A Study on Direction of Arrival Estimation of Low Frequency Electromagnetic Signal with a SQUID Array

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1. Introduction

A SQUID(**S**uperconducting **QU**antum **I**nterference **D**evice) is considered to be an extremely high-sensitivity and wide-band (DC～MHz) magnetic-flux density sensor. The equivalent circuit of a DC-SQUID is shown in Fig.1 A SQUID array has been used in MEG(**M**agneto**e**ncephalo**g**raphy) in shielded stationary operation. Magnetic antennas for submarine communications in unshielded mobile operations using LTS (Low-Temperature Superconducting)-SQUIDs were studied in the late 1970's.[1]～[3] Recently, HTS (High-Temperature Superconducting)-SQUIDs cooled by nitrogen were developed, and used in applications such as geophysical exploration and nondestructive inspection.[4],[5] Furthermore, magnetic anomaly detection and localization algorithms have been proposed.[6],[7] To date, there have been no studies that have attempted to utilize a SQUID array to examine in the problem of DOA(direction of arrival) estimation for low frequency electromagnetic signals. Fig.2 shows that a SQUID has cos(θ) directional characteristics in output amplitude. In this paper, On the basis of these characteristics, we propose two methods to estimate the direction of the H-field with a SQUID array for the case in which each SQUID is unable to detect the phase difference of the electromagnetic signal, because the SQUID array is too small compared to the wavelength of the arriving signal. The DFT(**D**iscreet **F**ourier **T**ransformation) method and SARSIC (**S**QUID **A**rray **R**oot **SI**gnal **C**lassification) method used to estimate the DOA, are proposed in this paper. In particular, the SARSIC method is able to efficiently estimate the DOA by solving the denominator of the MUSIC(**MU**ltiple **S**ignal **C**lassification) spectrum without a nonlinear search. This paper presents numerical comparisons of these two DOA estimation methods, which confirm that the SARSIC method is more efficient.

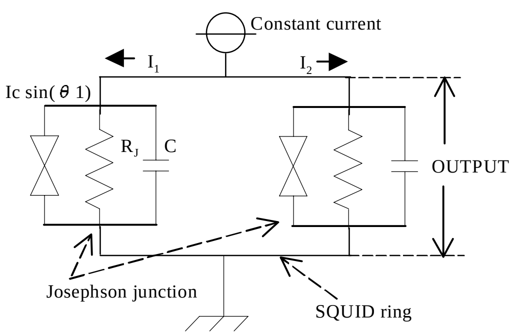

Fig.1 Equivalent circuit of DC-SQUID

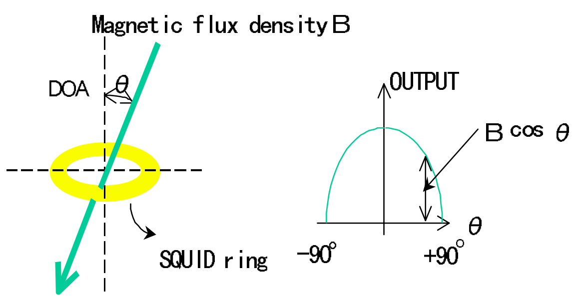

Fig.2 Directional characteristics of SQUID

2. Formulation of a SQUID array

A SQUID is a magnetic-flux density sensor. The direction of electromagnetic signal propagation is orthogonal to the H-field and the E-field. In this paper, hereafter, we call the direction of the H-field the DOA. Our SQUID array is mounted with the SQUIDs having different normal directions to each other, as shown in Fig.3 The observation data matrix X is denoted as follows.

$$X = A(\theta)\cdot S + N \qquad (1)$$

$$X = [x_1(t) \quad x_2(t) \quad \cdot \quad x_k(t) \quad \cdot \quad x_n(t)]^T \qquad (2)$$

$$x_k(t) = [x_k(1) \quad x_k(2) \quad \cdot \quad x_k(T)]^T \qquad (3)$$

where X, S and N denote the observation data, electromagnetic signals and noise, respectively. n denotes the number of SQUIDs and $A(\theta)$ is the array manifold. The wavelength is considered to be large sufficiently compared to the intervals of SQUIDs. In other words, the SQUID array is configured with a small equipment size. Therefore, the signal phases are approximately equal in each SQUID, and the array manifold is given by the matrix (4) in which the elements are the exact cos(θ) characteristics.

