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FPGA-based Implementation of Digital Spike Maps

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Abstract

This paper studies realization of desired digital spike-trains based on a simple evolutionary algorithm. Presenting a basic FPGA-based circuit, desired spike-trains are confirmed experimentally.

1. Introduction

This paper studies realization of desired digital spike-trains based on a simple evolutionary algorithm. Spike-trains is related to various systems: spiking neuron models, image processing systems and communication systems[1].

In order to visualize dynamics of spike-trains, we introduce a digital spike map(Dmap)[2]-[3]. The Dmap is a digital version of analog one-dimensional maps. The domain of Dmap consists of finite number of lattice points. The steady state must be a periodic spike-train(PST). The Dmap is related to various digital dynamical systems including cellular automata and dynamic binary neural networks.

In order to realize desired spike-trains, we present a simple evolutionary algorithm(SEA)[4]. The individuals refer to their past history, communicate to each other, and try to find a solution.

In order to realize operation the Dmap, we introduce the digital spiking neuron(DSN)[4]. The DSN is constructed by two shift registers connected by a wiring. Depending on wiring patterns, the DSN can generate various spike-trains. The dynamics of the DSN corresponds to Dmap. We present a FPGA based circuit, typical operations are confirmed experiementry.

2. Digital Spike Map

Fig.1 shows a digital spike map(Dmap) and a spike-train. The spike-train is defined by

$$Y(\tau) = \begin{cases} 1 & \text{for } \tau = \tau_n \\ 0 & \text{for } \tau \neq \tau_n \end{cases} \qquad \tau_n = \theta_n + (n-1). \tag{1}$$

Let τ_n denote the n-th spike position where $Y(\tau) = 1$. Let θ_n denote the n-th spike-phase: $\theta_n = \tau_n \bmod 1$.

The Dmap visualize dynamics of spike-trains. The Dmap is defined by

$$\theta_{n+1} = f(\theta_n),\ \theta_n \in \{1, 2, \cdots, N\} \equiv L_N \tag{2}$$

The Dmap is represented by the characteristic vector

$$\boldsymbol{d} \equiv (d_1, \cdots, d_N),\quad d_i = Nf(l_i) \in \{1, 2, \cdots, N\} \tag{3}$$

Foe example, a Dmap in Fig. 1 is represented by $\boldsymbol{d} = (8, 5, 2, 6, 7, 2, 3, 6)$.

Since the domain L_N of the Dmap consists of a finite number of points. The steady state must be a periodic spike-trains(PST). We give basic definitions on Dmap.

Definition 1: A point $p \in L_N$ is said to be a periodic point (PEP) with period k if $p = f^k(p)$ and $f(p)$ to $f^k(p)$ are all different where f^k is the k-fold composition of f. A sequence of the PEPs $\{p, f(p), \cdots, f^{k-1}(p)\}$ is said to be a periodic orbit (PEO) with period k. A PEO with period k is equivalent to a PST with period k. For example, in Fig. 1, the PEO with period 4 is equivalent to the PST with period 4.

Definition 2: A point $q \in L_M$ is said to be an eventually periodic point (EPP) with step k if the q is not a periodic point but falls into some periodic point p after k steps: $f^k(q) = p$. An EPP with step 1 is referred to as a direct eventually periodic point (DEPP): $f(q) = p$. An EPP corresponds to an spike-position of a transient spike-train to the PST.

3. Simple Evolutionary Algorithm

In order to realize a Dmap that can generate a desired PST, we introduce the simple evolutionary algorithm(SEA). We give several definitions for the SEA.

The i-th individual at generation g is denoted by

$$\begin{aligned} &\boldsymbol{\delta}^i(g) = (\delta^i_1(g), \cdots, \delta^i_N(g)), \\ &i \in \{1, 2, \cdots, K(g)\}, \\ &g \in \{0, 1, 2, \cdots, g_{max}\},\ K(g) \leq K_{max} \end{aligned} \tag{4}$$

where $K(g)$ is the number of individuals at generation g. The individual in the SEA corresponds to a characteristic vector $\boldsymbol{d}$ of the Dmap. We denote the maximum value of number of individual and generation as k_{max}, g_{max}. The individuals is evaluated by

