U^5(u) - 20U^3(u) + 5U(u) \\ &\vdots \end{aligned} \qquad (5)$$

Assume that the performance function is described by

$$\begin{aligned} J(u) &= \boldsymbol{\Phi}(u)^T \boldsymbol{C} + w_2 \\ &= C_0 + C_1\Phi_1(u) + C_2\Phi_2(u) + \cdots \\ &\quad + C_N\Phi_N(u) + w_2 \end{aligned}$$

so that

$$\begin{aligned} y &= J(u) + w_1 \\ &= \boldsymbol{\Phi}(u)^T \boldsymbol{C} + w \end{aligned}$$

where

$$C_0 = \frac{1}{\pi} \int_{-1}^{1} \frac{y}{\sqrt{1 - U^2}} dU$$

$$C_r = \frac{2}{\pi}\int_{-1}^{1}\frac{y\Phi_r}{\sqrt{1-U^2}}dU \quad (r \neq 0) \tag{6}$$
$$\boldsymbol{C} = [C_0, C_1, C_2, \cdots, C_N]^T$$
$$\boldsymbol{\Phi}(u) = [1, \Phi_1(u), \Phi_2(u), \cdots, \Phi_N(u)]^T$$
$$w = w_1 + w_2$$
$$w_2 \text{ is error.}$$

Therefore, we approximate the performance function at time t as

$$\begin{aligned}\hat{J}_t(u) &= \boldsymbol{\Phi}(u)^T\hat{\boldsymbol{C}} \\ &= \hat{C}_0 + \hat{C}_1\Phi_1(u) + \hat{C}_2\Phi_2(u) + \cdots \\ &\quad +\hat{C}_N\Phi_N(u).\end{aligned} \tag{7}$$

4.2. Peak Seeking

Let $u^*(t)$ be estimate of the peak seeking point or the optimal operating point u^* at time t.

We search for the maximum point of the performance function $\hat{J}_t(u)$ by comparing the values at $(L+1)$ points as follows.

$$\hat{J}_t(u^*(t)) = \max_u\{\hat{J}_t(u) : U(u) = p(2j/L-1) + m, \quad j = 0, 1, 2, \cdots, L\} \tag{8}$$

where L is the number of division of $D_0 = [-1, 1]$.

In a special case of $N = 2$, the $u^*(t)$ is analytically solved as follows.

From Eqs.(4) and (5), Eq.(7) becomes

$$\begin{aligned}\hat{J}_t(u) = \hat{C}_0 + \hat{C}_1(\hat{u}-m)/p \\ +\hat{C}_2(2(\hat{u}-m)^2/p^2 - 1)\end{aligned} \tag{9}$$

so that $\partial\hat{J}_t(u)/\partial u = 0$ derives

$$u_t^* = m - \frac{p\hat{C}_1}{4\hat{C}_2}. \tag{10}$$

The coefficient may be approximated by Eq.(6) as

$$\begin{aligned}\hat{C}_1/\hat{C}_2 &= \int_{-1}^{1}\frac{y\cdot U}{\sqrt{1-U^2}}dU/\int_{-1}^{1}\frac{y\cdot(2U^2-1)}{\sqrt{1-U^2}}dU \\ &\simeq \int_{t-T}^{t}\frac{y\cdot U}{\sqrt{1-U^2}}\frac{dU}{dt}dt/\int_{t-T}^{t}\frac{y\cdot(2U^2-1)}{\sqrt{1-U^2}}\frac{dU}{dt}dt\end{aligned} \tag{11}$$

4.3. Correction

Let a correction term to the ξ be

$$\mu(t) = k_2\cdot u^*(t)\cdot\beta\sin\omega t \tag{12}$$

where k_2 is a weight, $u^*(t)$ is by Eq.(8) or (10), and $\beta\sin\omega t$ is the perturbation. This $\mu(t)$ tries to accelerate an action of ξ because the ξ provides the increment coefficient of control $\hat{u}$.

Therefore, the proposed extremum seeking control becomes

$$u(t) = k(\xi+\mu)/s + \beta\sin\omega t \tag{13}$$
$$= u(0) + k\int_0^t(\xi(\tau) + k_2\beta u^*(\tau)\sin\omega\tau)d\tau + \beta\sin\omega t \tag{14}$$

4.4. Materialization

In this section we consider an easy realization as one of examples.

The control formula of Eq.(13) should be materialized so as to work even in noise circumstances.

Note that $\hat{J}_t(u)$ of Eq.(9) is identified by the data accumulated during the period T.

Introduce the following approximations in Eqs.(4) and (11).

$$m = \frac{1}{T}\int_{t-T}^{t}\hat{u}(\tau)d\tau$$
$$p = \gamma\left\{\frac{\Delta}{T}\sum_{i=0}^{t/\Delta}(\hat{u}(t-\Delta i) - m)^2\right\}^{1/2} + \varepsilon_p \tag{15}$$
$$\hat{U}(t) = (\hat{u}(t) - m)/p$$
$$U(t) = \begin{cases} U_{max} & if\ \hat{U}(t) \geq U_{max} \\ U_{min} & if\ \hat{U}(t) \leq U_{min} \\ \hat{U}(t) & otherwise\end{cases}$$
$$\hat{C}(t) = \left\{\int_{t-T}^{t}\frac{y(\tau)U(\tau)}{\sqrt{1-U^2(\tau)}}\frac{U(\tau)-U(\tau-\Delta)}{\Delta}d\tau\right\}$$
$$\times\left\{\int_{t-T}^{t}\frac{y(\tau)(2U^2(\tau)-1)}{\sqrt{1-U^2(\tau)}}\frac{U(\tau)-U(\tau-\Delta)}{\Delta}d\tau + \varepsilon_c\right\}^{-1}$$
$$\hat{C}_1/\hat{C}_2(t) = \begin{cases} \hat{C}_{max} & if\ \hat{C}(t) \geq \hat{C}_{max} \\ \hat{C}_{min} & if\ \hat{C}(t) \leq \hat{C}_{min} \\ \hat{C}(t) & otherwise\end{cases} \tag{16}$$