$$A(\theta) = [a(\theta_1) \quad a(\theta_2) \quad \cdot \quad a(\theta_J)] = \begin{bmatrix} \cos(\theta_1+\phi_1) & \cos(\theta_2+\phi_1) & . & \cos(\theta_J+\phi_1) \\ \cos(\theta_1+\phi_2) & \cos(\theta_2+\phi_2) & . & \cos(\theta_J+\phi_2) \\ . & . & . & . \\ \cos(\theta_1+\phi_n) & \cos(\theta_2+\phi_1) & . & \cos(\theta_J+\phi_n) \end{bmatrix} \qquad (4)$$

where $\phi_1,\cdot\cdot,\phi_k,\cdot\cdot,\phi_n$ are the angles between a normal of SQUID and a nominal axis. θ_J is the DOA of the Jth signal. For simplification, SQUIDs were mounted in a circle at equal angle intervals $\Delta\theta$, as shown in Fig.3. In this case, ϕ_k is given by $\phi_k = (k-1)\cdot\Delta\theta$

3. DOA methods using a SQUID array

We estimate the DOA using the array manifold $A(\theta)$ denoted by equation (4) using each SQUID's observation data. Here, we focus on only one arriving signal. The DOA problem is reduced to estimate the initial phase of one periodic cosine signal in element space owing to SQUID's cos(θ) directional characteristics. In this capter, we propose two novel methodsfor DOA estimation using multiple snapshots as follows.

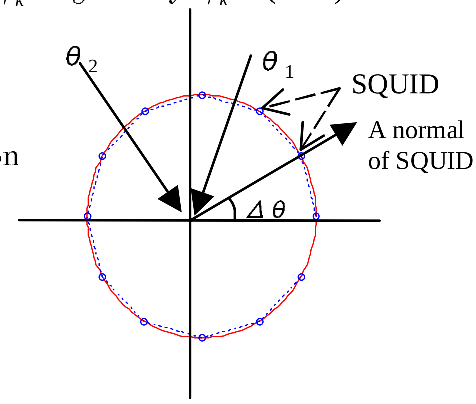

Fig.3 Configuration of SQUID array

1) DFT method

First, by reducing the noise and interference, each SQUID's observation data (columns of matrix X) which is the sample data's snapshots is processed by DFT in time domain as given by equation (5).

$$F_k(f) = \left|DFT(x_k)_f\right| \cdot \sin\{\arg(DFT(x_k)_f)\} \qquad (5)$$

where $DFT(x_k)_f$ is the complex output at frequency f . When SQUIDs were mounted in a circle, $F_k(f)$ given by equation (5) is expected to be a cosine functional form with respect to SQUID number k. Therefore, we can obtain the DOA as an initial phase of a fundamental component in DFT relating to $F_k(f)$ in element domain without a nonlinear search. The subscript 1 in equation (6) denotes the fundamental component.

$$DOA_{DFT} = \arg[\{DFT(F_k)\}_1] \qquad (6)$$

Furthermore, when the number of sampling is mth(m is integer) power of 2 and the sampling interval is equal, FFT processing is applicable in both time and element domain and allows the estimation of the DOA with low computational costs.

2) SARSIC method

This method is based on maximizing the MUSIC spectrum (7) derived from noise eigenvectors *En* of a sampling covariance matrix $R = X \cdot X^T$ as well as in a field of adaptive array.

$$MSC(\theta) = \max_{(\theta)} \left[\frac{A^T \cdot A}{A^T \cdot En \cdot En^T \cdot A} \right] \qquad (7)$$

In the case of a SQUID array mounted with equal angle intervals, $A^T \cdot A$ becomes to be constant. Minimizing the denominator $D(\theta) = A^T \cdot En \cdot En^T \cdot A$, we can estimate the DOA without a nonlinear search.

ROOT MUSIC based on the Pisarenko method has been proposed for the case that the signal phase can be measured. Detailed discussion on ROOT MUSIC can be found in reference [8]. In the case of the low frequency DOA problem with the SQUID array, only signal amplitudes are observable. We can treat the array manifold *A* as a real matrix. $D(\theta)$ is given as follows.

$$D(\theta) = c_1 \cdot \cos^2(\theta) + c_2 \cdot \sin(\theta) \cdot \cos(\theta) + c_3 \sin^2(\theta) \qquad (8)$$

where $Rn = \left[Rn_{i,j} \right] \equiv En \cdot En^T$

$$c_1 = \sum_{i=0}^{N-1} \left[\cos(i \cdot \Delta\theta) \left(\sum_{j=0}^{N-1} Rn_{i,j} \cdot \cos(j \cdot \Delta\theta) \right) \right]$$

$$c_2 = \sum_{i=0}^{N-1} \left[\sin(i \cdot \Delta\theta) \left(\sum_{j=0}^{N-1} Rn_{i,j} \cdot \cos(j \cdot \Delta\theta) \right) \right] + \sum_{i=0}^{N-1} \left[\cos(i \cdot \Delta\theta) \left(\sum_{j=0}^{N-1} Rn_{i,j} \cdot \sin(j \cdot \Delta\theta) \right) \right]$$

$$c_3 = \sum_{i=0}^{N-1} \left[\sin(i \cdot \Delta\theta) \left(\sum_{j=0}^{N-1} Rn_{i,j} \cdot \sin(j \cdot \Delta\theta) \right) \right] \qquad (9)$$

