

# 3-D FDTD Simulation of Reverse Refraction and Refocusing of Negative-Index Metamaterials

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## 1. Introduction

Left-handed, negative index metamaterials (NIMs) have been shown to exhibit several interesting electromagnetic properties and allow greater control over electromagnetic waves than has been previously possible. Vaselego in 1960's predicted, although NIMs were not available at that time, some interesting properties of NIMs such as the reversals of Snell's law, reversed Doppler effect, and the phase velocity pointing opposite to the direction of the group velocity. Following Pendry's initial research on plasmons in conducting wires and loops, Smith demonstrated experimentally the first NIMs made of split-ring resonators (SRRs) and conducting wires arranged in a periodic array. Since then, many experimental and theoretical attempts were made to investigate the unique electromagnetic properties of NIMs.

Most man-made versions of NIMs are currently constructed with dielectric or metallic materials in periodic structures working at microwave frequencies although nano-NIM structures are being attempted for operations at optical frequencies. NIMs exhibit negative permittivity and permeability within particular frequency bands and only in certain spatial directions. They are scalable and can operate over a wide range of frequencies. These qualities are very desirable for a variety of applications such as radar, communication devices, and sensors.

Due to the negative index of the metamaterials, electromagnetic waves incident to the planar NIM slab may refract to the same side of the interface normal as predicted by Snell's law,  $n_r \sin(\theta_r) = n_l \sin(\theta_l)$ , where  $n_r$  and  $n_l$  are the indices of refraction in the right-handed and left-handed regions, respectively and  $\theta_r$  and  $\theta_l$  are the refraction angles with respect to the interface normal. Snell's law also predicts that the NIM slab can refocus electromagnetic waves emanating from a point source. Just as most optical systems employ prisms and lens made of high index optically transparent substances, negative index of refraction of NIMs make possible microwave lens and prisms for unique applications at microwave frequency ranges.

Traditionally, the analysis of the electromagnetic properties of NIMs relied heavily on the unique properties of infinite periodic structures, similar to that used to describe crystal diffraction. However, for real applications, the finite dimensions and boundaries have to be included in the analysis to account for their impact on NIM characteristics. To accomplish this requires a direct numerical simulation of the finite NIM structures.

We have used a Finite-Difference Time-Domain (FDTD) numerical code to characterize NIM structures to analyze the refraction of electromagnetic waves from NIM prisms and the refocusing of electromagnetic waves through finite NIM structures at microwave frequencies.

## 2. Numerical Verification of Reverse Refraction and Refocusing of Negative Index Metamaterials

We have numerically investigated reverse refraction from a NIM prism and refocusing of electromagnetic waves from a slab of NIM structures. The NIMs are composed of a periodic array of split-ring resonators (SRRs) and an array of thin wires. A finite array of rectangular split ring resonators (SRRs) and wires are shown in Fig. 1. SRRs are located on the top side of a circuit board

and the wires on the opposite side of the board. Every other board is a wires-only board to enhance the negative permittivity. The SRRs can produce an effective negative magnetic permeability and the thin wires an effective negative electric permittivity. Figure 2 shows the geometry of a SRR. The computational volume is terminated with uniaxial perfectly matched layers (UPMLs). The NIM wedge is sandwiched between 2 conducting planes with a planar current source (shown in figure 3) similar to a parallel plate waveguide. In order to implement the detailed 3-D structural parameters of the NIMs into the FDTD simulation, we have used large parallel computers at various High-Performance Computer (HPC) sites. Our codes employ parallel programming techniques based on OpenMP and the Message Passing Interface (MPI) so that very large numerical grids may be used to model electromagnetic phenomena in very large computational domains. Typical simulations may use from several hundred million to over a billion FDTD cells and require 16-64 processors to model NIM and electromagnetic band gap (EBG) structures in 3 dimensions.

