# Nanovortices around Resonant Optical Nanoantennas with Higher Directivities

<sup>#</sup>Andrea Alù and Nader Engheta Department of Electrical and Systems Engineering, University of Pennsylvania 200 South 33<sup>rd</sup> Street – ESE 203 Moore, Philadelphia, PA 19104, U.S.A. andreaal@ee.upenn.edu, engheta@ee.upenn.edu

## 1. Introduction

For many years, plasmonic resonances supported by nanoparticles have been investigated in different fields of research, spanning optics, microscopy, biology and nanotechnology (see [1] and references therein). Although this physical phenomenon has been well understood, various studies on resonant nanoparticles and plasmonic materials are being conducted by various groups worldwide, due to the novel possibilities that the use of nano-scale metamaterials and plasmonic materials may offer. This includes phenomena such as sub-wavelength imaging [2], sub-diffractive propagation [3], optical nano-circuits [4] and metamaterial transparency [5]. Of other exciting features of such sub-wavelength resonances, the idea of exciting higher-order resonances in a sub-wavelength system for the purpose of realizing nanoantennas with higher directivities at infrared and optical frequencies has been recently proposed by our group [6]. Here we report our recent results on the investigation of near-field vortices in the power-flow distribution in the vicinity of plasmonic nanoparticle antennas with higher resonances. These results may provide novel insights into the resonance of sub-wavelength particles as nanoantenna arrays, and it may represent a first step towards employing these anomalous power vortices for moving or positioning nano-probes and nano-antennas for various nanooptical applications.

#### 2. Theoretical Analysis

It is well known that the field distributions near spherical nanoparticles are fully expressed by using the Mie theory [7-8]. Following our notation in [9] with an  $e^{-i\omega t}$  time dependence, for a homogenous spherical scatterer of radius *a*, permittivity  $\varepsilon$ , and permeability  $\mu_0$  (assumed to be the same as that of the background medium), the conditions for obtaining resonances of a generic azimuthal order *n* are given by the following expressions for  $TM^r$  and  $TE^r$  polarizations:

$$TM : \quad \varepsilon j_{n}(ka) [(k_{0}a) y_{n}(k_{0}a)]' = \varepsilon_{0} [(ka) j_{n}(ka)]' y_{n}(k_{0}a),$$
  

$$TE : \quad j_{n}(ka) [(k_{0}a) y_{n}(k_{0}a)]' = [(ka) j_{n}(ka)]' y_{n}(k_{0}a),$$
(1)

where  $k = \omega \sqrt{\varepsilon \mu_0}$  and  $k_0 = \omega \sqrt{\varepsilon_0 \mu_0}$  are, respectively, the wave numbers inside and outside the sphere and the functions  $j_n$  and  $y_n$  are spherical Bessel functions [10] (their derivatives are taken with respect to the argument). The use of plasmonic materials or metamaterials allows fulfilling the conditions for resonance (1) for sub-wavelength particles. For the case at hand, Eq. (1) is simplified in the "small-radii" approximation as the following well known approximate dispersion condition:

$$\varepsilon = -\frac{n+1}{n}\varepsilon_0\,,\tag{2}$$

which is valid only for  $TM^r$  resonances. Strict conditions on the possibility of exciting these nanoresonances in the presence of reasonable losses may be derived (see e.g., [6, 11]).

The first order resonance (n=1) has been studied by various groups over many years. Here we present our theoretical results for the field and power-flow distributions, not limiting our analysis to the dipolar resonances, with particular attention to the higher-order resonances. We study the power flow inside and around nano-spheres at resonance with higher order n. As we discussed in [6], the excitation of such resonances may provide the possibility of synthesizing higher-directive plasmonic nanoantennas at infrared and optical frequencies. This may present a new approach to scaling down the antenna size and increasing the frequency of operation into the IR and optical domains, with enhanced bandwidth. Here we study how such higher-order resonances may be established in a nanoparticle, we explore their power flows around and inside the particles. The anomalous power flow and its "nano-vortices" around the particle may also be useful for positioning of nano-particles and future design of optical tweezers.

#### **3. Numerical Results**

We have performed full-wave numerical simulations using the Mie theory to evaluate the power flow distribution around resonant plasmonic nano-particles illuminated by a plane wave. The results we have derived are fairly different from some of those available in the literature even for the lower order dipolar resonance, mainly because we have considered the appropriate spherical harmonics without utilizing any Taylor series approximation in calculating the interaction among the different multi-polar harmonics. As we have shown in details in our theoretical analysis (this will be reported in a more extensive upcoming manuscript), at the resonance of these plasmonic nanoparticles the near-field distribution is sensitive to the coupling of low-order dominant harmonics with higher-order harmonics.

Fig. 1 reports, as an example, the power-flow inside and around a plasmonic nanoparticle at its electric dipole (a) and quadrupole (b) resonance, which happen, respectively, at the frequency for which the electric permittivity is  $\varepsilon_{dip} = -2.223\varepsilon_0$ ,  $\varepsilon_{quad} = -1.533\varepsilon_0$ , consistent with Eq. (1). Both

results have been obtained for a nanoparticle with  $k_0 a = 0.3$ .

From this figure, it is clear how in the dipolar case an anomalous power flow distribution is established inside and around the nanoparticle. The power flow of the plane wave, traveling along z with electric field linearly polarized along x, is affected over a cross section that is much larger than the physical cross section of the resonant plasmonic particle, as clearly seen in Fig. 1a. Moreover, the power flow is dominated by nano-vortices around the particle, which bring back to the inside of the particle a portion of the power flow outside of its surface. This is an intrinsic peculiarity of these kinds of plasmonic sub-wavelength resonances, which explains the dramatic increase in the scattering cross section of the particle, qualitatively consistent with the discussion in [12].

Figure 1b refers to the higher-order quadrupolar resonance, which, as anticipated in [6], may provide the possibility of higher directivity in a nanoantenna setup. It is evident that in this case the nanovortices around the particle are even more pronounced and they affect a larger region of space around the particle. Singular lines and points that connect E and H planes of polarization arise in this higher-order resonant scenario.

We have applied this technique to take into account particles with small amount of material losses, and we have shown that although the material loss can affect some of the features of power flow around the particle, the presence of some losses may still allow, under proper conditions, the nanovortices to be present around particles close to their resonances.



Figure 1: Full-wave power flow distribution near and around a homogeneous spherical particle at its (a) dipolar and (b) quadrupolar resonance.

These nano-vortex distributions may be appealing in some potential applications such as moving and positioning small non-resonant nano-particles that weakly interact with the resonant particles.

### 4. Conclusions

In this contribution, we have presented our recent results on the anomalous power-flow distribution inside and around nano-particles at their multipolar resonances. We have found that in the dipolar resonance (lower order) anomalous power-flow is expected around the surface of the particle and inside its volume. These anomalous features are even amplified dramatically at the resonance of higher-order modes, which may be applied to higher-directive nano-antenna systems.

These results may have interesting applications at optical frequencies, where plasmonic materials are naturally available.

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