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Modeling of Agent-based Artificial Auction Markets based on the Genetic Programming and its Applications

Yoshikazu IKEDA† and Shozo TOKINAGA‡

†Faculty of Economics, Shinshu University
3-1-1 Asahi, Matsumoto-shi, Nagano 390-8621 Japan
‡Graduate School of Economics, Kyushu University
6-19-1 Hakozaki, Higashi-ku, Fukuoka 812-8581 Japan
Email: ikeda@econ.shinshu-u.ac.jp, tokinaga@en.kyushu-u.ac.jp

Abstract—Many real auctions involve complicated trader such as asymmetric bidder, then theoretical analysis becomes very hard. In this paper, we show the agent-based simulation of artificial auction markets by using the Genetic Programming (GP) and its applications. By assuming multi-agents as bidders who learn from past experiences based on the GP, we can analyze the capability to learn successful auctions by agents, and the change of profit of agents in various conditions of auctions. Considering two types of auctions, we can apply the same GP procedure to model learning of agents. In the simulation studies, the effects of parameters such as the private evaluation function are discussed.

1. Introduction

Auction mechanisms have been attracting increasing attentions in recent years motivated by selling systems over the Internet as well as conventional ones [1]-[3]. Most of theoretical researches in auction theory assume that bidders will be making competitive bids, and these bidders are symmetrical in size and are risk neutral. However, many real auctions involve many complicated trades such as an oligopoly of asymmetric bidder who meet and bid for the same commodity.

In this paper, we show the agent-based simulation of artificial auction markets by using the Genetic Programming (GP) and its applications [4][5]. By assuming multi-agents as bidders who learn from past experiences based on the GP, we can analyze the capability to learn successful auctions by agents, and the change of profit of agents in various conditions of auctions.

Different from conventional simulation studies of auction markets, we incorporate learning mechanism in a kind of community by using the co-evolutionary GP. Then, the situation of reactions among various kind of bidders in market is easily realized.

In the paper, we consider two types of auctions, namely, the sealed-bid auction and English auction. Even though the systems of two auctions are different, we can apply the same GP procedure to model learning of agents.

In the simulation studies, the effects of parameters such as the private evaluation function are discussed.

2. System configuration

2.1. Two types of auction models

We assume well-known two types of auction models in the following [1]-[3]. The first one is the sealed-bid auction where bidders can exhibit price for successful bid (bid price) only once, and they cannot know prices of other bidders. The auction model is employed in many bidding of construction of public utilities. In the scheme, a bidder who exhibits the highest bid price can win the bid.

The other type of auction is the English auction where the auction is carried out in real time (sometime the auction model is called as online auction, and bidder can know current highest price of bidding, and they can exhibit bid price repeatedly. Usually, a bidder exhibiting the highest bid price can win the bid, but it is also assumed that a bidder exhibiting second highest bid price can win the bid (called the Vickrey auction).

Originally, there are three types of agents in the auction market, namely, bidders who wish to suppress the bid price as low as possible, sellers who wish higher bid price, and the auctioneer who manages the auction. For simplicity, we assume that the seller and the auctioneer is the same agent in the following. We also assume that only one seller agent shows only one item (commodity) in the market, and is traded by many bidders (agents).

2.2. Agents' learning using the GP

In the following, we assume that agents learn to find appropriate bid price for future auction based on the GP. The GP is a extension of Genetic Algorithm (GA), but the representation of applicable solutions (called as individual) is not a array of bits, but the tree structure of functions [4]-[6]. Each agent has a pool of individuals each of which corresponds to the estimation of appopriate bidding for the next time point. The individuals are represented by using arithmetic operators, comparative operators, and the observation of past successful bid. Since the ability (called fitness) of each individual can be evaluated after the bidding is realized (ended), agents can improve the estimation of individuals by applying the GP operations (crossover and mutation) to the pool of individuals.