where $0 < \varepsilon_p \ll 1,\ 0 < \varepsilon_c \ll 1,\ 0 < \Delta \ll T,\ 0 < \gamma < 3,\ -1 \leq U_{min} < U_{max} \leq 1,\ -\infty < \hat{C}_{min} < \hat{C}_{max} < \infty$.

Therefore, from Eqs.(10)(14) and (16) we have

$$\begin{aligned}u(t) &= u(0) \\ &+k\int_0^t\left(\xi(\tau) + k_2\beta\left(m - \hat{C}_1/\hat{C}_2(\tau)\cdot p/4\right)\sin\omega\tau\right)d\tau \\ &+\beta\sin\omega t\end{aligned} \tag{17}$$

5. Simulations

Consider the problem of optimizing the yield for a bioreactor which is described by the Monod model[6–8]:

$$\dot{x}_1 = f_1(x,\alpha,u) = x_1\left(\frac{x_2}{(\alpha+x_2)} - u\right) \tag{18}$$

$$\dot{x}_2 = f_2(x, \alpha, u) = u(1 - x_2) - \frac{x_1 x_2}{(\alpha + x_2)} \quad (19)$$

$$y = h(x, u) + v = x_1 \cdot u + v \quad (20)$$

where $x = [x_1, x_2]^T$, $0 \leq u \leq 1$, and v is a noise.

The steady-state output (performance function) [6,8] is

$$J(u) = \frac{u(1 - (1 + \alpha)u)}{1 - u} \quad (21)$$

which is derived by substituting a solution (x, u) of $\dot{x}_1 = \dot{x}_2 = 0$ into Eqs.(18) and (19) to Eq.(20), where α is fixed and $v = 0$.

We should note that the equations of (18)~(21) are unknown during experiments when designed the control of Eq.(17).

The unknown parameter α is initially set to $\alpha = 0.02$, but it is changed to $\alpha = 0.1$ at $t = 500(sec)$ and then returned to $\alpha = 0.02$ at $t = 900(sec)$. The optimal operating value and the maximum output are $u^* = 0.860$ and $y^* = J(u^*) = 0.754$ when $\alpha = 0.02$, and $u^* = 0.698$ and $y^* = J(u^*) = 0.537$ when $\alpha = 0.1$, though they are unknown during the experiments.
The initial value is $u(0) = 0.6$. The parameters are set as follows. $\beta = 0.03$, $\omega = 0.08$, $\omega_h = 0.15$, $\omega_l = 0.02$, $k = 5$, $N = 2$, $T = 6(sec)$, $\Delta = 1(sec)$, $k_2 = 0.01$, $\gamma = 2$, $\hat{C}_{max} = -\hat{C}_{min} = 1$, $U_{max} = -U_{min} = 0.99$, $\varepsilon_p = 0.05$, and $\varepsilon_c = 0.01$.

In case of noiseless system of $v = 0$ in Eq.(20), figure 3 shows a comparison between the Krstić type(OLD) and our proposed approach(NEW) for the time responses of the extremum seeking control u and the output y.
Figure 4 shows those in case of noise system, whose v is the band-limited white noise of $psd(v) = 0.01$.

These results indicate that this new extremum seeking control approach shown in Fig.2 enables the system to regulate to the optimal operating point more swiftly than the original approach shown in Fig.1.

6. Conclusions

This paper has proposed a modification of the standard extremum seeking control, which aims a speedy reaction at an unknown extremum point for nonlinear systems. It is equipped with a continuous-time accelerator based on the Chebyshev polynomial identification. This new approach shall be studied of more reasonable materialization, applications to other systems, and its stability proofs in future works.

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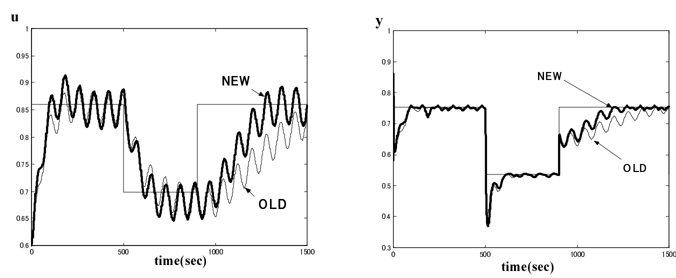

Figure 3: A noiseless case (v=0).

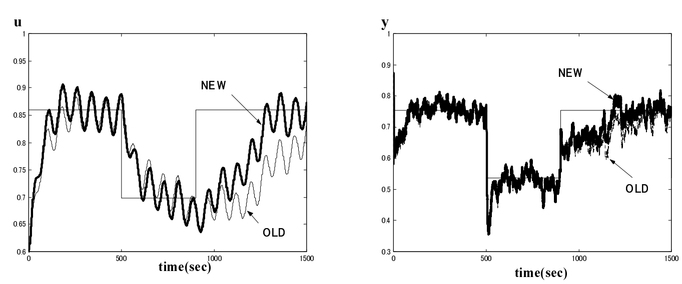

Figure 4: A noise case ($v \neq 0$).