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RADIATION MODE EXPANSIONS OF FIELDS IN THE HALF SPACE PARTITIONED BY A CONDUCTOR-BACKED DIELECTRIC LAYER

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Abstract - Radiation modes are defined for the half space partitioned by a conductor-backed dielectric layer. The modes have orthogonality properties of the Dirac delta type and constitute the basis modes in terms of which three-dimensional fields in the layered half space can be expanded uniquely. The specific functional expressions of radiation modes are given together with the specific mode expansion expressions of fields. Also presented are the equations for obtaining the expansion coefficients when the source field distribution is given.

1. Introduction

The problem of radiation from source fields in the half space partitioned by a conductor-grounded dielectric substrate is a problem typical of radiation from microstrip patch antennas, except that the dimensions of ground plane and dielectric substrate used in practical antennas are finite. To obtain the radiated field from microstrip antennas, it is necessary first to get the source field distribution and then to calculate the field radiated into the half space. It is usually not so easy to obtain the rigorous source field distribution of microstrip patch antennas. Theoretically, however, it is also not so easy to calculate rigorously the field radiated into the half space with the dielectric substrate layer, even if the rigorous source field distribution is given. This is because conventionally the radiated fields are calculated using integral representations involving source fields and appropriate Green's functions, the exact expressions of which usually have very complicated form and require very time-consuming computation.

This paper presents a new approach to the problem of calculating radiated fields into the half space partitioned by a conductor-grounded dielectric layer, a typical problem encountered in calculating radiation from microstrip antennas. In this method, fields are expressed in terms of spectral integrals of orthogonal modes, and Green's functions are not used. The fields of orthogonal modes are determined so that boundary conditions on the interface between the dielectric substrate region and the air region as well as those on the ground plane are satisfied. The expressions relating mode expansion coefficients with the source distribution of fields can be easily derived owing to the orthogonality properties of the modes. The basic idea of the present method is the same as that used in radiation mode expansions of fields in the three-dimensional space[1].

2. General expressions of radiation mode fields

Consider the half space of the region $y \geq 0$ shown in Fig.1, where the region $d \geq y \geq 0$ is the dielectric substrate of permittivity ε_1 and the plane $y = 0$ is grounded, or has conducting boundary condition.

We give general expressions of fields of radiation mode p as

$$\mathbf{E}_p = (\mathbf{e}_{tp} + \mathbf{i}_z e_{zp})e^{-j\beta_p z} = \mathbf{e}_p e^{-j\beta_p z} \tag{1}$$

$$\mathbf{H}_p = (\mathbf{h}_{tp} + \mathbf{i}_z h_{zp})e^{-j\beta_p z} = \mathbf{h}_p e^{-j\beta_p z} \tag{2}$$

where the subscript t stands for "transverse" and

$$\mathbf{e}_p = \tilde{\mathbf{e}}_p e^{-j\alpha_p x} = (\tilde{\mathbf{e}}_{tp} + \mathbf{i}_z \tilde{e}_{zp})e^{-j\alpha_p x} \tag{3}$$

$$\mathbf{h}_p = \tilde{\mathbf{h}}_p e^{-j\alpha_p x} = (\tilde{\mathbf{h}}_{tp} + \mathbf{i}_z \tilde{h}_{zp})e^{-j\alpha_p x} \tag{4}$$

with $\tilde{e}_{yp}$ and $\tilde{h}_{yp}$ satisfying the equation

$$\left(\frac{\partial^2}{\partial y^2} + \gamma_{pi}^2\right)\left(\begin{array}{c} \tilde{e}_{yp} \\ \tilde{h}_{yp} \end{array}\right) = 0 \tag{5}$$

where

$$\gamma_{pi}^2 = k_i^2 - \xi_{pi}^2 = -\bar{\gamma}_{pi}^2 \qquad \xi_{pi}^2 = \alpha_p^2 + \beta_p^2 \tag{6}$$

Two types of independent modes always exist for one spectrum. We choose as one type the E-type mode for which $h_y = 0$ and as the other type the H-type mode for which $e_y = 0$. For brevity, we direct our attention only to the E-type modes hereafter. For each type mode, we have two different kinds of modes depending upon the range of the spectrum. One is for the spectrum γ_{0p} being imaginary number($k_0 < \xi_{0p} < k_1$) and the other is for γ_{0p} being real number(ξ_{0p} is imaginary number or $0 < \xi_{0p} < k_0$). The former kind of mode is the one which corresponds to the dielectric slab waveguide mode and its field is trapped around the substrate region. We call this mode the substrate mode. On the other hand, the latter kind of mode is the one which includes both the fields coming toward the substrate and the fields going away from the substrate. We call this mode the space mode.