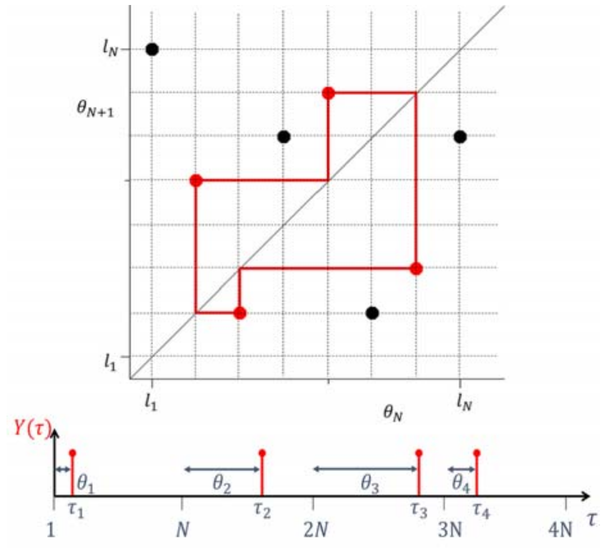

Figure 1: Digital spike map. The PEO with period 4(red point) corresponds to the PST with step 4. $\boldsymbol{d} = (8, 5, 2, 6, 7, 2, 3, 6)$

the cost function $F_c(\delta^i(g))$.

$$F_c(\boldsymbol{\delta}^i(g)) \geq 0,\ i \in \{1, 2, \cdots, K(g)\}. \tag{5}$$

Let $G_b(g)$ be the global best at generation g. We present the procedure of the SEA.

Step1 (Initial individual setting): Let $g = 0$ and let $K(g) = 1$. We prepare an initial indivudual $\delta^1(0)$ has PEO with period k. Let $G_b(0) = F_c(\boldsymbol{\delta}^1(0))$.

Step2 (Creating candidates): We focus on the elements that compose the PEO with period k of the i-th indivudual. One of those elements is changed to another integers in $\{1, \cdots, N\}$. It makes $k \times (N-1) \times K(g)$ candidates of individuals for the next generation. The j-th candidate is denoted by $\boldsymbol{\beta}^j(g)$, where $j \in \{1, \cdots, k \times (N-1) \times K(g)\}$

Step3 (Evaluation of candidates): The candidates that has PEO with period k are evaluated by the cost function F_c. The global best is updated:

$$\begin{cases} G_b(g) \leftarrow F_c(\boldsymbol{\beta}^j(g)) & \text{if } F_c(\boldsymbol{\beta}^j(g)) < G_b(g) \\ G_b(g) \leftarrow G_b(g) & \text{otherwise} \end{cases} \tag{6}$$

If $G_b(g)$ is the minimum value of the cost function F_c, the algorithm is terminated. If $G_b(g)$ is not the minimum value, the candidate is declared as a new individual. The number of individuals $K(g)$ is updated to the number of the new individuals K_N. If K_N exceeds K_{max}, the new individuals are selected at random from the candidates of the global best.

Step4 (Mutation): We focus on an element other than those composing PEO corresponds to an EPP. The element is changed to another elements in $\{1, \cdots, N\}$. randomly with mutation probability MR.

Step5: Let $g \leftarrow g + 1$, go to step 2 and repeat until the maximum generation g_{max}.

We apply the SEA to realize the desired spike-train. Our goal is to realize the PST with low autocorrelation. The autocorrelation (period p) is defined by

$$R_{YY}(d) = \sum_{\tau=1}^{p} Y_p(\tau)Y_p(\tau + d) \ \text{ for } d \in \{0, \cdots, p\} \tag{7}$$

The cost function represents the second peak of autocorrelation

$$F_c(\boldsymbol{d}) = \max_d R_{YY}(d) \ \text{ for } d \in \{1, \cdots, p-1\},\ F_c(\boldsymbol{d}) \geq 1 \tag{8}$$

where $\boldsymbol{d}$ is a characteristic vector whose Dmap has the PST with period p. The minimum value is $F_c(\boldsymbol{d}) = 1$. We apply the SEA to this cost function with the following parameters

$$M = 32,\ p = 10,\ K_{max} = 30,\ g_{max} = 20,$$
$$MR \in \{0, 1, 5, 10, 30\}$$

In the case of $MR = 10$, the individuals for each generation are as the following

$$\boldsymbol{\delta}(0) = (2, 3, 4, 5, 6, 7, 8, 9, 10, 1, 16, 9, 25, 14, 4, 6, 26, 21, 17, 25, 26, 4, 17, 8, 7, 16, 21, 29, 6, 9, 23, 1)$$

$$\boldsymbol{\delta}(6) = (5, 10, 17, 20, 1, 9, 24, 18, 5, 3, 18, 30, 9, 12, 27, 21, 19, 11, 2, 15, 25, 2, 3, 16, 29, 27, 21, 16, 24, 13, 25, 13)$$

Fig. 2 shows Dmap for $F_c(\boldsymbol{d}) = 1$. Fig. 3 shows evolution process for the global best, number of individuals.

We have executed 100 trials where different random numbers are used in individual initialization in step 1. We shows success rate and average number of generations and individuals against the mutation rate in Table. 1.

Table 1: SEA performance in 100 trials.

