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NONLINEAR WAVE PROCESSES IN PLASMA LAYER. HAMILTONINAN APPROACH.

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ABSTRACT.

A hamiltonian formalism for description of nonlinear wave processes in plasma layer is developed. The general nonlinear set of equations, which is suatable for description of both weak turbulence effects (decay, scattering) and strong turbulence effects (modulational instability, soliton-type solutions etc) is derived. In the framework of these equations it is possible to investigate a wide sphere of questions of nonlinear dynamics and kynetics not only for surface or volume waves in plasma layer but also for their interactions.

INTRODUCTION.

A development of investigations in plasma physics, radiophysics, solid state physics etc as wel as a search of a new types of devices for trestment and transmission of information, for generation and amplification of microwaves etc stimulated an interest for nonlinear wave processes in spatially bounded structures [1,2].

However, considerable difficulties under calculations which are typical under their theoretical description, even under some simplifying assumpsions, as well as the equality of many of them from oscillating-wave point of view. Stimulated the use of universal mathematical tool - hamiltonian formalism for their description [3,4]. It was earlier used mainly in bounded media. The exceptions are wave motions in ocean. After introduction of canonical and transmission to normal variables the media equations acquire standard, hamiltonian form. This allows to eleborate the universal approach for investigation of various phenomena of an arbitrary physical nature.

The aim of the present paper is the introduction and the use of hamiltonian formalism for description of nonlinear phenomena, which are due to to interaction of high-frequency (HF) and low-frequency (LF) surface and volume waves in plasma layer. As an example, the potential waves of homogeneous and isotropic plasma are suggested. This really simplest model allows to demonstrate more clearly a way to treat the formalism and its possibilities. But the way, it is highly productive, because it is a base for construction of nonlinear theory of modulational instability and collapse of langmiur waves [5] in bounded plasma.

Among the works, which are devoted to selfinteraction of surface and interaction between surface and volume waves, it is possible to point out the works [6,7] (see also a rewiews therein), where some particular cases of these processes are considered.

BASIC EQUATIONS.

Suppose that wave properties of plasma layer $-L \le z \le L$, bounded by dielectric $\varepsilon = const.$, are described by following set of equations:

$$\frac{\partial \vec{V}_\alpha}{\partial t} + (\vec{V}_\alpha \vec{\nabla})\vec{V}_\alpha = -\frac{e_\alpha}{m_\alpha}\vec{\nabla}\varphi - V_{T\alpha}^2 \vec{\nabla} \ln \frac{\tilde{n}_\alpha + n_{o\alpha}}{n_{o\alpha}} \qquad (1)$$

$$\frac{\partial n_\alpha}{\partial t} + \mathrm{div}(n_\alpha \vec{V}_\alpha) = 0 \ ; \quad \alpha = e, i \qquad (2)$$

$$\Delta \varphi = -\sum_\alpha e_\alpha n_\alpha \ ; \quad V_{T\alpha}^2 \ln \frac{\tilde{n}_\alpha + n_{o\alpha}}{n_{o\alpha}} \equiv w_\alpha \qquad (3)$$

Here all notations are common-used. These equations are possible to be written in hamiltonian form [8].

$$\frac{\partial P_\alpha}{\partial t} = \frac{\delta H}{\delta q_\alpha} \quad \frac{\partial q_\alpha}{\partial t} = -\frac{\delta H}{\delta p_\alpha} \quad p_\alpha = \begin{pmatrix} \Phi_\alpha \\ \mu_\alpha \end{pmatrix} \quad q_\alpha = \begin{pmatrix} n_\alpha \\ \lambda_\alpha \end{pmatrix} \qquad (4)$$

Where (Φ_α, n_α), $(\mu_\alpha, \lambda_\alpha)$ are canonically conjugated pairs of variables, Φ_α - velocity potential, μ_α, λ_α - are variables which determine vorticle part of velocity, symbols $\delta/\delta p_\alpha$; $\delta/\delta q_\alpha$ denote variational derivatives. Hamiltonian H is an energy of the system (1-3), expressed through the canonical variables. It can be easily obtained by integration of energy consideration law over all space with a fufillment of condition of divergention equality zero at the interfaces. Let us call this condition as conservativity condition and its appearense can be expressed as follow:

$$I_b \equiv \int d\vec{z}_\perp \left[-\sum_\alpha V_{\alpha z}\left(m_\alpha n_\alpha \frac{V_\alpha^2}{2} + m_\alpha n_\alpha w_\alpha + e_\alpha n_\alpha \varphi\right)\Big|_{-L}^{L} - \frac{1}{4\pi}\left\{ \varepsilon\varphi \frac{\partial^2 \varphi}{\partial z \partial t} \right\}\Big|_{-L}^{L} \right] = 0 \qquad (5)$$

where $\vec{z}_\perp$ are variables in a plane of interface (x,y), figure brackets denote a jump of corresponding substance at both boundaries. The most surprising and important thing is in fact that conservativing condition completely determine all necessary boundary condition, including so called extra boundary conditions [1].

