

BACKSCATTERING ENHANCEMENT FOR COMPLEX CONCAVE-CONVEX TARGETS  
IN RANDOM MEDIA FOR H-WAVE INCIDENCE

H. El Ocla and M. Tateiba

Department of Computer Science and Communication Engineering, Kyushu University  
6-10-1 Hakozaki, Higashi-ku, Fukuoka 812-8581, Japan  
E-mail:hosam@green.csce.kyushu-u.ac.jp

### 1 Introduction

Problems of wave propagation and scattering in random media have been investigated by many researchers for a long time. However, the problem of wave scattering by targets in random media has not been analyzed as a boundary value problem. Recently, the scattering problem has been solved by presenting a method that shows effects of random media on the manner of scattered waves from arbitrary shape targets [1]. Using this method, authors have presented numerical results for calculating radar cross-section (RCS) of convex targets such as circular and elliptic cylinders in random media [2, 3]. It was found out that the spatial coherence length of incident wave on the targets (SCL) is playing a leading role in determination of the RCS behavior. When authors considered targets as conducting cylinders with concave-convex shapes in random media, they pointed out that there are many anomalies in the behavior of normalized RCS (NRCS) defined as the ratio of RCS in random media to RCS in free space [4, 5, 6]. Also, we have found out a large increase in the NRCS when a H-wave propagating in random media illuminates a convex portion of the target. Moreover, these anomalies are affected largely by the target parameters. This leads to the fact that backscattering enhancement (BE) in RCS of conducting targets in random media is not only dependent on both effects of SCL of incident waves and the double passage effect. It is, therefore, extended to take account of target parameters. To conduct a clear understanding of BE phenomenon for concave-convex targets in random media, we have evaluated the RCS of these targets with different parameters. In this work, we will concentrate on H-wave incidence since anomalies in NRCS occur more strongly than an E-wave case. In this regard, we consider the case that the incident wave becomes incoherent enough around a conducting cylinder although it keeps a finite SCL. We present numerical results of NRCS for H-wave incidence on conducting targets of concave-convex shapes with different sizes and also different values of target shape parameter: concavity index, to observe the behavior of NRCS of such complex targets in random media.

The time factor  $\exp(-i\omega t)$  is assumed and suppressed in the following section.

### 2 Formulation

Geometry of the problem is shown in Figure 1. A random medium is assumed as a sphere of radius  $L$  around a target of the mean size  $a \ll L$ , and also to be described by the dielectric constant  $\epsilon(\mathbf{r})$ , the magnetic permeability  $\mu$ , and the electric conductivity  $\sigma$ . For simplicity  $\epsilon(\mathbf{r})$  is expressed as

$$\epsilon(\mathbf{r}) = \epsilon_0[1 + \delta\epsilon(\mathbf{r})] \tag{1}$$

where  $\epsilon_0$  is assumed to be constant and equal to free space permittivity and  $\delta\epsilon(\mathbf{r})$  is a random function with

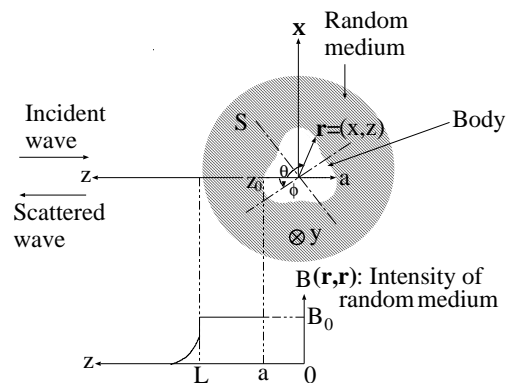


Figure 1: Geometry of the problem of wave scattering from a conducting cylinder in random media.

$$\langle \delta\varepsilon(\mathbf{r}) \rangle = 0, \quad \langle \delta\varepsilon(\mathbf{r}) \delta\varepsilon(\mathbf{r}') \rangle = B(\mathbf{r}, \mathbf{r}') \quad (2)$$

and

$$B(\mathbf{r}, \mathbf{r}) \ll 1, \quad kl(\mathbf{r}) \gg 1 \quad (3)$$

Here, the angular brackets denote the ensemble average and  $B(\mathbf{r}, \mathbf{r})$  is the local intensity of random medium. Also,  $\mu$  and  $\sigma$  are assumed to be constant;  $\mu = \mu_0$ ,  $\sigma = 0$ .

