CONVENTIONAL AND MODERN METHODS FOR THE ANALYSIS OF LARGE FINITE PHASED ARRAYS

Prabhakar H. PATHAK[1], Özlem A. ÇIVI[2], Do-Hoon KWON[1], and Jacob J. KIM[3]

[1] The Ohio State University ElectroScience Laboratory, Dept. of Electrical Eng., 1320 Kinnear Rd., Columbus, Ohio 43212, U.S.A. (pathak.2@osu.edu; kwon.22@osu.edu)

[2] Middle East Technical University, Dept. of Electrical Eng., Ionu Bulvari, 06531 Ankara, Turkey (ozlem@metu.edu.tr)

[3] Raytheon Systems Co., 2501 W. University Dr., MS 8019, McKinney, Texas 75070, U.S.A. (jkim@raytheon.com)

1 Introduction

Large phased arrays are finding an increasing use in radar and satellite communications, as well as in other applications. The conventional brute force numerical solution approaches for the analysis of such large phased arrays are generally versatile and robust, but the number of unknowns to be solved numerically in such methods becomes exorbitantly large and computationally expensive as the electrical size of the array becomes large. On the other hand, some recently developed ray solutions for analyzing the radiation and scattering from large planar finite arrays [1, 2] provide a very attractive alternative because they are computationally highly efficient and physically appealing. However, the UTD solutions for planar finite arrays in [1, 2] have been developed for uniform array current distributions. Some extensions to include symmetrically tapered array distributions have been obtained via UTD slope diffraction [3], but that extension does not appear to work sufficiently accurately for highly tapered and possibly non-symmetric practical array distributions for the design of low sidelobe phased arrays. More recent work has therefore centered on the development of new hybrid methods which systematically combine the conventional numerical approaches with the UTD for the efficient and physically appealing analysis of the radiation/scattering from practical large finite arrays [3, 4]. The number of unknowns which must be solved numerically is drastically reduced in the hybrid approach.

In this paper, the conventional numerical approach as well as the modern hybrid approach are briefly reviewed for the analysis of large finite arrays in Section 2, and some conclusions are provided in Section 3.

2 Analysis of Large Planar Arrays

2.1 Conventional Numerical Approach

Consider a large planar traveling wave slot array as shown in Fig. 1(a). This array consists of 28 rectangular waveguides stacked on top of each other. Each guide contains a set of tilted slots in the narrow wall, and they are spaced approximately 0.5 wavelength apart. The total number of slots is 2052, and they are tilted as shown in Fig. 1(a). The slot tilts together with the power fed to each guide are chosen to obtain a sinusoidal feed distribution in both the $z = 0$ plane and the $x = 0$ plane of the entire array aperture. Also, the guides operate in the dominant TE_{10}

mode, and the slots are sufficiently short and thin electrically so that one can also assume the usual sinusoidal electric field distribution along the thin slot. Furthermore, the minimum slot tilt is set to 80° and the maximum at 100° from the waveguide axis. Also, the tilts are chosen to provide a zero cross-polar radiation pattern in the $y = 0$ (or x-z) plane. The operating frequency is chosen in the S band to provide a peak at a desired scan angle of $-10°$ (so $z < 0$ and $x > 0$) from the boresight (x-axis) for this traveling wave slot array. An integral equation (IE) formulation which is solved by the moment method (MoM) yields the matrix equation

$$([Y_{\text{ext}}] - [Y_{\text{int}}])\,[V] \;=\; [I]\,. \tag{1}$$

The $[V]$ represents the unknown array electric field amplitude distribution in the presence of array mutual coupling for an array feed distribution $[I]$ that is here assumed known (as a sinusoidal variation in the y and z directions). The solution for $[V]$ for a given $[I]$ requires one to calculate $([Y_{\text{ext}}] - [Y_{\text{int}}])^{-1}$, and such a task is computationally very expensive since the order of the $[Y]$ matrix to be inverted is huge, namely, 2052×2052. In the present work, Gauss elimination is employed which is more efficient but still computationally expensive. The $[Y_{\text{ext}}]$ and $[Y_{\text{int}}]$ refer to the external and internal slot mutual coupling admittance effects; the external effects ignore the presence of the outer edges of the waveguide and assumes for convenience that the slots are in an infinite ground plane for the exterior problem. The co-polar and cross-polar radiation patterns are shown for the azimuthal (x-z plane) cut, respectively, in Fig. 1 which also shows the effects on the pattern if array mutual coupling effects are ignored.

