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PROPAGATION OF LOCALIZED WAVE BEAMS IN THE WAVEGUIDE EARTH-IONOSPHERE

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1 Introduction

It is known that the expansion of field on the Gaussian beams is an effective method for the analysis of propagation of radiation in the waveguide Earth-Ionosphere. A such approach speeds up an analysis of field structure and an estimation of wave effects [1]. However the Gaussian beams are optimal only in the waveguides with a symmetric distribution of dielectric constant. But the waveguide Earth-Ionosphere has an asymmetric distribution of dielectric constant [1].

In this paper the solutions of wave equation localized along the rays trajectories are finded for the asymmetric distribution of dielectric constant of Earth-Ionosphere waveguide. These solutions are the known in quantum mechanics coherent states in a form of localized wave packets with a minimum possible width and diffractional angle divergence. These states allow naturally to introduce the concept of width of ray and clearly to observe a connection between the wave and ray descriptions.

2 Formulation of the problem

It is known the Maxwell equations for monochromatic component of field E(x,y,z) in a weakly inhomogeneous medium can be reduced to the scalar Helmholtz equation

$$\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \kappa^2 \varepsilon(x,y,z) E = 0, \qquad (1)$$

where $\kappa = 2\pi/\lambda$ is the wavenumber, λ is the wavelength characteristic of the free space, $\varepsilon(x,y,z)$ is the dielectric constant. Note, that the dielectric constant equal to square of the refractive index of medium $n(x,y,z)$.

Equation (1) for weakly inhomogeneous media (i.e. such that $\Delta n/2\pi n \ll 1$ for distances of the order of λ) in a paraxial approximation [2] can be reduced to the equivalent Schrodinger equation for the reduced field $\Psi(x,y,z) = n_0^{1/2} E(x,y,z) \exp(-i\kappa \int_0^z n_0 dz)$

$$\frac{i}{\kappa} \frac{\partial \Psi}{\partial \xi} = -\frac{1}{2\kappa^2}\left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2}\right) + \frac{1}{2}\left(n_0^2 - n^2\right)\Psi \qquad (2)$$

where $n_0 = n(0,0,z)$ is the refractive index on the z axis of medium, $\xi = \int_0^z n_0^{-1} dz$.

In equation (2) time takes the part of the longitudinal coordinate ξ, potential takes the part of the refractive index $V = (1/2)*(n_0^2 - n^2)$, and in place of Planck's constant $\hbar$ we have the free space wavelength. The methods of quantum mechanics for the solution of that equation can be used. For example, in [3] the integrals-of-motion method, the coherent states methods and the density matrix formalism are used for the description of paraxial optical beams propagating in weakly inhomogeneous graded index media.

3 Rays and modes in the Earth-Ionosphere waveguide

Let us consider as a model of Earth-Ionosphere waveguide an inhomogeneous waveguide with an asymmetric distribution of dielectric constant:

$$\mathcal{E}(x,y,z) = \mathcal{E}_0 - \omega^2(z)x^2 - \frac{g^2}{x^2}, \quad x > 0 , \tag{3}$$

where $\mathcal{E}_0$ is the dielectric constant on the axis of waveguide, ω is the gradient parameter, g is the parameter describing an asymmetric distribution of $\mathcal{E}$,x is the coordinate in the transverse plane,z is the longitudinal coordinate.
A such profile of dielectric constant is most close to real [1].Besides equation (2) with a such potential have exact solutions.Solutions in the form of wave packets describing rays and modes in the waveguide (3) are of practical interest.Evident ray solution in inhomogeneous waveguide using the coherent states methods may be obtained:

$$\Psi_\alpha(x,\xi) = B\cdot\exp\left(\frac{ik\dot{\mu}}{\mu}x^2 + \frac{\mu^*}{2\mu}\alpha^2\right)J_a\left(\frac{x\alpha(2k)^{1/2}}{\mu}\right),$$
$$B = \frac{|\alpha^2|^{a/2}}{\alpha^a\left(I_a(|\alpha^2|)\right)^{1/2}}\left(\frac{2kx}{\mu^2}\right)^{1/2}, \tag{4}$$

where a function $\mu(\xi)$ is the solution of equation $\ddot{\mu}+\omega^2(\xi)\mu = 0$;
$\alpha = \sqrt{k\omega/2}\,x_0 + i\sqrt{k/2\omega}\,\sin\varphi_0$; x_0 , φ_0
are an initial coordinate and an initial angle,accordingly,
$a = (1/2)\cdot(1+8gk^2)^{1/2}$, $\tilde{x}_0 = (2g/\omega^2)^{1/4}$ is an coordinate of waveguide axis.Here $J_a(x)$ is the Bessel function, $I_\alpha(x)$ is the modified Bessel function.
By direct substitution may be convinced,that the expression (4) is the solution of equation (2).In the case of longitudinal homogeneous waveguide a function μ is equal to $\mu = \omega^{-1/2}\exp(i\omega z - i\chi)$.
The solution (4) is the generating function for the waveguides modes $\Psi_n(x,z)$,which across the Laguerre polynomial expressed.

