A New High Resolution DOA Method for Uniform Circular Array

Wang Yongliang Chen Hui Wan Shanhu
Key Research Lab, Wuhan Radar Academy, Wuhan, 430010, China
E-mail: baoxqing@public.wh.hb.cn

ABSTRACT

A new DOA estimation method is presented, which is used by the virtual array transformation and the improved spatial smoothing algorithm. The new method not only overcomes the weakness of the ambiguity of DOA estimation of arbitrary array, but also improves the abilities of resolution and de-correlation. It is proven to be effective by theoretical analyses and computer simulations. What is more, the method can improve the estimation and resolution of DOA under the condition of sparse practical array.

Ⅰ. INTRODUCTION

Direction-finding techniques based on eigendecomposition of the covariance matrix of the vector of the signals received by an array of sensors have received considerable attention since 80's. The MUSIC-like techniques typically provide asymptotically unbiased estimates and have proved effective in several applications. The non-uniform array can extend the aperture in the condition of the same sensor number in comparison with the uniform linear array (ULA). However, the non-uniform array has some faults. For example, the ambiguity of DOA estimation will be brought out and the technique of spatial smoothing can not be directly applied. However, the faults can be overcome by arranging the non-uniform array. A method of linear array introduced by Zoltowski *et al.* [1] appears to have some strict restrictions. Another method of arranging linear array is minimum redundant array introduced by S.Haykin [2]. It can not only overcome the ambiguity problem, but also improve the resolution. An attempt to generalize the spatial smoothing technique to arbitrary array geometries by using the interpolated idea is the main topic in [3] by B.Friedlander. The methods keep the aperture and the sensor number. The performance and accuracy of the third method are best among three methods. In this paper we propose a new method to solve the DOA problem of arbitrary array geometry.

Ⅱ. ARRAY SIGNAL MODEL

Consider an arbitrary array composed of m sensors. Let d narrowband plane waves, centered at frequency ω_0, impinge on the array. Using complex signal representation, the received signal at i th sensor can be expressed as

$$x_i = \sum_{k=1}^{d} g_{ik} e^{-j\omega_0 \tau_{ik}} s_k(t) + n_i(t) \qquad i = 1,2,\cdots,m \tag{1}$$

where g_{ik} is the complex response of the sensor to the kth wave front, $s_k(t)$ is the signal associated with the kth wave front, and $n_i(t)$ is the additive noise at the i th sensor. Rewriting (1) in vector notation, assuming for simplicity that the sensors are omnidirectional with unit gain, we obtain

$$X = AS(t) + N(t) \tag{2}$$

where $X(t) = [x_1(t), x_2(t), \cdots, x_m(t)]^T$ is $m \times 1$ vector of received signal, $A = [a(\theta_1), \cdots, a(\theta_d)]$ is the $m \times d$ matrix, and $a(\theta)$ is the steering vector of the array in the direction θ. Thus, the covariance matrix of the received signals can be obtained

$$R = E[X(t)X^H(t)] = AR_S A^H + R_N \tag{3}$$

In which, $R_S = E[S(t)S^H(t)]$ is the signal covariance matrix, $S(t) = [s_1(t), s_2(t), \cdots, s_d(t)]^T$ is a $d \times 1$ vector of the signal, and $R_N = E[N(t)N^H(t)]$ is noise covariance matrix. Note that R_S is diagonal when the signals are uncorrected, nondiagonal and nonsingular when the signals are partially correlated, and nondiagonal but singular when some signals are perfectly correlated (coherent).

III. VIRTUAL ARRAY TRANSFORMATION

In this section we describe a new method, which can be called the virtual array transformation (VAT) method. Let us divide the field of view of the array into sectors, then choose the step of angle of sector. Define the sector as the interval $[\theta_l, \theta_r]$, where θ_l is the left boundary, and θ_r is the right boundary. Assume that signals lie in the sector Θ. Thus, we have

$$\Theta = [\theta_l, \theta_l + \Delta\theta, \theta_l + 2\Delta\theta \cdots, \theta_r] \tag{4}$$

where $\Delta\theta$ is the step of angle. Then the steering vector of actual array can be obtaine

$$A = [a(\theta_l), a(\theta_l + \Delta\theta), a(\theta_l + 2\Delta\theta), \cdots, a(\theta_r)] \tag{5}$$

And the steering vector of virtual array is

$$\overline{A} = [\overline{a}(\theta_l), \overline{a}(\theta_l + \Delta\theta), \overline{a}(\theta_l + 2\Delta\theta), \cdots, \overline{a}(\theta_r)] \tag{6}$$

In other words A is the response of the real array to signals arriving from directions θ_r, and $\overline{A}$ is the response of the interpolated array to the same signals. The spacing of th virtual array sensors is a half wavelength. Then, there exists a constant matrix B between the actual array and the virtual array, which can be written as

$$BA = \overline{A} \Rightarrow B = \overline{A}A^{-1} \tag{7}$$

Assume that the actual data covariance matrix is $\hat{R}$, and the noise covariance matrix R_N. Hence, the data covariance matrix of the virtual array is

$$\overline{R} = B\hat{R}B^H = B(AR_SA^H + R_N)B^H = BAR_SA^HB^H + BR_NB^H \tag{8}$$

When the noise is white, we can obtain

$$BR_NB^H = \sigma^2 BB^H \tag{9}$$

From equation (8), we know that the white noise has become the color noise. Therefore, when the noise received is independence, the covariance matrix is

$$\hat{R} = \frac{1}{N}\sum_{i=1}^{N} X(i : L - N + i - 1) * X^H(i + 1 : L - N + i) \tag{10}$$

where, N is time-domain average time, L is snapshot and $X(i{:}j)$ is the data from i th to j th snapshot. The noise received is independence, the noise covariance matrix therefore equal to zero.