Reverse refraction is investigated by studying a NIM prism made of 16 left-handed material (LHM) boards of varying length and stacked to form a 45 degree wedge. Figure 4 shows the transmission spectrum. A narrow left-handed band exists at about 13.4 GHz. Figure 5a shows the electric field in the NIM prism. Plane waves are incident from the left at a frequency of 13.25 GHz. Waves leaving the wedge undergo negative refraction since they are bent down from the normal (indicated by the dashed line) at about -34 degrees. This implies an index of refraction of about -.8 using Snell's Law. At frequencies where both the effective permeability and permittivity are negative, the index of refraction becomes negative and the group and phase velocities will have opposite signs. Snell's law requires that for negative index materials refracted waves be bent to the same side of the normal as the incident waves unlike right-handed materials (RHMs), which bend waves to the opposite side of the normal. The higher amplitude areas indicated in red over the circuit boards are indications of resonances in the SRR elements.

We have also investigated refocusing of electromagnetic waves through a NIM slab. The NIM lens is composed of 32 circuit boards having arrays of 14 by 12 SRRs, but with every other board having only wires. The computational domain for this case is 2000 by 350 by 640 cells with a grid spacing of .125 mm. A cylindrical wave is generated by a line source positioned 2 wavelengths away from the slab with a frequency of 13.4 GHz. Fig 5b shows the electric field after 30000 time steps, indicating refocused electromagnetic waves on the other side of the slab. Work is underway using a 2D geometry composed of both horizontal and vertical interlocking boards to form a 2D isotropic NIM structure, as opposed to the 1D structure employed here.

These types of large-scale simulations illustrate the power of the FDTD method when combined with parallel processing to study electromagnetic wave propagation in complex structures. They will be useful in designing and optimizing unit cell geometry, lattice parameters and determine stop-band, pass-bands of EBG and other metamaterial structures. The unusual characteristics of NIMs make them candidates for use in super lenses, cloaks of invisibility, and filters.

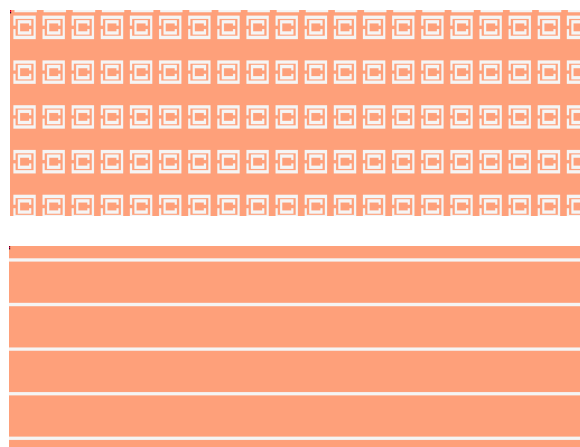


Figure 1 – Section of circuit board show an array of split-ring resonators and wires. Front side of the circuit board (top) has the split-ring resonators and the back side (bottom) has conducting wires.

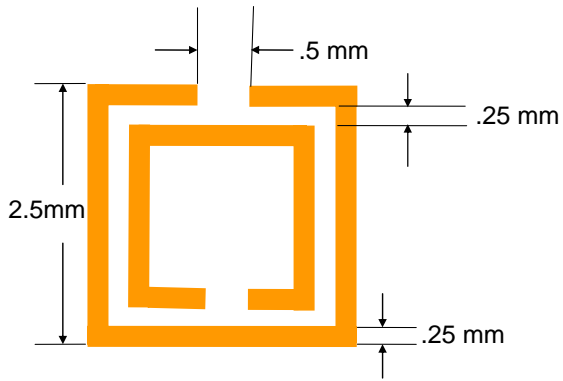


Figure 2 –Dimensions of the SRR used in this paper.