For practical turbulence the conditions  $B(\mathbf{r}, \mathbf{r}) \ll 1$ ,  $kl(\mathbf{r}) \gg 1$  may be satisfied, where  $k = \omega \sqrt{\varepsilon_0 \mu_0}$  is the wavenumber in free space and  $l(\mathbf{r})$  is the local scale-size of random medium, and therefore we can assume the forward scattering approximation and the scalar approximation. Consider the case where a directly incident wave is produced by a line source  $f(\mathbf{r}')$  distributed uniformly along the  $y$  axis. Then the incident wave is cylindrical and becomes plane approximately around the target because the line source is very far from the target. Here, let us designate the incident wave by  $u_{in}(\mathbf{r})$ , the scattered wave by  $u_s(\mathbf{r})$ , and the total wave by  $u(\mathbf{r}) = u_{in}(\mathbf{r}) + u_s(\mathbf{r})$ . The target is assumed as a conducting cylinder. The cross-section of the cylinder is expressed by

$$r = a[1 - \delta \cos 3(\theta - \phi)] \quad (4)$$

where  $\delta$  is the concavity index and  $\phi$  is the rotation index. We can deal with this scattering problem two dimensionally under the condition (3); therefore, we represent  $\mathbf{r}$  as  $\mathbf{r} = (x, z)$ . Assuming an incident H-wave, we can impose the Neumann boundary condition for  $u(\mathbf{r})$  on the cylinder.

$$\frac{\partial}{\partial \mathbf{n}} u(\mathbf{r}) = 0 \quad (5)$$

where  $\partial/\partial \mathbf{n}$  denotes the outward normal derivative at  $\mathbf{r}$  on  $S$ .

According to our method [1], using the current generator  $Y_H$  and Green's function in random medium  $G(\mathbf{r} | \mathbf{r}')$ , we can express the scattered wave as

$$u_s(\mathbf{r}) = - \int_S d\mathbf{r}_1 \int_S d\mathbf{r}_2 \left[ \left( \frac{\partial}{\partial \mathbf{n}_2} G(\mathbf{r} | \mathbf{r}_2) \right) Y_H(\mathbf{r}_2 | \mathbf{r}_1) u_{in}(\mathbf{r}_1 | \mathbf{r}_t) \right] \quad (6)$$

Here,  $Y_H$  is the operator that transforms incident waves into surface currents on  $S$  and depends only on the scatterer. The current generator is expressed in terms of wave functions which satisfy Helmholtz equation and the radiation condition. That is, for H-wave incidence, the current generator is obtained as

$$Y_H(\mathbf{r} | \mathbf{r}') \simeq - \frac{\partial \Phi_M^*(\mathbf{r})}{\partial \mathbf{n}} A_H^{-1} \ll \Phi_M^T(\mathbf{r}') \quad (7)$$

Here, the basis functions  $\Phi_M$  are called the modal functions and constitute the complete set of wave functions satisfying the Helmholtz equation in free space and the radiation condition;  $\Phi_M = [\phi_{-N}, \phi_{-N+1}, \dots, \phi_N]$ ,  $\phi_m(\mathbf{r}) = H_m^{(1)}(kr) \exp(im\theta)$ , and  $A_H$  is a positive definite hermitian matrix given by

$$A_H = \begin{pmatrix} \left( \frac{\partial \phi_1}{\partial \mathbf{n}}, \frac{\partial \phi_1}{\partial \mathbf{n}} \right) & \cdots & \left( \frac{\partial \phi_1}{\partial \mathbf{n}}, \frac{\partial \phi_M}{\partial \mathbf{n}} \right) \\ \vdots & \ddots & \vdots \\ \left( \frac{\partial \phi_M}{\partial \mathbf{n}}, \frac{\partial \phi_1}{\partial \mathbf{n}} \right) & \cdots & \left( \frac{\partial \phi_M}{\partial \mathbf{n}}, \frac{\partial \phi_M}{\partial \mathbf{n}} \right) \end{pmatrix} \quad (8)$$

in which its  $m, n$  element is the inner product of  $\phi_m$  and  $\phi_n$ :

$$\left( \frac{\partial \phi_m}{\partial \mathbf{n}}, \frac{\partial \phi_n}{\partial \mathbf{n}} \right) \equiv \int_S \frac{\partial \phi_m}{\partial \mathbf{n}} \frac{\partial \phi_n^*}{\partial \mathbf{n}} d\mathbf{r} \quad (9)$$

In (7),  $\ll \Phi_M^T$  denotes the operation (10) of each element of  $\Phi_M^T$  and the function  $u_{in}$  to the right of  $\Phi_M^T$

$$\ll \phi_m(\mathbf{r}), u_{in}(\mathbf{r}) \gg \equiv \phi_m(\mathbf{r}) \frac{\partial u_{in}(\mathbf{r})}{\partial \mathbf{n}} - \frac{\partial \phi_m(\mathbf{r})}{\partial \mathbf{n}} u_{in}(\mathbf{r}) \quad (10)$$

