2005 International Symposium on Nonlinear
Theory and its Applications (NOLTA2005)
Bruges, Belgium, October 18-21, 2005

High-Resolution Image Interpolation Using Two-Dimensional Lagrange-Type Variable Fractional-Delay Filter

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Abstract

We have theoretically proved the Liu-Wei's closed-form formula for computing the coefficients of one-dimensional (1-D) variable fractional-delay (VFD) finite-impulse-response (FIR) digital filter derived from N-th order interpolating polynomial. In this paper, we extend the 1-D VFD filter design to the two-dimensional (2-D) case and show that the image interpolation using VFD filtering can achieve higher resolution image than the conventional interpolation techniques such as zero-order interpolation, bilinear interpolation, and 6-term polynomial interpolation.

1. Introduction

The digital filters with variable fractional group-delay are referred to as variable fractional-delay (VFD) digital filters, which have been found useful in various signal processing applications such as comb filter design, digital communications, high-performance speech coding, modeling of music instruments, and sampling rate conversion. Among the existing methods for designing VFD filters [1]-[9], the frequency-domain approaches can achieve higher design accuracy than the time-domain ones using N-th order interpolating polynomials [1]. However, because the polynomial interpolator can be used to derive the simple Lagrange-type VFD FIR filter, and the Lagrange-type VFD filter exhibits the maximally flat delay and satisfactory frequency characteristics in the low frequency band, the Lagrange-type VFD filter is still an attractive candidate for many applications where the digital signal to be delayed (or interpolated) contains relatively low frequency components [1].

In [10], we have theoretically proved the closed-form formula for computing the coefficients of one-dimensional (1-D) VFD filters with general even-order $N = 2M$. In this paper, we extend the 1-D VFD filter to the two-dimensional (2-D) case, and demonstrate that the image interpolation (often called image resolution conversion) using 2-D VFD filter can achieve higher-resolution images than using the most commonly used conventional image interpolation techniques such as zero-order interpolation, bilinear interpolation, and 6-term polynomial interpolation.

2. Two-Dimensional VFD Filter

As shown in Fig. 1, it is easy to delay a 2-D discrete signal $x(n_1T_1, n_2T_2)$ by integer multiples of the sampling periods T_1 and T_2 through using the 2-D unit delay element $z_1^{-1}z_2^{-1}$. For simplicity, we assume $T_1 = 1, T_2 = 1$. However, it is difficult to delay the 2-D signal by fractional multiples of the sampling periods. If we can construct an ideal fractional-delay element $z_1^{-p_1}z_2^{-p_2}$ as shown in Fig. 2, where p_1 and p_2 are fractional numbers, then the output of the delay element is exactly the delayed $x(n_1, n_2)$ by p_1 and p_2 in n_1 and n_2 directions, respectively, i.e.,

$$y(n_1, n_2) = x(n_1 - p_1, n_2 - p_2).$$

The output $y(n_1, n_2)$ can also be viewed as the signal value of the original band-limited 2-D analog signal $x(s, t)$ evaluated at $(s, t) = (n_1 - p_1, n_2 - p_2)$. Therefore, by using the ideal 2-D fractional-delay element $z_1^{-p_1}z_2^{-p_2}$, we can perfectly re-construct the original 2-D analog signal $x(s, t)$. Since the ideal 2-D fractional-delay element $z_1^{-p_1}z_2^{-p_2}$ is separable as shown in Fig. 2, the input signal $x(n_1, n_2)$ can be first filtered in n_1 direction with n_2 fixed, which produces the output $x(n_1 - p_1, n_2)$, then the 2-D signal $x(n_1 - p_1, n_2)$ is filtered in n_2 direction with $(n_1 - p_1)$ fixed, which generates the final output

$$y(n_1, n_2) = x(n_1 - p_1, n_2 - p_2).$$

That is, filtering the 2-D signal $x(n_1, n_2)$ can be performed in n_1 direction and n_2 direction separately. In practice, it is impossible to construct an ideal 2-D fractional-delay element $z_1^{-p_1}z_2^{-p_2}$, and only approximation can be done. Below, we extend the 1-D VFD filter to the 2-D case, and demonstrate that the resulting 2-D VFD filter can achieve higher-resolution image interpolation than the conventional image interpolation techniques.