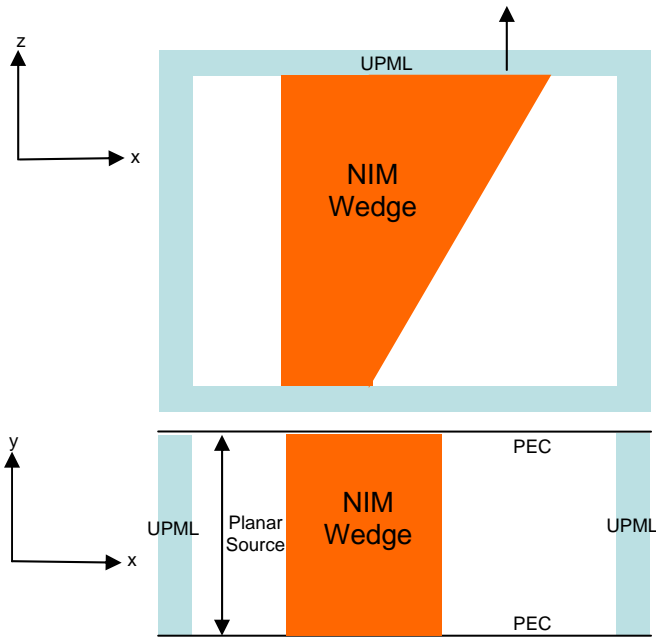


Figure 3 – Configuration for FDTD simulations

### Transmission Spectrum

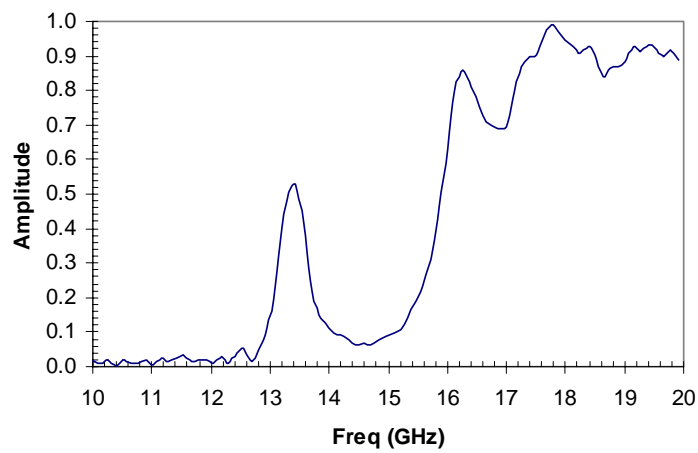


Figure 4 -Transmission spectrum of NIM slab

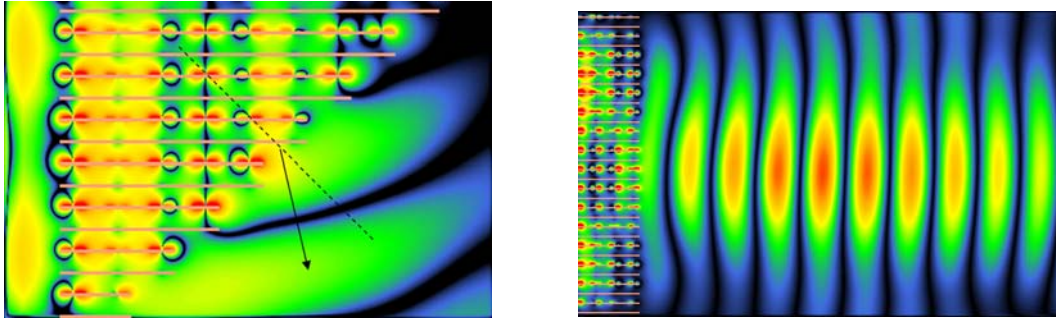


Figure 5a and 5b – (a) Negative refraction from a NIM wedge (b) Focusing from a flat NIM lens

### 3. Summary

We have numerically investigated reverse refraction from a NIM prism and refocusing of electromagnetic waves from a slab of NIM structures. In order to implement the detailed 3-D structural parameters of the NIMs into the FDTD simulation, we have used large parallel computers at various High-Performance Computer (HPC) sites. Our numerical results indicate that the SRRs and wires exhibit reverse refraction in a finite NIM prism since they are bent down from the normal unlike right handed materials (RHMs). We have proved numerically refocusing of electromagnetic waves through a NIM slab. These simulations illustrate the power of the FDTD method when combined with parallel processing to study electromagnetic wave propagation in complex structures.

### Acknowledgments

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### References

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