RECURSIVE NON-ORTHOGONAL DECOMPOSITION WITH HYPERBOLIC WAVELETS FOR SUBSURFACE RADAR SIGNAL PROCESSING

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1 Introduction

Noise reduction is among most important issues in the signal processing of subsurface radars. We have developed a noise reduction scheme based on a two-dimensional parabolic wavelet transform, which is designed to detect hyperbolic features associated to subsurface radar images[1]. The proposed algorithm showed a superior performance compared to conventional wavelet noise reduction schemes which do not make use of such features. The problem we encountered was the non-orthogonality of the parabolic wavelet basis, which limits the reconstruction capability of the algorithm.

Here we propose a scheme based on similar 2-D wavelet bases, but employing the recursive non-orthogonal decomposition algorithm known as the matching pursuit[2]. The idea is to repeat the procedure of fitting waveforms given in a redundant dictionary to the given waveform, and subtracting the best matched one recursively. The advantage is that the desired signal can be retrieved from a very noisy data if the waveform is included in the dictionary. The inherent problem of this procedure is a heavy computational load because a large number of iteration is needed.

We develop schemes to substantially reduce the computation by customizing the algorithm to the signal processing of subsurface radars, and by taking into account the characteristics of the desired signals. The capability of the proposed algorithm in detecting various targets buried in noise and clutter is evaluated based on simulated data for an attenuating and dispersive medium.

2 Algorithm

Input image $f(x,y)$ can be expressed in terms of the waveforms in a dictionary of images as[2]

$$f(x,y) \quad = \quad C^0 \cdot g_{\gamma 0}(x,y) + R^0 f(x,y) \ , \tag{1}$$

where $g_{\gamma n}(x,y)$ is the best matched waveform at the n-th iteration in the dictionary g_γ, and $R^n f(x,y)$ is the residual of $f(x,y)$. The correlation coefficient C^n between $f(x,y)$ and $g_{\gamma n}(x,y)$ is obtained by

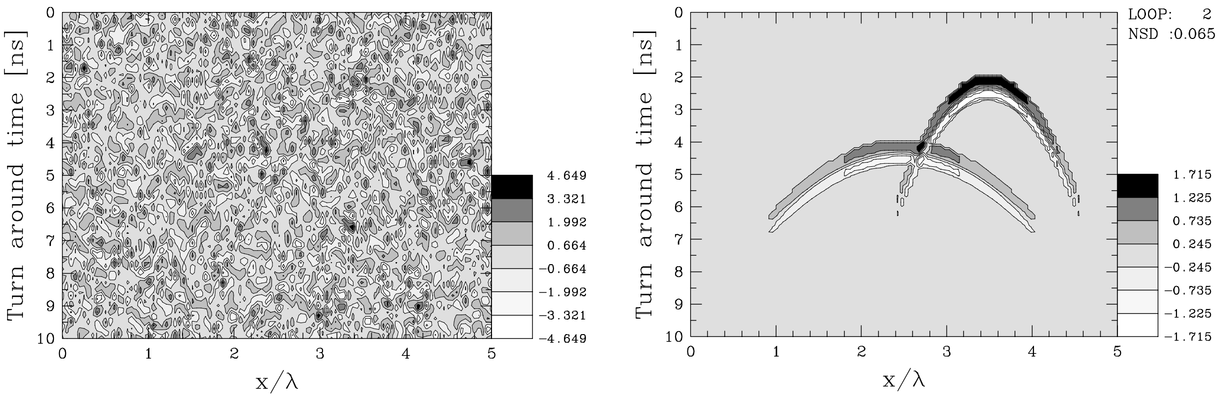
$$C^n \quad = \quad \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R^{n-1} f(x,y) \cdot g_{\gamma n}(x,y) \,\mathrm{d}x \mathrm{d}y \ , \tag{2}$$

and $g_{\gamma n}(x,y)$ is defined as the waveform which maximizes $|C^n|$. Recursive decomposition using various waveforms in the dictionary thus gives

$$f(x,y) \quad = \quad \sum_{n=0}^{m-1} \{C^n \cdot g_{\gamma n}(x,y)\} + R^m f(x,y) \ , \tag{3}$$

which means that if the set of m images is adequately chosen, $R^m f(x,y)$ becomes sufficiently small, and the input image can be expressed as a linear combination of the waveforms. It should be noted that the orthogonality between the waveforms is not required in this procedure, although poor orthogonality results in a larger number of iterations.

The major problem of this method is the computational time to search through the dictionary of waveforms which can be quite large for the case of two-dimensional images such as subsurface radar data. The dictionary should contain all possible images with arbitrary shift in x- and y-directions.



