

CUTOFF FREQUENCIES OF RECTANGULAR-COAXIAL LINE WITH FINITE THICKNESS AND OFFSET INNER CONDUCTOR

P. J. Luypaert and N. Wang

ESAT/TELEMIC, Dept. of Electrical Engineering
Katholieke Universiteit Leuven, B-3001 Leuven, Belgium

INTRODUCTION

Rectangular-coaxial transmission lines are being widely used in such areas as radio frequency radiation dosimetry, biological effect, EMC testing, calibration of radiation survey meters and electric field probes[1][2]. In all these applications it is desirable that only TEM-mode can be propagated. However, the appearance of higher order modes limits the usable TEM-mode frequency range, therefore the determination of cutoff frequencies of higher modes are of great importance. Many different methods have been used to determine the cutoff frequencies of rectangular coaxial transmission lines[3]-[5]. All these analysis are only concentrated on transmission types with center inner conductor of very thin thickness (rectangular stripes). The data for arbitrary thickness and offset inner conductor are not available and neglected in the literatures. This type of transmission line has been used in some EMI measurement system in which the device under test (DUT) is located between the inner and outer conductors and in many cases the inner conductor is offset in order to accommodate larger DUT's [6].

The purpose of this paper is to determine the cutoff frequencies of higher order modes of rectangular-coaxial lines with offset and finite thickness inner conductors. The method of lines[7], which is a powerful method for hybrid-mode analysis of planar structures, is extended to perform the analysis and calculations. The proposed technique for this problem has an easy formulation and simple convergence behavior as well as fast algorithm with small memory requirement.

The validity of the method is verified by the comparison of our results with the available theoretical and measured data for rectangular stripes.

THEORY

The cross section of the rectangular-coaxial lines investigated in this paper is that of Fig.1. The finite thickness inner conductor is arbitrarily situated but is parallel to the x axis. We assume the air-filled waveguide's walls and the inner conductor are perfectly conducting. The higher order modes of this structure could be TE or TM modes. For TE modes, the electromagnetic field in each homogeneous region can be described by longitudinal field components H_z , which satisfies the Helmholtz equation

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (k^2 - \beta^2)H_z = 0 \quad (1)$$

and the boundary conditions.

Discretizing the x-variable in the partial differential equation (1), a system of coupled ordinary differential equations are obtained. These equations can be transformed into a system of uncoupled equations by the eigenvalue techniques [7] and yield:

$$\frac{d^2 \bar{H}_z}{dy^2} + \text{diag}[K^2] \bar{H}_z = 0 \quad (2)$$

where \bar{H}_z is the transformed longitudinal field components H_z and $\text{diag}[K^2]$ is the diagonal eigenvalue matrix of the region involved.

With the general solution to (2) which given the relations between \bar{H}_z and its normal derivative in two continuity plane, \bar{H}_z can be transformed from one interface into another. Considering the boundary conditions, the relations between the transformed fields of region 1 at interface A and region 3 at interface B are given by:

$$\begin{bmatrix} \bar{H}_{zA1} \\ \bar{H}_{zB3} \end{bmatrix} = \begin{bmatrix} \bar{Y}_{11} & \\ & \bar{Y}_{22} \end{bmatrix} \begin{bmatrix} \bar{E}_{xA1} \\ \bar{E}_{xB3} \end{bmatrix} \quad (3)$$

For the region 2, The transformed fields at the interfaces can be expressed as:

$$\begin{bmatrix} \bar{H}_{zA2} \\ \bar{H}_{zB2} \end{bmatrix}_{\text{slot}} = \begin{bmatrix} \bar{Y}'_{11} & \bar{Y}'_{12} \\ \bar{Y}'_{21} & \bar{Y}'_{22} \end{bmatrix}_{\text{slot}} \begin{bmatrix} \bar{E}_{xA2} \\ \bar{E}_{xB2} \end{bmatrix}_{\text{slot}} \quad (4)$$