MR	Sucess rate	Ave.g	Ave.K(g)
0	18	2.56	8.20
1	74	6.12	84.4
5	98	4.91	39.8
10	96	4.79	37.6
30	95	3.46	18.6

4. Digital Spiking Neuron

In order to realize operation the Dmap, we intoroduce the digital spiking neuron(DSN). The DSN is constructed by two registers connected by a wiring as shown in Fig. 4(a). The right and left shift registors

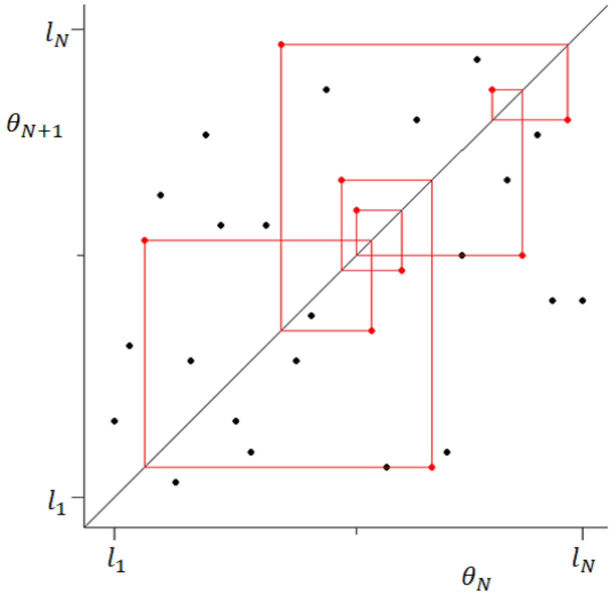

Figure 2: Optimized Dmap by the SEA. $g = 6$, $G_b(6) = 1$.

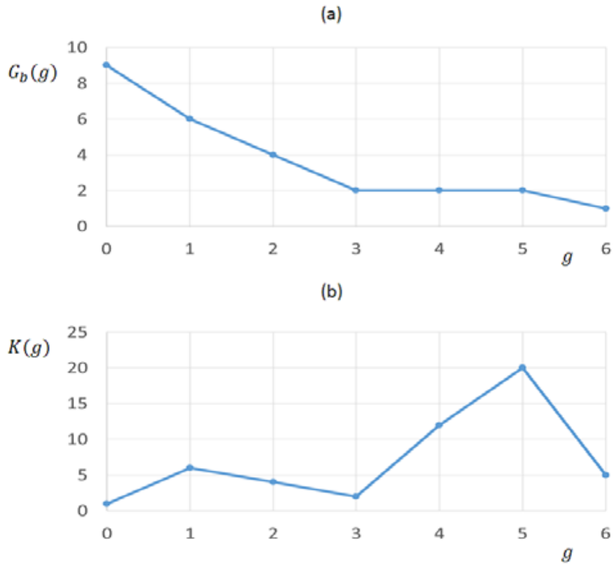

Figure 3: The process of the SEA. (a)Global best. (b)Number of individuals.

are refferd to P-cells and X-cells in respectively. The P-cells consist of N elements. The P-cells operate as a pacemaker. Only one element can be 1 with period N, all the other elements 0.

$$\begin{array}{l} P(\tau) \equiv (P_1(\tau), \cdots, P_N(\tau)) \\ \tau \in \{1, 2, 3, \cdots\},\ P_i(\tau) \in \{0, 1\} \end{array} \tag{9}$$

The X-cells consist of M elements. The X-cells construct a state variable vector corresponding to the membrane potential in analog neuron models. Only one element can be 1 with period N, all the other elements 0.

$$\begin{array}{l} X(\tau) \equiv (X_1(\tau), \cdots, X_M(\tau)) \\ \tau \in \{1, 2, 3, \cdots\},\ X_i(\tau) \in \{0, 1\} \end{array} \tag{10}$$

The P-cells and X-cells are connected by the wiring. The wiring is characterized by the wiring vector:

$$\begin{array}{l} W = (w_1, w_2, \cdots, w_j, \cdots, w_N), \\ w_i = j \text{ if } P_i \text{ is connected to } X_j. \end{array} \tag{11}$$

The wiring vector determines the base signel

$$B(\tau) = (B_1(\tau), B_2(\tau), \cdots, B_j(\tau), \cdots, B_N(\tau)) \tag{12}$$

Fig. 4(b) shows the base signal. The base signals are determined by Dmap.

The dynamics of X-cells described by

$$X_{j+1}(\tau+1) = \begin{cases} 1 & \text{if } X_j(\tau) = 1 \text{ for } j = 1 \sim M \\ 1 & \text{if } X_M(\tau) = 1 \text{ and } B_{j+1}(\tau) = 1 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

If top elements of X-cell is 1, the DSN outputs a spike $Y(\tau+1) = 1$

$$Y(\tau+1) = \begin{cases} 1 & \text{if } X_M(\tau) = 1 \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

Fig. 4(b) shows the spike. For simplicity, let number of elements of the P-cell be $M = 2N - 1$. In this condition, the DSN outputs one spike per one clock priods.