3. Substrate mode

We define the orthonormalization condition for E-type mode to be

$$\frac{1}{2}\int_S (\mathbf{e}_{tm} \times \mathbf{h}_{tn}^*) \cdot \mathbf{i}_z ds = \frac{\beta_n}{|\beta_n|}\delta_{mn}\delta(\alpha_m - \alpha_n) \tag{7}$$

where S is a plane perpendicular to the z-axis, the subscripts m and n are the integers

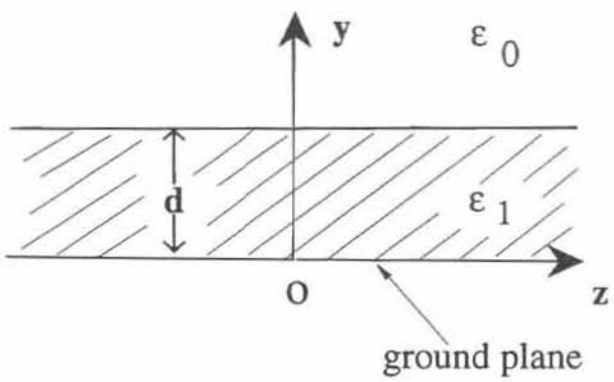

Fig.1. Geometry of the problem.

corresponding to the dielectric slab waveguide mode number, * denotes complex conjugate, and δ_{mn} and $\delta(\cdot)$ stand for the Kronecker's delta and Dirac delta functions, respectively. The function $\tilde{e}_{yn}$ for E type mode satisfying (7) and the boundary conditions on $y = 0$ and $y = d$ planes are determined as

$$\tilde{e}_{yn} = C_n^e e^{-\bar{\gamma}_{0n}(y-d)} \qquad , y \geq d \tag{8}$$

$$\tilde{e}_{yn} = C_n^e \frac{\varepsilon_0 cos\gamma_{1n} y}{\varepsilon_1 cos\gamma_{1n} d} \quad , d \geq y \geq 0 \tag{9}$$

where

$$tan\gamma_{1n} d = \frac{\varepsilon_1 \bar{\gamma}_{0n}}{\varepsilon_0 \gamma_{1n}} \tag{10}$$

$$C_n^e = \frac{\sqrt{2}\xi_{0n}}{\sqrt{\omega\varepsilon_0\pi|\beta_n|}} \left(\frac{\varepsilon_0 d}{\varepsilon_1 cos^2\gamma_{1n} d} + \frac{k_1^2 - k_0^2}{\bar{\gamma}_{0n}\gamma_{1n}^2} \right)^{-\frac{1}{2}} \tag{11}$$

With $\tilde{e}_{yn}$ thus given, all the field components can be determined.

4. Space mode

The function $\tilde{e}_{yp}$ of the E-type space mode satisfying orthonormalization condition

$$\frac{1}{2}\int_S (\mathbf{e}_{tp} \times \mathbf{h}_{tq}^*) \cdot \mathbf{i}_z ds = \frac{\beta_q^*}{|\beta_q|}\delta(\alpha_p - \alpha_q)\delta(\gamma_{0p} - \gamma_{0q}) \tag{12}$$

and the appropriate boundary conditions are determined as

$$\tilde{e}_{yp} = F_p^e[e^{-j\gamma_{0p}(y-d)} + B_p^e e^{j\gamma_{0p}(y-d)}] \qquad , y \geq d \tag{13}$$

$$\tilde{e}_{yp} = F_p^e \frac{2\varepsilon_0\gamma_{0p} cos\gamma_{1p} y}{\gamma_{0p}\varepsilon_1 cos\gamma_{1p} d - j\gamma_{1p}\varepsilon_0 sin\gamma_{1p} d} \qquad , d \geq y \geq 0 \tag{14}$$

with

$$B_p^e = \frac{\gamma_{0p}\varepsilon_1 cos\gamma_{1p} d + j\gamma_{1p}\varepsilon_0 sin\gamma_{1p} d}{\gamma_{0p}\varepsilon_1 cos\gamma_{1p} d - j\gamma_{1p}\varepsilon_0 sin\gamma_{1p} d} \qquad F_p^e = \frac{\xi_{0p}}{\pi} \cdot \frac{1}{\sqrt{2\omega\varepsilon_0|\beta_p|}} \tag{15}$$