Since $D(\theta)$ is a differentiable function, DOA minimizing the equation (8) is obtained by solving the equation (10) in which the derivative of equation (8) is equal to zero. In this paper, we termed this novel method the SARSIC.

$$\frac{\partial D(\theta)}{\partial \theta} = 2 \cdot c_2 \cdot \cos^2(\theta) - 2 \cdot (c_1 - c_3) \cdot \sin(\theta) \cdot \cos(\theta) - c_2 = 0 \qquad (10)$$

The solution of equation (11) is easily derived by following equation (11).

$$DOA_{SARSIC} = \frac{1}{2} \tan^{-1} \left(\frac{c_2}{c_1 - c_3} \right) \qquad (11)$$

We can distinguish the minimum and maximum of equation (9) by the sign of the second derivative $\frac{\partial^2 D(\theta)}{\partial \theta^2}$. The SARSIC method is computationally less demanding than MUSIC and other methods(for example, the maximum likelihood method)

4. Computer simulation

The dependency of the number of SQUIDs, the S/N (Signal to Noise Ratio) and the snapshot in the Gaussian-distributed white noise were examined by numerical simulation in the case of only one arriving signal. The simulation conditions were as follows. The DOA was10deg. The frequency of the signal and the sampling frequency was 10kHz and 100kHz, respectively. Each simulation under the same condition and different seeds for random number involved a total of 100 runs. Fig.4 shows the resultant standard deviation of the estimated DOA as a function of the S/N. The number of SQUID was selected to be either 5 or 15. The number of snapshots, *Nss* taken was 100. In Fig.4, CRB stands for Cramer-Rao Bound for the SQUID array.[9],[10] As shown in Fig.4, the standard deviation of the SARSIC method was approximately one-half that of the FFT method. This was only 25% larger than that of CRB. Fig.5 shows the result of the snapshot dependency for S/N=5dB and number of SQUID=15. The S/N and the snapshot dependency in each methods showed the same tendency as that of CRB. Furthermore, we confirmed that the results of the SARSIC method were equal to that of Maximum likelihood method.[11] The standard deviation was found to be proportional to $Nss^{-1/2}$ and

S/N$^{-1/2}$. It is worth noting that the standard deviations of CRB with only the signal amplitude available and CRB with both the signal amplitude and phase available are proportional to $n^{-1/2}$ and $n^{-3/2}$, respectively, for the number of elements n.

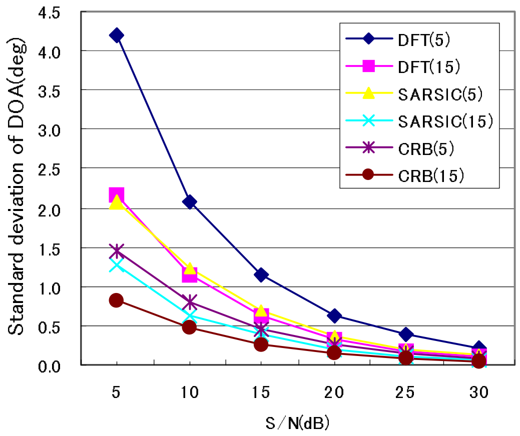

Fig.4 S/N dependency

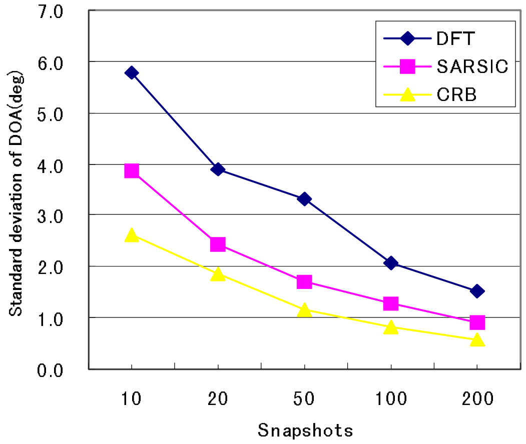

Fig.5 Snapshot dependency

5. Conclusion

The SQUID array configuration and the DOA estimation methods for applications such as underwater communications and geophysical exploration were proposed.

We confirmed the following results, the SARSIC method had a standard deviation of approximately one-half that of the DFT method. This was only 25% larger than that of CRB. The results of the SARSIC method which has low computational costs were equal to those of Maximum likelihood method. The DFT method and SARSIC methods are unlikely to able to resolve the influence of coherent interference. It is well known that space smoothing processing is used in adaptive arrays. A means of settling in coherent interference remains under investigation. Nevertheless, these proposed methods are also applicable to conventional E-field antennas.

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