In order to match the field at the interface A and B, the transformed fields of each region have to transform back into the original domain with the corresponding transformation matrices. since the tangential electric fields vanish on the metallic strip, a reduce matrix equation which is inverse transformation of (3) is obtained:

$$\begin{bmatrix} H_{zA1} \\ H_{zB3} \end{bmatrix}_{\text{slot}} = \begin{bmatrix} Y_{11} & \\ & Y_{22} \end{bmatrix}_{\text{slot}} \begin{bmatrix} E_{xA1} \\ E_{xB3} \end{bmatrix}_{\text{slot}} \quad (5)$$

and the inverse transformation of (4) is

$$\begin{bmatrix} H_{zA2} \\ H_{zB2} \end{bmatrix}_{\text{slot}} = \begin{bmatrix} Y'_{11} & Y'_{12} \\ Y'_{21} & Y'_{22} \end{bmatrix}_{\text{slot}} \begin{bmatrix} E_{xA2} \\ E_{xB2} \end{bmatrix}_{\text{slot}} \quad (6)$$

Follow the (5) and (6), we have the field matching in the spatial domain:

$$\begin{bmatrix} Y'_{11} - Y_{11} & Y'_{12} \\ Y'_{21} & Y'_{22} - Y_{22} \end{bmatrix}_{\text{slot}} \begin{bmatrix} E_{xA2} \\ E_{xB2} \end{bmatrix}_{\text{slot}} = 0 \quad (7)$$

From the nontrivial solution of this homogeneous equation the characteristics of the field and the cutoff frequencies of TE modes are obtained. In the same way, TH modes can be determined by a equation similar to (7).

RESULTS

It has been shown that the first higher order mode propagating in the structure is determined by TE modes [5]. We take equal dimension of a and b to show the calculation. In this case TE_{01} mode is always the first higher mode. The normalized cutoff wavelength changing with the dimensions and location of the inner conductor are given in

Fig.2-4. For the same inner conductor it is found that the normalized cutoff wavelength increases with the decrease of b_1 and a_2 (Fig.2-3). That means the cutoff frequencies of rectangular-coaxial line with offset inner conductor is lower than that with center inner conductor. By increasing the length or the thickness of inner conductor, the same tendency can also be demonstrated (Fig.3). The physical explanations for these are that when dimension of inner conductor are relative larger or it is placed closer to the walls of waveguide, the capacitive coupling between the inner conductor and side walls are increasing.

In order to examine the validity of the method used in this paper, we have calculated the normal cutoff wavelength of rectangular strips and compared with the theoretical results of [3] and [5] and experimental results of [3] the agreement are excellent as shown in table 1.

CONCLUSION

In this paper, the method of lines has been extended to analysis the rectangular-coaxial lines with offset and finite thickness inner conductors. The cutoff frequencies of first higher-order mode propagating in this structure has been calculated and discussed. Valuable results concerning the limit of useable TEM-mode frequency range has been obtained. The accuracy of results has been examined by comparing with published theoretical and experimental data for rectangular strips.

REFERENCE

- [1] B. S. Yarman and H. J. Carlin, "A simplified "real frequency" technique applied to broad-band multistage microwave amplifiers," IEEE Trans. MTT. vol.30, pp.2216-2222, Dec. 1982.
- [2] W. T. Joines, C. F. Blackman, and M. A. Hollis, "Broadening of the RF power-density window for calcium-ion efflux from brain tissue," IEEE Trans. BEM. vol.28, pp.568-573, Aug. 1981.
- [3] R. Lampe, P. Klock, D. Tanner, and P. Mayes, "Analysis and experiment concerning the cutoff frequencies of rectangular striplines," IEEE Trans. MTT. vol.34, pp.898-899, Aug. 1986.
- [4] C. M. Weil and L. Gruner, "High-order mode cutoff in rectangular striplines," IEEE Trans. MTT. vol.32, pp.638-641, June 1984.
- [5] L. Gurner, "Estimating rectangular coax cutoff," Microwave J. vol.22, pp.88-92, Apr. 1979.
- [6] J. C. Tippet, D. C. Chang, "Characteristics impedance of a rectangular coaxial line with offset inner conductor," IEEE Trans. MTT. vol.26, pp.876-883, Nov. 1978.
- [7] F. J. Schmuckle and R. Pregla, "The Method of lines for the analysis of planar waveguides with finite metallization thickness," IEEE Trans. MTT. vol.39, pp.107-110. Jan. 1991.