The  $Y_H$  is proved to converge in the sense of mean on the true operator when  $M \rightarrow \infty$ . We can obtain the RCS ( $\sigma$ ) by calculating  $\langle |u_s|^2 \rangle$  from (6).

$$\sigma = \langle |u_s(\mathbf{r})|^2 \rangle \cdot k(4\pi z)^2 \quad (11)$$

### 3 Numerical Results

We analyze numerically the NRCS by changing the target in shape and size. First we restrict the shape and size to  $\delta = 0.1, 0.2$  and  $0.1 \leq ka \leq 10$ , respectively (see figure 2).

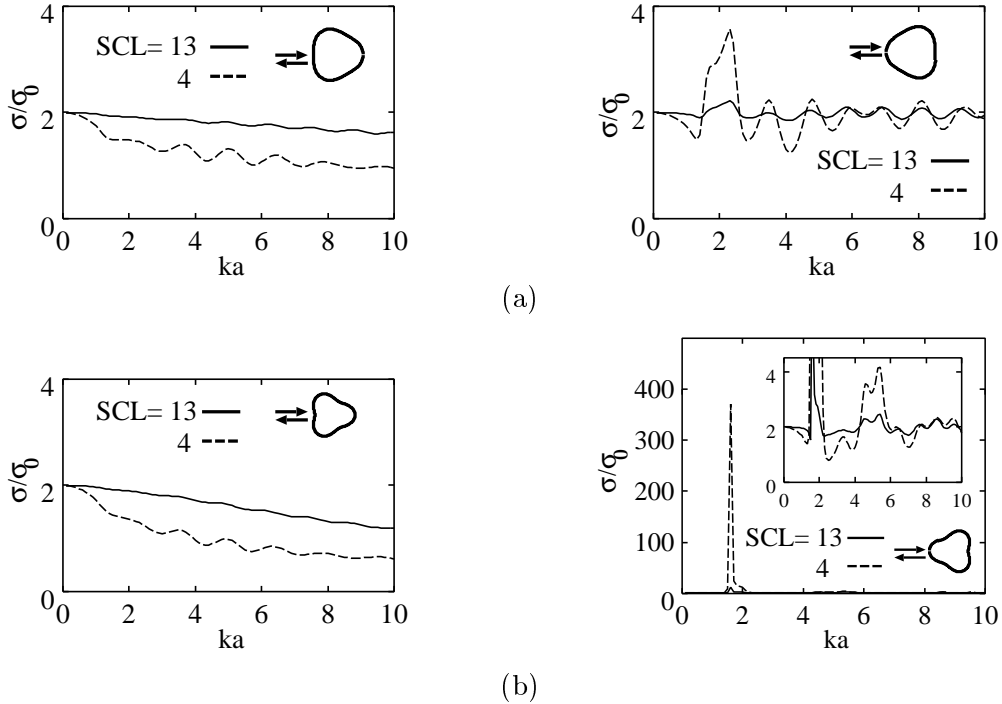


Figure 2: Normalized RCS vs. target size at two different incident angles and SCLs for H-wave incidence where (a)  $\delta = 0.1$ , (b)  $\delta = 0.2$ .

By analyzing numerically the NRCS in three regions of  $ka$  compared to SCL, we have found that for  $ka \ll SCL$ , the NRCS equals two due to the double passage effect of waves in random media. This value of NRCS is realized, independent of incident angle. For  $ka \approx SCL$ , in case of convex portion illumination, NRCS oscillates largely around value of two. At certain values of target parameters ( $ka = 1.6$  and  $\delta = 0.2$ ), this oscillation becomes resonant and the value of oscillation increases dramatically and reaches a maximum. Here in our numerical results, the resonance occurs at certain shape of target ( $\delta = 0.2$ ). In addition, it happens when the target size is close to the wavelength and becomes comparable with SCL of incident wave. To describe the circumstances related to the resonance in more details, we will handle this problem in a separate work. However, we should note that the resonance becomes acceptable by considering the scattering of beam wave with the spot size  $\approx SCL$  in free space. In case of concave portion illumination, NRCS decreases gradually due to the effect of SCL; and it fluctuates, while it is decreasing, due to the difference of creeping waves effects between both of free space and random media. On the other hand, in case of convex portion illumination and for  $ka > SCL$ , the NRCS tends to two with increasing  $ka$ . To make clear the effect of target parameters on NRCS, we calculated the NRCS of targets with  $\delta = 0.25$  and  $\delta = 0.3$  for the interval  $0.1 \leq ka \leq 3$ . Figure 3 shows the change in NRCS with  $ka$ , SCL, and illumination portion; this change agrees with that of figure 2.