To derive a 2-D VFD filter starting from a 2-D interpolating polynomial, assume that we want to find a 2-D polynomial $\widehat{x}(s, t)$ that passes through a set of discrete points $(s_{m_1}, t_{m_2}, x_{m_1m_2})$ defined in the 3-D space, where s_{m_1} and t_{m_2} are the equally-spaced samples of s and t, respectively, m_1, m_2 are integers,

$$\begin{array}{l} m_1 \in [-M_1, M_1] \\ m_2 \in [-M_2, M_2] \end{array}$$

and the central point of the grid is (s_0, t_0) with coordinate (n_1, n_2). Fig. 3 shows the case $M_1 = 1, M_2 = 1$. Thus, s_{m_1} and t_{m_2} can be expressed as

$$\begin{aligned} s_{m_1} &= s_0 + m_1 = n_1 + m_1 \\ t_{m_2} &= t_0 + m_2 = n_2 + m_2. \end{aligned}$$

We begin with the 2-D interpolating polynomial

$$\widehat{x}(s,t) = \sum_{m_1=-M_1}^{M_1} \sum_{m_2=-M_2}^{M_2} x_{m_1 m_2} L_{m_1 m_2}(s,t) \qquad (1)$$

where

$$x_{m_1 m_2} = x(s_{m_1}, t_{m_2})$$

are the uniformly sampled values of the 2-D analog signal $x(s,t)$ at the discrete points (s_{m_1}, t_{m_2}), and

$$L_{m_1 m_2}(s,t) = \prod_{i_1=-M_1, i_1\neq m_1}^{M_1} \left(\frac{s - s_{i_1}}{s_{m_1} - s_{i_1}}\right) \times \prod_{i_2=-M_2, i_2\neq m_2}^{M_2} \left(\frac{t - t_{i_2}}{t_{m_2} - t_{i_2}}\right) \qquad (2)$$

is the 2-D Lagrange polynomial. It is easy to verify that the 2-D interpolating polynomial $\widehat{x}(s,t)$ passes through the given discrete points $(s_{m_1}, t_{m_2}, x_{m_1 m_2})$ in the 3-D space due to

$$L_{m_1 m_2}(s_l, t_k) = \begin{cases} 1 & \text{if } (l,k) = (m_1, m_2) \\ 0 & \text{if } (l,k) \neq (m_1, m_2). \end{cases} \qquad (3)$$

As in deriving the 1-D VFD filter, the polynomial value of $\widehat{x}(s,t)$ at (s,t)

$$\begin{cases} s = s_0 - p_1 = n_1 - p_1 \\ t = t_0 - p_2 = n_2 - p_2 \end{cases}$$

can be determined as

$$\begin{aligned} \widehat{x}(s,t) &= \widehat{x}(n_1 - p_1, n_2 - p_2) \\ &= \sum_{m_1=-M_1}^{M_1} \sum_{m_2=-M_2}^{M_2} x_{m_1 m_2} L_{m_1 m_2}(s,t) \\ &= \sum_{m_1=-M_1}^{M_1} \sum_{m_2=-M_2}^{M_2} a_{m_1}(p_1) b_{m_2}(p_2) x(n_1 - m_1, n_2 - m_2) \\ &= \sum_{m_2=-M_2}^{M_2} b_{m_2}(p_2) \tilde{x}(n_1 - p_1, n_2 - m_2) \end{aligned} \qquad (4)$$

with

$$a_{m_1}(p_1) = \frac{\prod_{i_1=-M_1, i_1\neq m_1}^{M_1} (p_1 - i_1)}{(-1)^{M_1 - m_1}(M_1 + m_1)!(M_1 - m_1)!}$$

$$b_{m_2}(p_2) = \frac{\prod_{i_2=-M_2, i_2\neq m_2}^{M_2} (p_2 - i_2)}{(-1)^{M_2 - m_2}(M_2 + m_2)!(M_2 - m_2)!}.$$

In (4),

$$\widetilde{x}(n_1 - p_1, n_2) = \sum_{m_1=-M_1}^{M_1} a_{m_1}(p_1) x(n_1 - m_1, n_2 - m_2)$$

can be viewed as the outputs (marks "◯" in Fig. 3) of the 1-D VFD filter

$$H_1(z_1, p_1) = \sum_{m_1=-M_1}^{M_1} a_{m_1}(p_1) z_1^{-m_1} \qquad (5)$$

through filtering the 2-D input signal $x(n_1, n_2)$ in n_1 direction with $(n_2 - m_2)$ fixed, where the 1-D VFD filter $H_1(z_1, p_1)$ approximates the ideal fractional-delay element $z_1^{-p_1}$. Then, the first-stage output signal $\widetilde{x}(n_1 - p_1, n_2)$ is further filtered by using another 1-D VFD filter

$$H_2(z_2, p_2) = \sum_{m_2=-M_2}^{M_2} b_{m_2}(p_2) z_2^{-m_2} \qquad (6)$$

to generate the final output (mark "□" in Fig. 3)

$$y(n_1, n_2) = \widehat{x}(n_1 - p_1, n_2 - p_2). \qquad (7)$$

Consequently, the above 2-D polynomial interpolation problem can be reduced to the 2-D VFD filtering problem. The input signal is $x(n_1, n_2)$, and the output signal $\widehat{x}(n_1 - p_1, n_2 - p_2)$ is the approximation of the true signal value $x(n_1 - p_1, n_2 - p_2)$.