Two parabolic images embedded in a random noise (S/N=−6 dB).

Retrieved image after two iterations of the non-orthogonal decomposition.

Figure 1: Denoising with the recursive non-orthogonal decomposition when the given images are included in the dictionary.

We solved this problem as follows. Firstly, we prepare only 12 parabolic waveforms with different curvature. In finding the best matched waveform, it is necessary to compute the correlation coefficient by shifting the waveform in x- and y-axes. This process of shifting and correlation can be performed efficiently by using 2-D FFT of 100x100 points in our current case. Thus a dictionary of 120,000 waveforms can be sought by only 12 set of forward and backward FFT operations. The dictionary matching gives the best result when the waveform in the dictionary precisely matches to the given one.

Left panel of Fig. 1 shows a numerical example of two parabolic images embedded in a random noise where signal-to-noise ratio is -6dB. While no signal is visible in the given image, the precise waveform is retrieved as shown on the right panel after two iterations.

Since the actual subsurface radar image basically consists of hyperbolic features, parabolic dictionary only gives a rough match. After finding the best match parabolic waveform, we switch the dictionary to a 12 set of hyperbolic waveforms with the same curvature at their apex as the best match parabola, but with a variety of asymptotes. We further switch the dictionary to 12 curves with a variety of attenuation after searching for asymptotes. This dictionary switching reduces a 3-D dictionary search into a series of three 1-D search, thus reducing the computational time drastically.

3 Numerical Simulations

Signal processing of the subsurface radars is characterized by the presence of strong noise and clutter, as well as heavy attenuation and dispersion of the signal in the medium. We generate realistic simulated data using the FD-FDTD (Frequency-Dependent Finite-Difference Time-Domain) method[3]. The grid interval is $\Delta s = 0.025\lambda$, and the time step is $\Delta t = 0.01T$, where λ and T are the wavelength and the period of the transmitted pulse at its center frequency. The domain size is 480×240 grids. The transmit/receive antenna scans 100 points located at $40 \leq x \leq 440, y = 200$ as shown in Fig. 2. At each antenna position, the received signal is computed for 1,000 time steps, and 100 points are sampled at 10-point intervals, thus generating the radar image of 100×100 points. The characteristics of the medium simulates those of typical dry rock, whose attenuation coefficient is 23 dB/m/GHz[4]. Fig. 3 shows the transmitted mono-cycle pulse waveform.

Three types of metallic targets are placed at the depth of 1.5λ beneath the scan line. They include a point target, and cylinders and plates of various sizes in the range of 0–2λ. As the target differs from a point target, the image deviates from hyperbolic shapes. If there is no additional noise, the deviation can be taken care of by recursive decomposition with multiple hyperbolic images. The residual can be made as small as desired by increasing the number of iteration. As the iteration proceeds, the magnitude of the correlation coefficient C^n decreases monotonically.

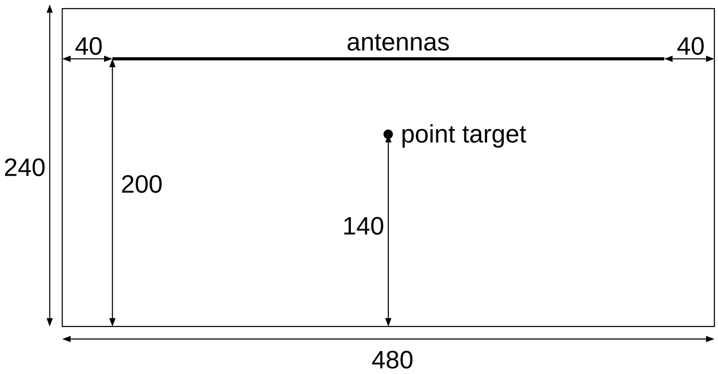


Figure 2: The computational domain and the location of T/R antennas.

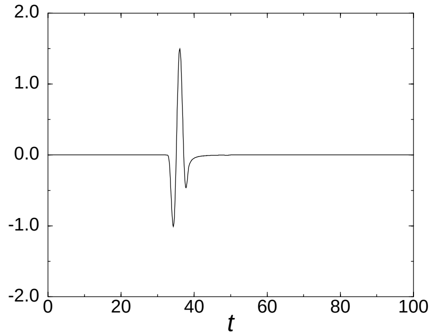


Figure 3: Transmitted waveform.