2.2 Modern Hybrid Approach

The potential of the hybrid UTD-MoM approach for a highly useful and efficient array analysis is presented here by treating the radiation/scattering by a large planar rectangular dipole array in free space. This simpler example is chosen to illustrate the concepts of the hybrid approach without undue complications in modeling. The effects of material substrates/superstrates, as well as slightly aperiodic element locations, are currently being implemented into the hybrid method and will be reported in the future. Note that the dissimilar slot tilts along any waveguide stick of Fig. 1 makes that array slightly aperiodic, such an array will be specifically treated by the hybrid approach in the near future. The present, simpler dipole array is located in the $z = 0$ (or $x - y$ plane) and consists of 45×45 elements. The elements are half wavelength, thin wire center fed dipoles oriented along the $\hat{y}$ direction. The IE based formulation for this dipole array can be solved by the MoM which yields the following matrix equation:

$$[Z][I] = [V] \tag{2}$$

in which the known, applied or feed voltages V_{nm} to each nm^{th} dipole in the array are given by $[V]$. Also, $[Z]$ is the mutual impedance matrix for the array, and $[I]$ are the unknown current amplitudes A_{nm} for the array of $(2M+1) \times (2M+1)$ elements (here $M = N = 22$). The current distribution $I_{nm}(y')$ on each nm^{th} dipole has a sinusoidal form with amplitude A_{nm}. Rather than solve the matrix equation of (2) by conventional numerical MoM where $[Z]$ is a $(45 \times 45)^2$ matrix here, the hybrid approach is used whereby the size of the original $[Z]$ is reduced to solving approximately 133 unknowns instead of 45×45 unknowns! This reduction in unknowns is possible because the array of 45×45 cells can be divided into a large inner part and a thin, outer boundary part, and then the UTD can be employed to relate the A_{nm} within the inner array as follows [3]

$$A_{nm} \sim \left\{ D\frac{V_{nm}}{V_{00}} e^{-j\beta_x n d_x} e^{-j\beta_y m d_y} + \sum_{e=1}^{4} \left(\left[\frac{A_e}{\sqrt{s_e}} + \frac{B_e}{\sqrt{s_e^3}} + \frac{F_e}{\sqrt{s_e^5}} \right] e^{-jks_e} e^{-j\beta_x x_e} e^{-j\beta_y y_e} \right) \right\} \tag{3}$$

where the periodic spacing is d_x and d_y in the $\hat{x}$ and $\hat{y}$ directions, respectively and (β_x, β_y) are the impressed (or excitation) phase. Eq. (3) ignores UTD corner diffraction effects within the inner array region, but they can be added if desired. For the thin outer boundary part [3]

$$A_{nm} = \begin{cases} C_{-n+N+1,m+M+1}, & \text{for corner cells of outer part} \\ E_{-n+N+1,m+M+1}, & \text{for edge cells of outer part} \end{cases} \tag{4}$$

The relatively few unknowns $C_{-n+N+1,m+M+1}$, $E_{-n+N+1,m+M+1}$, and A_e, B_e, F_e above can be solved by MoM. The amplitude of the geometrical optics based Floquet modal contribution is described by D, while V_{nm}/V_{00} describes the array feed taper, and the remaining terms in (3) are the edge diffraction effects. Also, s_e denotes the effective ray distance along the edge diffracted field measured from the e^{th} edge. A slightly different hybrid approach was developed in [4]. One notes that the UTD basis set in (3) does not contain any sum over Floquet modes, whereas that in [4] does. It can be shown that the coefficients E are not all independent in (4) thereby reducing the number of unknowns further. In [3] which dealt with uniform feed array distribution, the F_e term is neglected, but for tapered arrays F_e must be included for improved accuracy. Numerical results for the radiation pattern which are based on this hybrid approach are shown in Fig. 2(a) for a tapered feed distribution. The current amplitudes for this case are shown in Fig. 2(b) for the 3^{rd} and in 2(c) for the 23^{rd} rows. In Fig. 2, the hybrid solution is compared with the conventional numerical MoM solution which is used as a reference.

3 Conclusions

Both a conventional numerical type solution approach and a modern hybrid approach, respectively, are briefly reviewed for the analysis of large finite arrays. The hybrid approach utilizes the best features of the numerical approach which handles the array edge and corner effects more accurately, and combines it with the UTD ray approach which accurately and efficiently models the large array effects away from the neighborhood of edges and corners. In doing so, the hybrid approach requires far less number of unknowns to be solved as compared to the conventional numerical method because it models most of the array physics *a priori* in the hybrid formulation via the new UTD ray based solution for arrays. Thus the hybrid method also provides physical insights into the array radiation/scattering mechanisms that are generally masked in the purely numerical based conventional solution approaches. Work is presently in progress to make the hybrid approach as versatile as the conventional numerical approach for large finite array analysis.

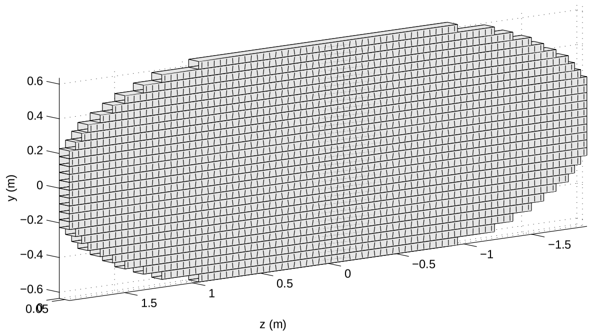
References

[1] L.B. Felsen and L. Carin, "Diffraction theory of frequency and time domain scattering by weakly aperiodic truncated thin wire gratings," *J. Opt. Soc. Am. A*, vol. 11, pp. 1291-1306, April 1994.