3. Image Interpolation

To demonstrate the effectiveness of the 2-D VFD filtering, we apply the 2-D VFD filter to image interpolation. The input images of 128×128 pixels are decimated from the original ones of 256×256 pixels. To compare the 2-D VFD filtering method with the widely used conventional zero-order interpolation, bilinear interpolation, and 6-term polynomial fitting [11], various test images are interpolated, where the 2-D VFD filter with $(M_1, M_2) = (1, 1)$ is used. To perform image interpolations, the boundary image data are set to zero. Table 1 lists the normalized root-mean-squared (RMS) interpolation errors for various images, which shows that the 2-D VFD filtering approach can achieve smaller interpolation errors than other well-known typical interpolation techniques.

Fig. 4 shows the input image (girl) of 128×128 pixels, and Fig. 5, Fig. 6, and Fig. 7 show the interpolated images from the zero-order interpolation, bilinear interpolation, and 2-D VFD filtering approach, respectively. It is observed that the interpolated image from the zero-order method is blocky, while the one from 2-D VFD filtering is smoother and exhibits improved appearance.

4. Conclusion

In this paper, we have extended the Lagrange-type 1-D VFD filter to the 2-D case and used image interpolation examples to demonstrate that the image interpolation using 2-D VFD filter can achieve higher resolution images than the most commonly used conventional interpolation techniques such as zero-order interpolation, bilinear interpolation, and 6-term polynomial interpolation..

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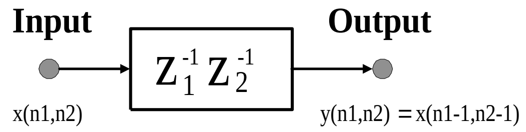

Fig. 1. Delaying 2-D signal using 2-D unit delay element.

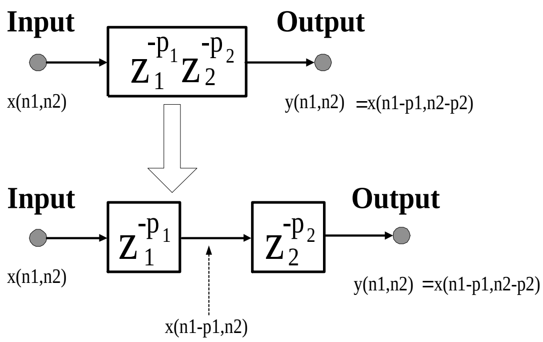

Fig. 2. Delaying 2-D signal using fractional-delay element.

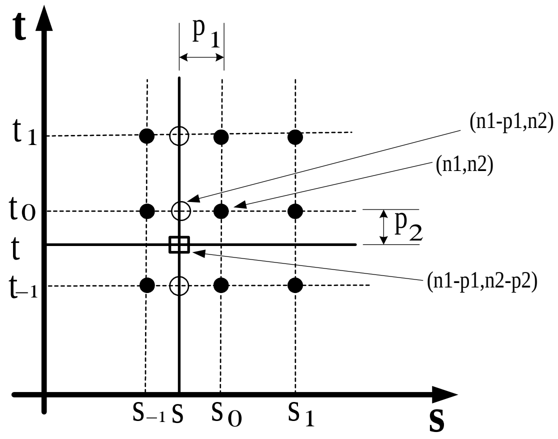

Fig. 3. 2-D VFD filtering.

Fig. 4. Input image.

Fig. 5. Interpolated image using zero-order interpolation

Fig. 6. Interpolated image using bilinear interpolation.

Fig. 7. Interpolated image using 2-D VFD filtering.

Table 1: Image Interpolation Errors [%]

	Zero-Order	Bilinear	6-Term Poly.	VFD
Girl	12.0968	11.0496	9.4393	8.6420
Woman	10.4128	8.4541	8.1865	7.7921
Lenna	10.9982	9.3472	8.2575	7.4514
Barbara	18.9479	16.1078	15.4797	15.1612
Cameraman	14.3648	12.2491	11.7016	11.0374
Boat	9.0182	8.4373	7.3395	7.0979
Airplane	9.6172	8.1401	7.6463	7.0193
Building	10.7976	10.0534	9.5636	8.9861
Bridge	20.2145	17.6128	16.6724	16.6371
Lighthouse	16.0835	14.7572	14.1634	13.7624
Text	17.6304	15.8615	13.5413	12.7177