4 Computation of trajectory and and width of beam

Trajectories of rays may be calculated from the integral:

$$\langle x\rangle_\alpha = \int_0^\infty \Psi_\alpha\, x\, \Psi_\alpha\, dx \tag{5}$$

Analogically the width of beam is determined

$$\Delta x_\alpha^2 = \langle x^2\rangle - \langle x\rangle^2 \tag{6}$$

Let us consider a behaviour of these values in the longitudinal homogeneous waveguide (3).For the simplicity we consider the case when a = 1/2 ,i.e. when the earth surface is a reflecting wall.
For the intensity of beam we obtain the expression

$$\left|\Psi_\alpha\right|^2=\frac{\sqrt{\kappa\omega}}{\sqrt{\pi}\,\mathrm{sh}|\alpha|^2}\exp(\varkappa^2-\delta^2)\exp(-\kappa\omega x^2)\left[\mathrm{ch}\left(2\sqrt{2\kappa\omega}\,\delta x\right)-\cos\left(2\sqrt{2\kappa\omega}\,\varkappa x\right)\right] \tag{7}$$

where $\delta^2+\varkappa^2=|\alpha|^2$, $\delta=\sqrt{\frac{\kappa\omega}{2}}\,x_0\cos\omega z+\sqrt{\frac{\kappa}{2\omega}}\sin\varphi_0\sin\omega z$,

$\varkappa=\sqrt{\frac{\kappa\omega}{2}}\,x_0\sin\omega z-\sqrt{\frac{\kappa}{2\omega}}\sin\varphi_0\cos\omega z$.

Substituting the function (4) to the integral (5) we obtain the evident expression for trajectory of beam

$$\langle x\rangle_\alpha=\frac{1}{\sqrt{2\kappa\omega}\,\mathrm{sh}|\alpha|^2}\left(e^{|\alpha|^2}\delta\,\mathrm{erf}(\sqrt{2}\,\delta)+e^{-|\alpha|^2}\varkappa\,\mathrm{erfi}(\sqrt{2}\,\varkappa)\right) \tag{8}$$

where $\mathrm{erf}(x)$ is the error function, $\mathrm{erfi}(x)$ is the error function with an imaginary argument.
The square of beam width in the dependence on the longitudinal axis z changes in the following way

$$\Delta x_\alpha^2=\frac{1}{\kappa\omega}\left[\frac{1}{2}+|\alpha|^2\mathrm{cth}|\alpha|^2+\delta^2-\varkappa^2\right]-\langle x\rangle_\alpha^2 \tag{9}$$

Below the calculation results of beams trajectory using the expressions (8) and (9) for $\omega=1.25*10^{-5}\ m^{-1}$ and $\lambda = 30$ m are presented.In Fig. the trajectories of beams with different initial values of height x_0 over the earth surface at given initial angle to the horizon $\varphi_0=2^0$ and an initial width $\Delta x_0 \simeq 1/\sqrt{2\kappa\omega} = 437$ m are shown (1 - x_0 = 200 m;2- x_0 = 2200 m; x_{max} = 3 km, z_{max} = 800 km).