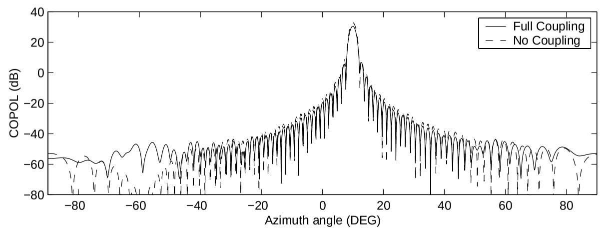
[2] F. Capolino, M. Albani, S. Maci and L.B. Felsen, "Floquet wave diffraction theory for truncated dipole arrays: Propagating and evanescent spectra," paper presented at URSI EMT meeting in Thessaloniki, Greece, May 1998.

[3] Ö.A. Çivi, P.H. Pathak and H.-T. Chou, "A hybrid UTD-MoM for efficient analysis of EM radiation/scattering from large finite planar arrays," paper presented at URSI EMT meeting in Thessaloniki, Greece, May 1998.

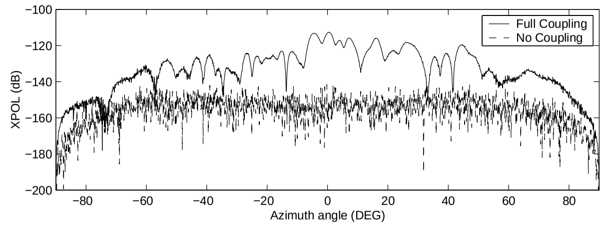
[4] A. Neto, S. Maci, G. Vecchi and M. Biagiotti, "Full wave analysis of a large rectangular array of slots," International IEEE AP-S/National Radio Science Meeting, Atlanta, Georgia, June 1998.



(a) Geometry

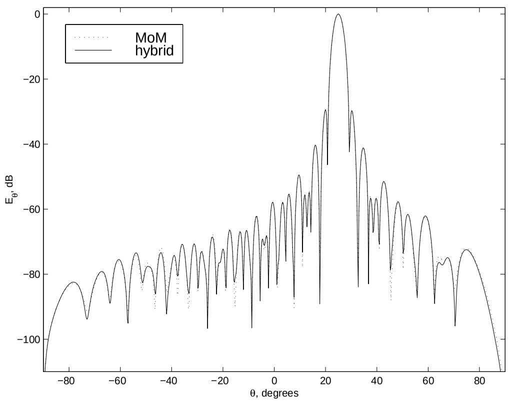


(b) Co-pol. pattern

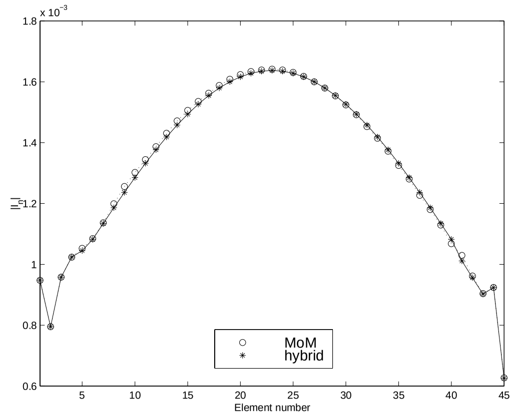


(c) Cross-pol. pattern

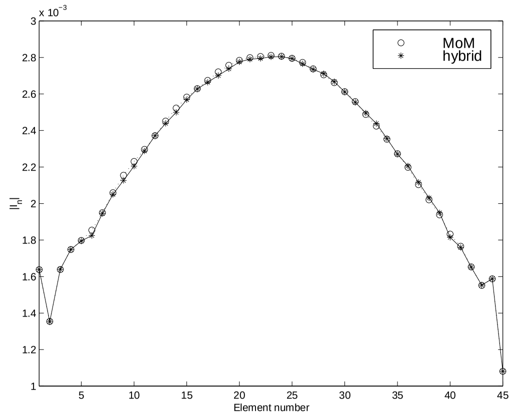
Figure 1: Traveling wave slot array antenna and radiation patterns in $x - z$ Plane



(a) Radiation pattern



(b) Current amplitudes: 3^{rd} row



(c) Current amplitudes: 23^{rd} row

Figure 2: Radiation pattern and current amplitudes for 45×45 dipole array. Here, $d_x = 0.3\lambda$, $d_y = 0.6\lambda$ and scan direction is $\theta_0 = 25°$, $\phi_0 = 40°$. The feed distribution is $\exp\left[\left(\frac{n}{N}\right)^2 \ln(p)\right] \exp\left[\left(\frac{m}{M}\right)^2 \ln(p)\right]$ with $p = 0.5$.