The transformation from $\hat{R}$ to $\overline{R}$ can be represented as $\overline{R} = F_1(\hat{R})$. A signal enhancement method presented in [4] by Cadzow *et al.* can be used in the case of low SNR. Assuming that the matrix of actual array is $\hat{R}$. The matrix $\hat{R}$ is Toplitz in the ideal condition. Therefore, $\hat{R}$ can be expressed by the vector of R_v.

$$r_v = \mathrm{vec}(\hat{R}) = AR_v \tag{11}$$

$\mathrm{vec}(\bullet)$ indicates transformation fro $\hat{R}$ to vector r_v, and $\mathrm{vec}^{-1}(\bullet)$ represents the revers transformation of $\mathrm{vec}(\bullet)$. The enhancement matrix $\hat{R}_A$ is

$$\hat{R}_A = vec^{-1}(P_A r_v) = A(A^HA)^{-1}A^H r_r \tag{12}$$

where P_A is the pseudo-revers A. The process can be expressed $\hat{R}_A = F_2(\hat{R})$.

$\hat{R}_r$ must be Toplitz in practical. Therefore, the covariance matrix can be expressed as

$$\hat{R}_{rA} = F_2(\overline{\hat{R}}) = F_2(F_1(\hat{R})) \tag{13}$$

Once the matrix $\hat{R}$ is obtained, the signal subspace $\hat{E}_S$ or noise subspace $\hat{E}_N$ can also be gotten. Then we can perform the DOA estimation the spectrum estimator $1/|\overline{a}^H(\theta)\hat{E}_N|^2$.

Ⅳ. Monte Carlo Simulations

In this section, we evaluate the performance of the VAT described above, using Monte Carlo simulations. And the method is compared with the Cramer-Rao bound of MUSIC presented by P.Stoica and A.Nehorai [5].

Experiment 1:The detecting the numbers of signals and de-correlation.

In this experiment we use a uniform circular array with 5 elements. The radius is 4λ and SNR=10dB. We use 10 Monte Carlo simulations. The bearings of signals are fixed at $5^\circ,10^\circ,15^\circ,23^\circ,28^\circ$ and 35°. Three signals of $5^\circ,15^\circ$ and 28° are coherent. The sector is $0^\circ \sim 40^\circ$. Fig.1 shows the performance of the virtual array with 16 elements and the spacing of elements is $\lambda/2$.

Experiment 2:The performance comparisons.

In this experiment we will compare the method C in [3] with the method proposed in this paper. We choose a uniform circular array with 8 elements. The radius is 3λ, SNR=10dB and snapshots 100. We use 10 Monte Carlo simulations. Two signals are coherent and their bearings are fixed at $5^\circ,25^\circ$. The sector is $0^\circ \sim 30^\circ$. The dotted line in Fig.2 shows the result of the method of B.Friedlander. The virtual array keeps the aperture $2r$, in which r is radius and the spacing of sensors is $2r/8$. The solid line in Fig.2 shows the result of the method of this paper. The virtual array has 16 elements and the spacing of elements is half wavelengt

Experiment 3:Statistics analysis.

In this experiment we use a uniform circular array (UCA) with 5 elements and a uniform linear array with 4 elements. The radius of UCA is 3λ, and the position vector of ULA is $X = 0.5\lambda[0,1,3,7]$. We use 100 Monte Carlo simulations and snapshots 100. The bearing signals are fixed at $5^\circ,15^\circ,25^\circ$. Two signals are coherent. The sector is $0^\circ \sim 30^\circ$. The virtual array with 8 elements is spaced $\lambda/2$ apart.

Fig.3 (a) shows the relation of probability and SNR. Fig.3 (b) shows the relation of mean square error (RMSE) and SNR. From Fig.3 (a)(b), we know the technique of signal enhancement can be applied under the condition of low SNR, but the precision will degrade with the SNR ascending.Fig.4 (a) shows the relation of probability and virtual sensors. The relation of RMSE and virtual sensor number is shown in Fig.4 (b). From Fig.4 (a)(b), we know the performance of VAT will degrade, when the sensor number is larger than three times sensor of actual array. With the sensor number increase, the probability of success will decrease gradually and the variance will increase apart from CRB.