It is assumed that the first N_1 times of biddings are used for learning for agents, and no commodity is delivered to bidder, and seller gets no money. In this learning period, each agent try to improve the estimation of individuals by using the GP procedure. Then, in successive N_2 times of bidding, agents apply the estimation using the pool of individuals. After N_2 times of auctions, the profit of each agent is determined.

3. Sealed-bid auction

3.1. Behavior of agents

We concisely summarize the learning of agents for sealed-bid auctions. It is assumed that each bidder agent has its own pool of functions (called as individuals) for deciding bid price. To simplify the simulation in reasoning of agents, we restricted ourselves to the cases where the functions can be represented in binary tree structures. But, the restriction has no serious effect on generalization of the method.

For example, an agent has following function including if-the rules.

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if (vi>72) then 1.2P1-0.1 else 0.9 MAX
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Fig.1 shows the corresponding tree structure of function. In this case, the agent exhibit bid price as 1.2P1-0.1 if $v_i > 72$, otherwise exhibit bid price as 0.9 MAX. We also show general form of functions using symbols R,M and T which mean the root node, intermediate node and terminal node, respectively.

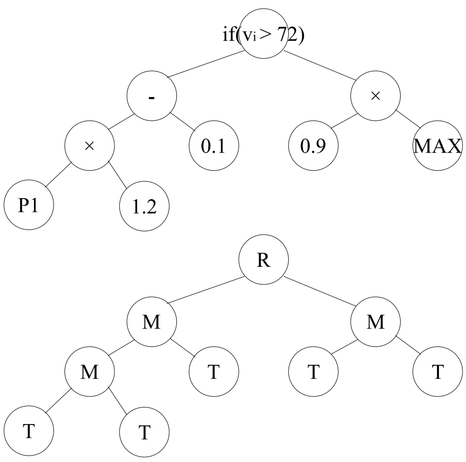

Figure 1: Example of function

The function includes various terminal symbols as well as constants. At first, we introduce the private evaluation v_i for ith agent as the terminal symbol. Each bidder agent i reacts to the commodity brought by seller, and assign a value v_i representing the private evaluation (preference). The function can have also the symbols P1=$CP(t-1)$ where $CP(t-1)$ is the successful bid price in previous auction. The function has also symbols AV,MAX and MIN defined by taking the average, maximum and minimum of successful bid pirces in previous t_1, t_2, t_3 time periods of auctions, respectively. The nodes of tree structure are composed of arithmetic operators $+, -, \times, /$ and comparative operators $=, \neq, <, >, \geq, \leq$

If the intermediate node n_I is a arithmetic operator (say +) having two nodes n_1, n_2 (having values x_1, x_2) below, then we do the calculation using x_1, x_2, and the result x_1+x_2 is stored in the intermediate node. If the intermediate node n_I is if-then-else node, then we see the logical value of if node, and the result is true, then we use the value x_1 of left node connected to N_I below, otherwise we use the value x_2 of right node.

3.2. Fitness of tree

The ability of individuals corresponding to the functions is defined as fitness. As the first ability measure, we use following value.

$$pr_{ik} = \sum_j [CP(j) - v_i(j)]/N^w_{ik} \tag{1}$$

where $v_i(j)$ is the ith agents's private evaluation of bidding in jth auction, and N^w_{ik} is the number of successful bid obtained by using kth individual in the pool. Then, the numerator of equation (1) corresponds to average profit obtained by successful biddings.

We also employ the second evaluation measure for fitness for kth individual in ith agent as follows.

$$r_{ik} = sb_{ik}/N^u_{ik} \tag{2}$$

where sb_{ik} means the number of successful bid, and N^u_{ik} means the number of time where the agent uses the kth individual.

Finally, by changing the weight ω_i among pr_{ik} and r_{ik}, we have fitness for kth individual as follows.

$$s_{ik} = \omega_i(pr_{ik} - \min_j pr_{ij})/R^{pr}_i + (1-\omega_i)(r_{ik} - \min_j r_{ik})/R^r_i \tag{3}$$

where R^{pr}, R_i mean the ranges of two measures to normalize the fitness.