The base signals are determined by Dmap. That is, the wiring vector $\boldsymbol{W}$ and the characteristic vector of Dmap $\boldsymbol{d}$ are rerated as the following

$$W_i = N - (d_i - i), i = 1 \sim N \tag{15}$$

Foe example, a Dmap in Fig. 4(a) is represented by $\boldsymbol{W} = (1, 5, 9, 6, 6, 12, 12, 10)$.

In order to observe the behaivior of DSN experimentally, we have designed a digital circuit as shown in Fig. 5.

We implements DSN based on the wiring vector. This circuit is implemented in FPGA. Fig.6 shows a laboratory measurement. The DSN is controlled by CLK.

5. Conclusion

Realization of desired digital spike-trains based on a simple evolutionary algorithm is studied in this paper. Presenting a basic FPGA-based circuit, desired spike-trains are confirmed experimentally. Future problems include development of the SEA into various digital dynamical systems and using multiple cost functions on the SEA.

References

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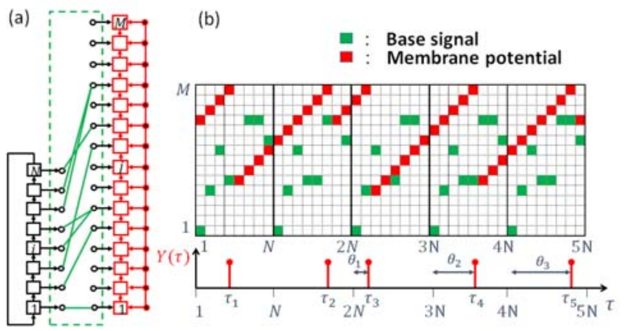

Figure 4: Digital spiking neurons. (a)$\boldsymbol{W} = (1, 5, 9, 6, 6, 12, 12, 10)$. (b)Time domain waveform.

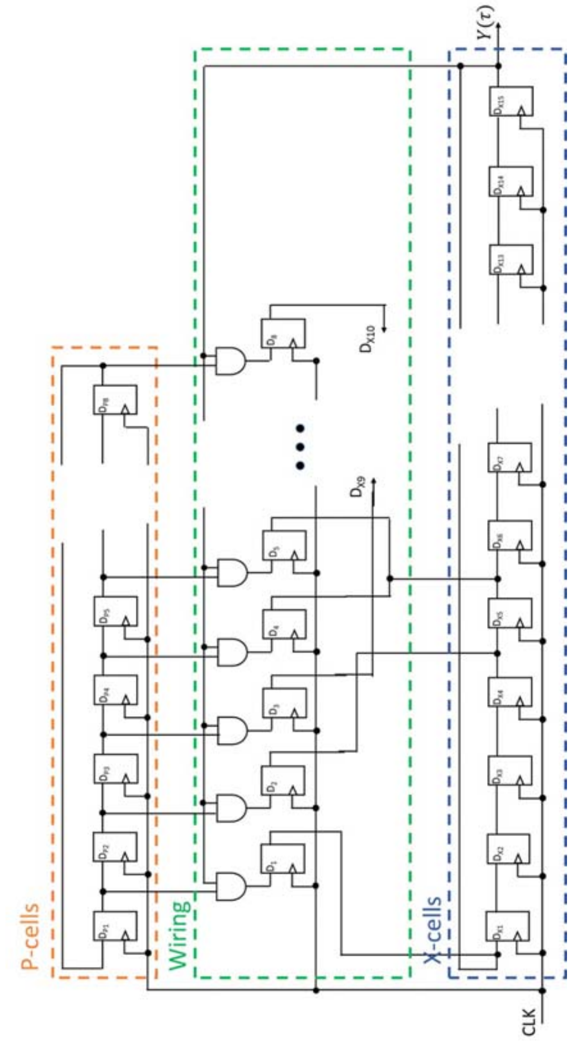

Figure 5: The circuit diagram of DSN. $\boldsymbol{W} = (1, 5, 9, 6, 6, 12, 12, 10)$.

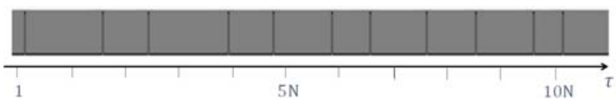

Figure 6: Desired PST in FPGA implemantion. $\boldsymbol{W} = (27, 23, 17, 15, 35, 28, 14, 21, 35, 38, 24, 13, 35, 33, 19, 26, 29, 38, 48, 36, 27, 51, 51, 39, 27, 30, 37, 43, 36, 48, 37, 50)$ (FPGA board: DIGILENT Inc. NEXYS3 Spartan-6. Logic Analyzer: Analog Discovery2.)