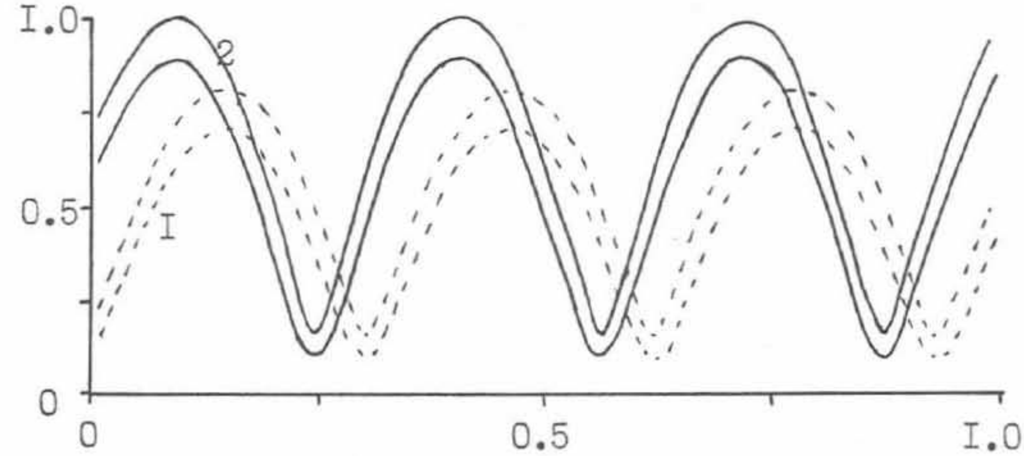

Fig.1

It is seen from figure,that the width of beams practically is not changed in the process of propagation.Period of beam trajectory oscillation does not depends on the initial height of source and makes up $L=\pi n_0/\omega \simeq$ 250 km.Amplitude of oscillation or the maximum height h_{max} of ray increases with a growth of initial height of source.Maximum height h_{max} increases with a growth of initial angle φ_0 .Note that the heights h_{max} for the beams with initial angles of opposite sign φ_0 and $-\varphi_0$ are the same.The period of oscillation L is decreased twice when the gradient parameter ω is increased twice (Fig.2).Amplitude of trajectory h_{max} in this case is decreased.Note that the expression $\int_0^\infty|\Psi_\alpha(x,z)|^2dx = 1$ is valid for all values of z, i.e. the functions $\Psi_\alpha(x,z)$ form a complete set on the interval $0 < x < \infty$.

5 Discussion

Results obtained may be useful for solution of re-esrtablishment problem of dielectric constant in Earth-Ionosphere waveguide. In particular the gradient parameter determines the oscillation period L of rays. Methods supposed allows to calculate an energy transformation between the modes on different inhomogenities, in particular, at the joint of waveguides "day-night".

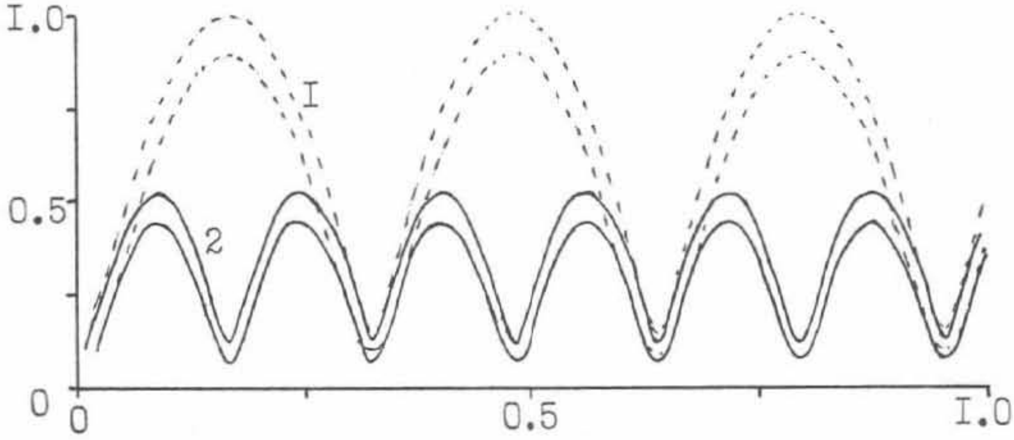

Fig.2. Trajectories of beams with the same values of x_0 = 20 m and $\varphi_0 = 2.5^\circ$, at different gradient parameters ω: 1 – ω = $1.25*10^{-5}$ m^{-1}; 2 – $\omega = 2.5*10^{-5}$ m^{-1}; $x_{max} = 3.92*10^3$ m; z_{max} = $8*10^5$ m.

Note that the paraxial approximation is valid only for the small angles φ_0 or the small heights x_0 as compared with the period L. Therefore the influence of nonparaxiality it is necessary to take into account for investigation of beam propagation at large angles to horizon. Nonparaxiality leads to the dependence of rays oscillation frequency on their amplitude and to the modulation of rays amplitude with an period larger than L, and besides these effects are accumulated with a distant.

Note that the results obtained may be continued on the case of homogeneous medium. It is known [4], that the localized wave beams in the homogeneous medium also exist.

Thus the wave beams which are not divergenced in the process of propagation in the Earth-Ionosphere waveguide are finded. The considered beams posses by optimal parameters and for transmission of radiation on the large distances may be used. The results obtained may be helpful also in the calculations of more complicated fields using the summing up method. Results obtained easily may be continued on the case of medium with absorption or amplification as it was made in paper [5] for optical waveguide.

References

1 A.V.Gurevich, E.E.Tsedilina. Superdistant propagation of short radiowaves. Nauka, Moscow, 1979.

2 J.A.Arnaud. Beam and Fiber Optics. New York, 1976.

3 S.G.Krivoshlykov, N.I.Petrov, I.N.Sisakyan. Opt. and Quant. Electr., 18, 253-264, 1986.

4. R.W.Ziolkowski. Phys. Rev. A., 39, N4, 2005-2033, 1989.

5 S.G.Krivoshlykov, N.I.Petrov, I.N.Sisakyan. Kvant. Electron., 13, N7, 1424-1436, 1986.