3.3. GP learning

To improve initial set of individuals, we apply following procedure.

A.Select private evaluation v_i.

B.Select one (kth) individual from the pool with the probability

$$P^s_{ik} = s_{ik}/\sum_{j=1}^{k} s_{ij} \tag{4}$$

C.Seller determines the successful bid $CP(t)$ at time t by observing bid prices given by bidders.
D.Reevaluate fitness of individuals using current $CP(t)$ and equations (1),(2),(3).
E.Iterate procedures from A through D for sufficient times, and then apply following GP.
F. Apply GP (crossover and mutation operations)

Select a pair of individual with probabilities proportional to equation (3), and then exchange portions of tree structure which are selected at random as shown in Fig.2.

In this example, a terminal node of Parent A and an intermediate node of Parent B are exchanged. We have two offsprings, and to keep the size of pool same, two individuals having lower fitness are replaced by two offfsprings.

Besides crossover operations, we use mutation operations with a certain probability by repalcing a portion of tree by another symbol (details are omitted here).

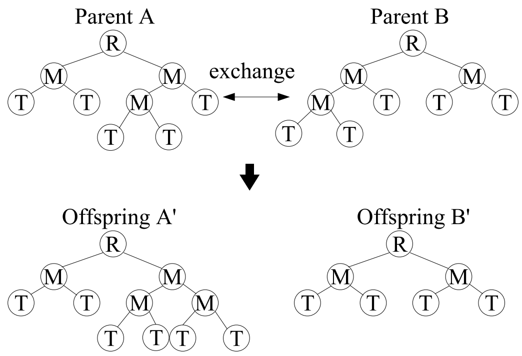

Figure 2: GP operations (crossover)

4. English auction

4.1. Behavior of agents

Different from sealed-bid auctions, agents in English auctions can exhibit bid price repeatedly at multiple times. The agents' behaviors are described by programs rather than functions. We also use tree structures to represents these programs, but their terminal nodes include action part of rules, and on their intermediate nodes if-then-else type rules are placed. As the result of rules, agents take one of two actions, namely, "wait" (no action) and "join" (exhibit bid price).

In case of "join", the agent must determine the bid price. Then, we assume that the agent use one of following two methods for decision.
(1) incremental price

By adding price increment inc to present price s by several times, $s + m \times inc$ will be the bid price.
(2) random selection of multiple

Assume set $[b_1, b_2, ..., b_l] = [1.1, 1.2, ..., 2.0]$, then the agents select one of these numbers to obtain bid price as $s \times b_i$.

The if-then-else type rules treated here are the same as used in the sealed-bid auction, but in place of terminal node we use "wait" or "join".

4.2. Interpretation of tree

The interpretation of trees (individuals) is slightly complicated. For example, in a tree structure in Fig.2, we start if-clause at the root node. if the condition is true, then we go to left branch and meet "div" node which means we go further to left branch. Then, we meet if-clause, and depending on the condition, we choose whether left branch of right branch. These two branches are denoting "join" showing the bid prices, and the action taken by the agent is terminated in this step.

In the next suction, the agent go backward to "div" node, and restart the action. Since the node is "wait", then the agent take no action for bidding. Further, in the next auction, the agent goes back to root node again, and select appropriate action.

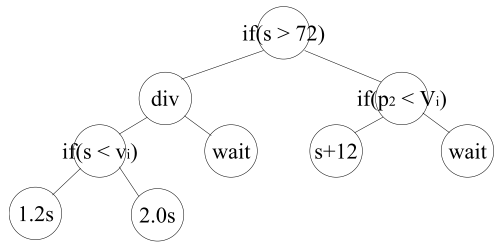

Figure 3: Individual structure in English auction

5. Applications

5.1. Sealed-bid auction

The parameters for simulation studies are selected as follows.
Number of bidder agents:10
$N_1 = 500000$ (apply GP for 1000 iterations)
$N_2 = 50000$,upper limit of bid price=150
Number of individuals for each pool=50
Maximum number of nodes in trees=50
Probability for applying if-then-else rules=0.2
Probability for "wait" is 0.6, and for bid is 0.1
Duration time of auction $T_E = 200$