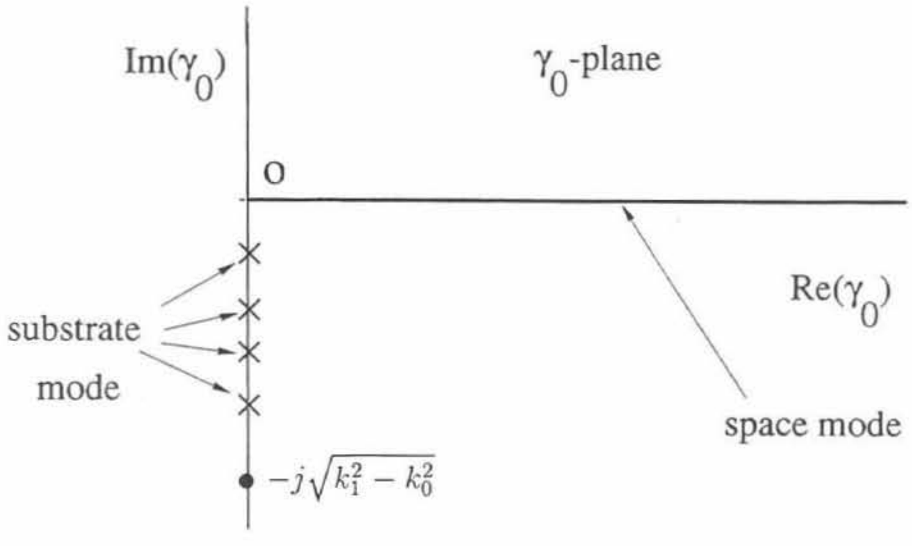

Fig.2. γ_0 - spectrum for the radiation modes.

With $\tilde{e}_{yp}$ thus given, all the field components can be determined.

5. Radiation from surface distributions of source fields

Electric field in the half space $y \geq 0$, $\mathbf{E}$, is generally expanded in terms of radiation modes as follows:

$$\begin{aligned}\mathbf{E} \;=\; & \int_{-\infty}^{\infty} d\alpha e^{-j\alpha x} \sum_{i=e,h} \left\{ \left[\sum_m a_m^{i(+)} \tilde{\mathbf{e}}_m^{i(+)} + \int_0^{\infty} d\gamma_0 b^{i(+)} \tilde{\mathbf{e}}^{i(+)}(\gamma_0) \right] e^{-j\beta z} \right. \\ + & \left. \left[\sum_m a_m^{i(-)} \tilde{\mathbf{e}}_m^{i(-)} + \int_0^{\infty} d\gamma_0 b^{i(-)} \tilde{\mathbf{e}}^{i(-)}(\gamma_0) \right] e^{j\beta z} \right\} \end{aligned} \quad (16)$$

The mode expansion form for magnetic field is given similarly. The terms expressed in terms of the single spectral integral together with the summation over m and those in terms of the double spectral integral correspond, respectively, to the substrate mode expansion and the space mode expansion. The symbols $(+)$ and $(-)$ express the field propagation direction with respect to the z-axis, the plus and minus directions, respectively. The γ_0 - spectrum range for both modes is illustrated in Fig.2 where the bold line represents the space mode range and the cross symbol $\times$ stands for the substrate mode location.

Assume that the source fields $\mathbf{E}_0$ and $\mathbf{H}_0$ are given on a surface S_0. Then, the expansion coefficient $a_m^{(i)(\pm)}$ in (16) are obtained, by applying Lorentz reciprocity theorem, to be

$$a_m^{(i)(\pm)} = -\frac{1}{4} \int_{S_0} (\mathbf{E}_m^{(\pm)*} \times \mathbf{H}^0 + \mathbf{E}^0 \times \mathbf{H}_m^{(\pm)*}) \cdot \mathbf{i}_n ds \quad (17)$$

and $b_m^{(i)(\pm)}$ in (16) are obtained to be

$$b^{(i)(\pm)} = -\frac{1}{4} \int_{S_0} (\mathbf{E}^{(\pm)}(\gamma_0)^* \times \mathbf{H}^0 + \mathbf{E}^0 \times \mathbf{H}^{(\pm)}(\gamma_0)^*) \cdot \mathbf{i}_n ds \quad (18)$$

when β is real, and

$$b^{(i)(\pm)} = -\frac{j}{4} \int_{S_0} (\mathbf{E}^{(\mp)}(\gamma_0)^* \times \mathbf{H}^0 + \mathbf{E}^0 \times \mathbf{H}^{(\mp)}(\gamma_0)^*) \cdot \mathbf{i}_n ds \quad (19)$$

when β is imaginary, where $\mathbf{i}_n$ is the unit normal to the surface S_0 and the upper or lower sign is to be taken throughout. The far radiated field expression is easily obtained from (16) by applying the conventional method of steepest descent.

6. Conclusion

A method of calculating radiation fields in the half space with a dielectric substrate backed by a conducting sheet is proposed. The efficiency of numerical calculation by means of this method is expected to be very high because the method does not use Green's functions that generally require very time-consuming computation. The next step of the study is practical application of the method to the calculation of radiation from microstrip patch antennas.

[1] N. Morita, "Expressions of fields in terms of radiation modes in the three-dimensional space", IEICE Tech. Rep., 91, 32, pp.29-34, May 1991.