Ⅴ. CONCLUSIONS

A new DOA estimation method is presented, which is used by the virtual array transformation and the improved spatial smoothing algorithm. The new method not only overcomes the weakness of the ambiguity of DOA estimation of arbitrary, but also improves the ability of resolution and de-correlation. Moreover the new method can estimate more sources since it increases the numbers of sensors (virtual sensors). To evaluate the performance of the proposed technique we ran a large number of Monte Carlo simulations for different array geometries and different system parameters. In summary, it is proven to be effective by theoretical analyses and computer simulations.

REFRENCES

1 Zoltowski M D, Mathews C P. Real-time frequency and 2-D angle estimation with sub-nyquist spatio-temporal sampling. IEEE Trans., 1994, SP-42(10): 2781~2794

2 S.Haykin, J.P.Reiliy, V.Kezys and E.Vertatschitsch, Some aspects of array signal processing, IEE Proc.-F, Vol. 139, No.1 FEBRUARY 1992: 1~25

3 Friedlander B. Direction finding using spatial smoothing with interpolated arrays. IEEE Trans., 1992, AES-28(2): 574~587

4 Cadzow J A. Signal Enhancement--a composite property mapping algorithm. IEEE Trans., 1988, ASSP-36 (1): 49~62

5 P.Stoica and A.Nehorai, MUSIC, Maximum likelihood, and Cramer-Rao bound, ASSP-37, 1989, IEEE Trans on ASSP-37(5): 720~741

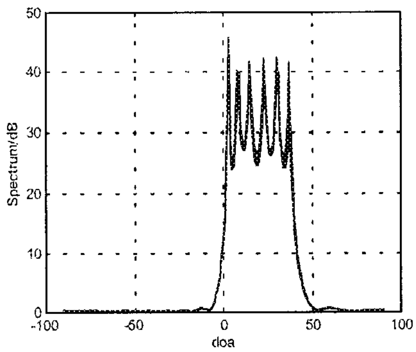

Fig.1

Fig.1 Actual UCA,5 elements,radius $r = 4\lambda$,100 Snapshots,6 signals SNR=10dB,DOA=5°,10°,15°,23°,28° and35° 3 coherence sources, Virtual sensors=16, sector=[0° 40°].

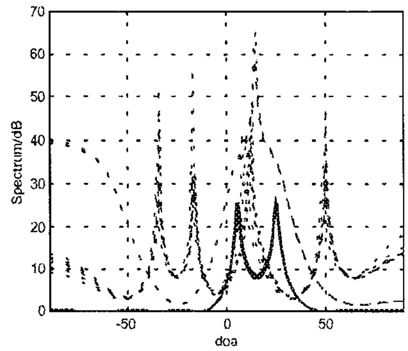

Fig.2

Fig.2 UCA with 8 elements, Radius $r = 3\lambda$,100 snapshots 2 coherence sources, DOA=5° and 25° , SNR=10dB Virtual sensors=8, sector=[0° 30°], Dotted line is the method of B.Friedlander. Solid line is the method of this paper

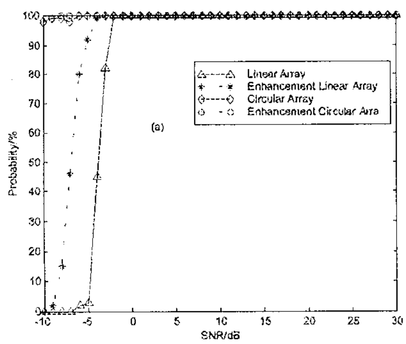

Fig.3(a)Probility with SNR

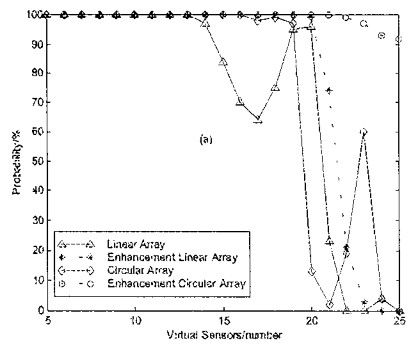

Fig.4(a) Probility with virtual sensors

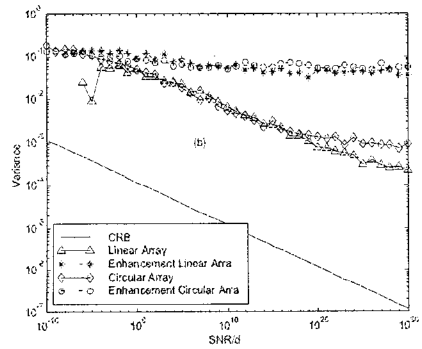

Fig.3(b)Variance with SNR

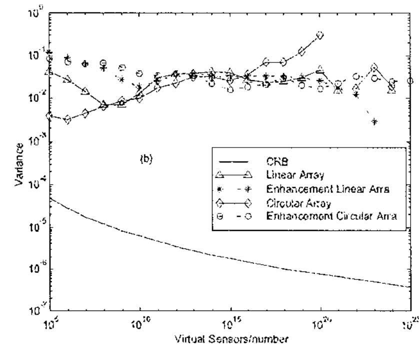

Fig.4(b) Variance with virtual sensors