We auume three cases for the definition of private evaluations v_i as follows.
(Case 1)identical:each agent has the same v_i
(Case 2)uniformly distributed:select one v_i from set $(90, 91, .., 100)$
(Case 3)piecewise constant:assign two values depending on agent, such as $V = (100, 100, ..., 100, 90, 90, ..., 90)$

Table 1 shows the result for average profit of bidders depending on the private evalutations. In Table 1, Prf1, Prf2 mean mean profit of agents in Case 1 and 2, and Prc2 mean the bid price in Case 2 (the result for case 3 is omitted here).

As is seen from the result, if $\omega = 1$, Prf1=0, then every bid prices greater than v_i are smoothly removed from the system, and no bid price greater than 100 is not realized. But if ω becomes less than 1, agents pay more attentions to the rate of successful bid, and the profit decreases.

In Case 2, we see also almost the same decrease of profit (then, the average price of bidding almost increases) along the decrease of ω form 1. The fact imply the random behavior of bidder agents help them to get more profit, and affect seller to decrease price.

Table 1: Simulation result

	$\omega = 1$	0.9	0.8	0.7	0.6
Prf1	0.00	-8.32	-13.43	-14.79	-46.48
Prf2	9.48	-0.20	-8.79	-19.65	-2.91
Prc2	90.04	105.45	109.33	119.32	107.69

5.2. English auction

The parameters for simulation studies are the same as in sealed-bid auction, and the definitions of Case 1,2, and 3 are also the same.

Table 2 shows the result for average profit of bidders depending on the private evaluations. As is seen from the result, if we choose $\omega = 1$ the profit of bidder is large compared to sealed-bid auction, while agents know the current bid price and decide to finish bidding earlier. However, if ω becomes 0.8, 0.7 or 0.6, the profit bocomes megative, an the result is similar to the cases of sealed-bid auction. The situation is more favirable to sellers. The fact reflects that from the sellers' viwepoint, it is a satisfactory bidding where bidder pay no attention to successful bid, and the supply of commodity is large enough.

Table 2: Simulation result of English auction

	$\omega = 1$	0.9	0.8	0.7	0.6
Prf1	98.99	17.37	-16.03	-6.15	-47.42
Prf2	98.53	0.055	-4.38	-6.55	-21.55
Prc2	1.00	106.08	105.99	106.10	121.18

6. Conclusion

In this paper, we showed the agent-based simulation of artificial auction markets using the GP where bidder agents learn from past experiences. Simulation studies for two types of auctions were given to show the ability of our method.

For further works, we must study the chaotic behavior of agents.

References

[1] H.Mizuta and K.Steiglitz,“ Agent-based simulation of dynamic online auction,” Proc.of 2000 Winter Simulation Conference, 2000.

[2] J.O.Kephart,J.E.Hanson and A.R.Greenwald, Dymanics pricing by software agents,2000.

[3] P.R.Milgrom and R.J.Weber,“ Pennies from eBay: the determinants of price in online auctions,” Working Paper, Vanderbilit University,Nashville, Tenessee, 2000.

[4] X.Chen and S.Tokinaga,“ Synthesis of multi-agent systems based on the co-evolutionary genetic Programming and its applications to the analysis of artificial markets”, Trans. IEICE,vol.E86-A,vol.10,pp.1038-1048,2003.

[5] Y. Ikeda and S.Tokinaga,“ Chaoticity and fractality analysis of an artificial stock market by the multi-agent systems based on the co-evolutionary Genetic Programming”,Trans. IEICE,vol.E87-A, no.9,pp.2387-2394,2004.

[6] J.R.Koza,Genetic Programming,MIT Press, 1992