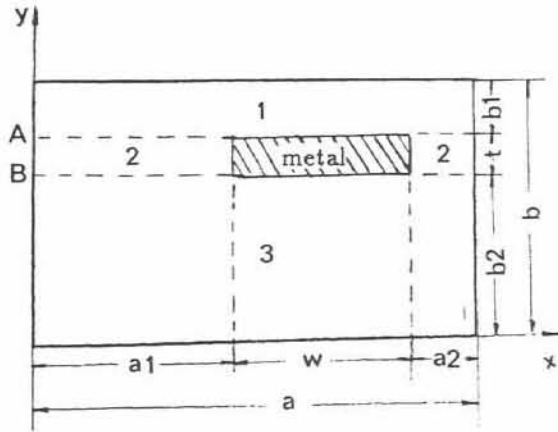


Fig.1 The cross section of the rectangular coaxial line.

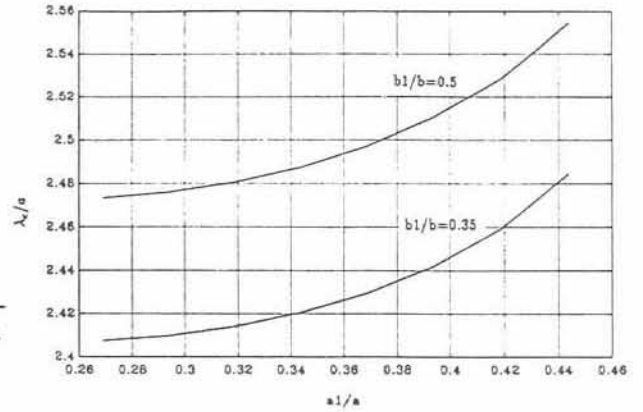


Fig.2 Normal cutoff wavelength versus the location of inner conductor.
($a=b$, $b_1=b_2$, $t=0$)

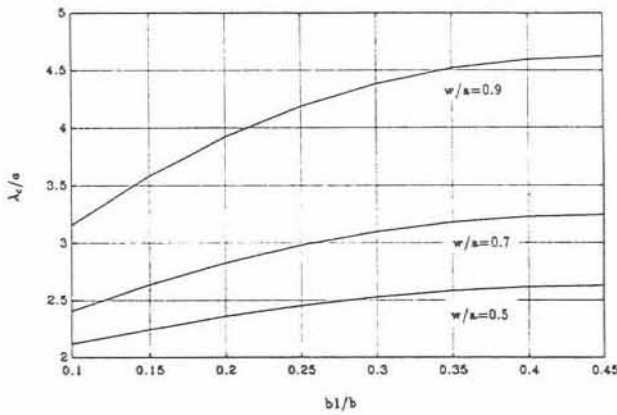


Fig.3 Normal cutoff wavelength versus location of inner conductor with different dimension.
($a=b$, $a_1=a_2$, $t/b=0.01$)

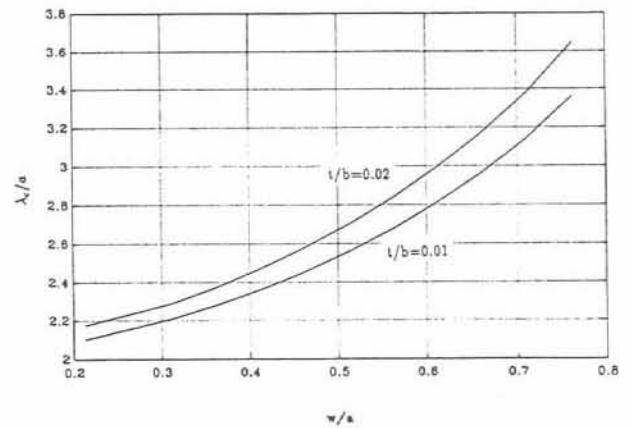


Fig.4 Normal cutoff wavelength versus dimension of the inner conductor.
($a=b$, $a_1=a_2$, $b_1/b=0.3$)

Table 1. Comparison of normal cutoff wavelength for rectangular strips.

w/a for $b_1 = b_2$	λ_c/a			
	Method of lines	Fourier expansion [5]	Method of Moments [3]	Experiment [3]
.20	2.066	2.065	2.062	2.06
.40	2.278	2.278	2.275	2.26
.60	2.662	2.665	2.666	2.63
.80	3.330	3.331	3.331	3.22