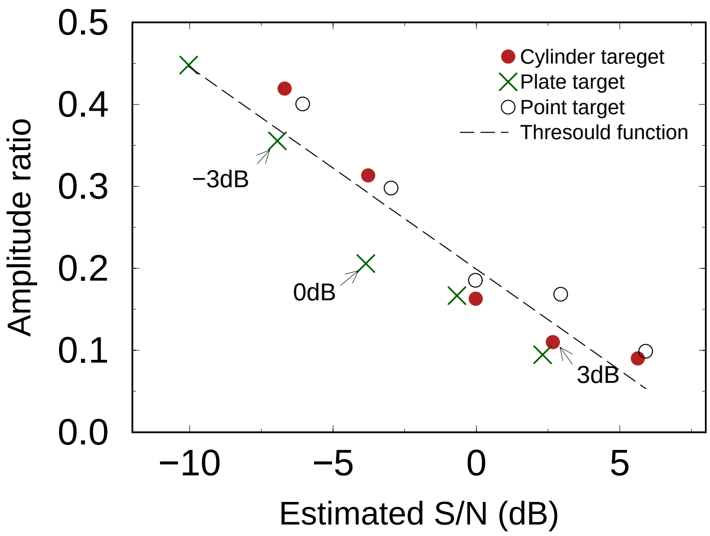


Figure 4: Relative amplitude of the decomposed wave at the optimum number of iteration as a function of the estimated S/N ratio.

When the noise becomes stronger, however, additional iterations are used up to express the fluctuations associated with the noise rather than to retrieve the details of the given image. Thus the optimum number of iteration is determined for a given S/N ratio. Fig. 4 shows $|C^m/C^0|$ for various targets at the optimum iteration number m as a function of the estimated S/N ratio, which is determined from $|C^0|$ and the variance of the input image outside the detected signal region. The dashed line gives the empirical threshold of $|C^m/C^0|$ at which the iteration should be terminated. It should be noted that a single threshold can be safely used regardless to the type of targets. It is thus possible to obtain the optimum result for unknown targets.

The proposed decomposition technique can be used to the case of distributed clutters as well as the white noise. Although there is no physical difference between the desired target and clutter, the present algorithm retrieves targets in the order of their magnitude. So if the echo from the desired target is stronger than clutter echoes, it is possible to suppress only the clutter echoes by properly choosing the threshold. It is also necessary to search for only those images whose apexes are located closely each other in order to retrieve the shape information of the target. Fig. 5 shows an example of retrieving cylindrical target in the presence of clutter echoes. However, determination of the optimum threshold is not yet automated for the case of clutters.

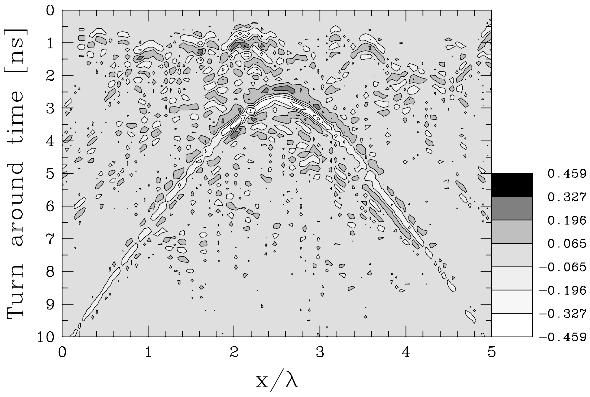


Figure 5: Simulated image of a cylindrical target in the presence of clutters.

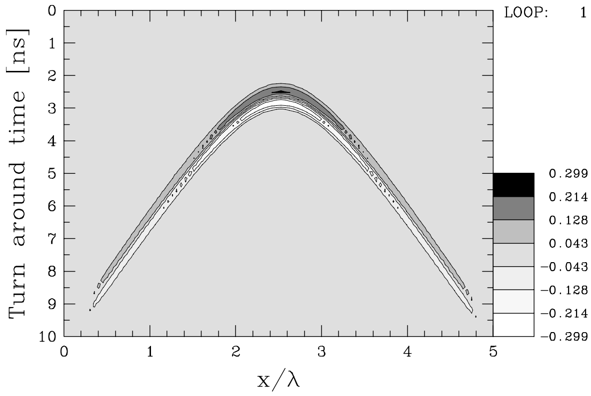


Figure 6: Retrieved image by the proposed algorithm.

4 Summary

A denoising algorithm based on the recursive non-orthogonal wavelet decomposition has been proposed. It is successfully applied to the synthetic data with strong noise and clutter. The target can be accurately detected from data with a very poor S/N ratio of -6dB. The desired target and the sources of clutter echoes, each of which is essentially a weak target, can be detected in turn by the recursive estimation. Also, targets with various shapes can be precisely reconstructed by combination of multiple waveforms when the S/N ratio is sufficiently high. Optimum threshold for terminating the iteration is given for the case of white noise.

References

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