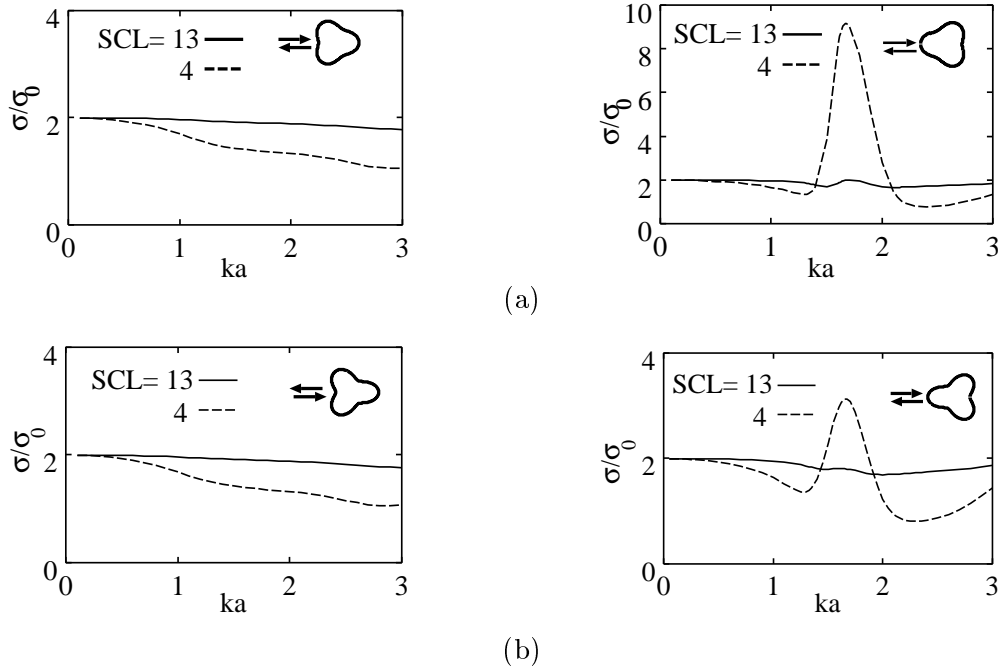


Figure 3: As figure 2, but with (a)  $\delta = 0.25$ , (b)  $\delta = 0.3$ , and  $0.1 \leq ka \leq 3$ .

## 4 Conclusion

Using the method that assumes a current generator together with Green's function to get the scattered waves in random media, we have evaluated the NRCS of complex concave-convex targets for the interval  $0.1 \leq ka \leq 10$ . In this work, we assume that waves around target are incoherent and the SCL keeps a finite value; this assumption is needed for backscattering enhancement. In this paper, we have investigated the backscattering enhancement of RCS of targets in random media by assuming a variety of target parameters for H-wave incidence. From numerical results, it is clear that SCL of incident waves on the target has a remarkable effect in estimating the RCS of partially concave targets. On the other hand, for the case of convex portion illumination, in both regions of  $ka \ll \text{SCL}$  and  $ka > \text{SCL}$ , the NRCS is two or tending to it with increasing  $ka$ . While for  $ka \simeq \text{SCL}$ , the NRCS oscillates around value of two with some anomalous increases in the NRCS, when the target size is close to the wavelength, due to resonance. Also it was found that the NRCS changes obviously with complexity of target and the incident angle as well.

## References

- [1] M. Tateiba, and E. Tomita, IEICE Trans. Electron., vol.E75-C, no.1, pp.101–106, Jan. 1992.
- [2] M. Tateiba, and L. Tsang, *Electromagnetic Scattering by Rough Surfaces and Random Media*, PIER 14, pp.317–361, PMW Pub., Cambridge MA, USA, 1996.
- [3] Z. Q. Meng, and M. Tateiba, *Waves in Random Media*, vol.6, pp.335–345, 1996.
- [4] H. El Ocla, and M. Tateiba, Proc. Asia Pacific Microwave Conf. APMC'98, Yokohama, Japan, vol.2, pp.937–940, Dec. 1998.
- [5] M. Tateiba, T. Matsuoka, and H. El Ocla, Proc. of 2nd Asia-Pacific Engineering Research Forum on Microwaves and Electromagnetic Theory, Fukuoka, Japan, pp.58–67, Dec. 1998.
- [6] M. Tateiba, T. Matsuoka, and H. El Ocla, Proc. of URSI, General Assembly, Toronto, Canada, p.193, Aug. 1999.