Further, it is necessary to make a Fourier transform on translationally invariant variables and to make canonical variables a series expantion of hamiltonian and conservation condition $H = H^{(2)} + H^{(3)} + H^{(4)} + \ldots$, $I_b = I_b^{(2)} + I_b^{(3)} + \ldots = 0$. Note, that following notations will be used further, e.g. $n_{\alpha k} \equiv n_\alpha(\vec{k}, z, t)$, and $k = \{k_x, k_y\}$. If we assume $H = H^{(2)}$ in equations (4) and take a corresponding condition $I_b^{(2)} = 0$ then our task becomes an ordinary linear boundary task, which follows from set (1-3). Solving it, it is not difficult to obtain field topographies and dispertion equations. And the analysis of them yields dispersions of all surface and volume waves of plasma layer.

RESULTS OF NONLINEAR CALCULATIONS.

Next principal step is to make a transition to normal variables (complex wave amplitude). It is convenient to chose it on a form:

$$\Phi_{\alpha k}=\sum_{S=1}^{\ell}\sum_{\nu=0}^{\infty} F^{S}_{k\nu}(z)\,\hat{C}^{S+}_{k\nu}\equiv\sum_{S=1}^{\ell}\sum_{\nu=0}^{\infty} F^{S}_{k\nu}(z)\left(a^{S}_{k\nu}-a^{S*}_{-k\nu}\right)$$

$$n_{\alpha k}=\sum_{S=1}^{\ell}\sum_{\nu=0}^{\infty} N^{S}_{k\nu}(z)\,\hat{C}^{S}_{k\nu}\equiv\sum_{S=1}^{\ell}\sum_{\nu=0}^{\infty} N^{S}_{k\nu}(z)\left(a^{S}_{k\nu}+a^{S*}_{-k\nu}\right) \tag{6}$$

$$\varphi_{k}=\sum_{S=1}^{\ell}\sum_{\nu=0}^{\infty} f^{S}_{k\nu}(z)\,\hat{C}^{S}_{k\nu}\equiv\sum_{S=1}^{\ell}\sum_{\nu=0}^{\infty} f^{S}_{k\nu}(z)\left(a^{S}_{k\nu}+a^{S*}_{-k\nu}\right)$$

Functions $F^{S}_{k\nu}(z)$, $N^{S}_{k\nu}(z)$, $f^{S}_{k\nu}(z)$ are corresponding solution of linear boundary task. Index is $S = 1,2..\ell$ denote a kind of solution, respectively: HF surface symmetrical, HF surface antisymmetrical, HF surface symmetrical, HF volume symmetrical, HF volume antisymmetrical, LF surface symmetrical etc, index is a number of discret volume mode.

It is not difficult to verify that the transform (6) is a canonical one and reducees hamiltonian $H^{(2)}$ to a diagonal form

$$H^{(2)}=\sum_{S=1}^{\ell}\sum_{\nu=0}^{\infty}\int d\vec{k}\,\omega^{S}_{k\nu}\,a^{S}_{k\nu}\,a^{S*}_{k\nu} \tag{7}$$

Set of equations (4) in nonlinear variables has the following form:

$$\frac{\partial a^{S}_{k\nu}}{\partial t}+i\omega^{S}_{k\nu}a^{S}_{k\nu}=-i\,\frac{\delta}{\delta a^{S*}_{k\nu}}\left(H^{(3)}+H^{(4)}\right)\equiv-i\,\frac{\delta H_{int.}}{\delta a^{S*}_{k\nu}} \tag{8}$$

and is the main equation in hamiltonian technique. All linear information about interacting waves is contained in dispertion law $\omega^{S}_{k\nu}$ and all nonlinear information is contained in interaction hamiltonian. The latter can be obtained by substitution of expressions (6) in hamiltonian $H^{(3)}$ and $H^{(4)}$, expansion:

$$H^{(3)}=\sum_{S_{123}}\sum_{\nu_{123}}\int d\vec{k}_{123}\,\hat{C}^{S_1}_{\nu_1}(\vec{k}_1)\,\hat{C}^{S_2}_{\nu_2}(\vec{k}_2)\,\hat{C}^{S_3}_{\nu_3}(\vec{k}_3)\,D^{S_1S_2S_3}_{\nu_1\nu_2\nu_3}(\vec{k}_1,\vec{k}_2,\vec{k}_3)\,\delta_{(\vec{k}_1+\vec{k}_2+\vec{k}_3)} \tag{9}$$

$$\begin{aligned}D^{S_1S_2S_3}_{\nu_1\nu_2\nu_3}(\vec{k}_1,\vec{k}_2,\vec{k}_3)\equiv\sum_{\alpha}\frac{1}{2m_\alpha}\int_{-L}^{L}dz\Big\{&N^{S_1}_{\alpha k_1\nu_1}\Big[\frac{\partial F^{S_2}_{\alpha k_2\nu_2}}{\partial z}\frac{\partial F^{S_3}_{\alpha k_3\nu_3}}{\partial z}-\\&-\vec{k}_2\vec{k}_3F^{S_2}_{\alpha k_2\nu_2}F^{S_3}_{\alpha k_3\nu_3}\Big]-\frac{1}{3}\frac{m_\alpha V^2_{T\alpha}}{n^2_{0\alpha}}N^{S_1}_{\alpha k_1\nu_1}N^{S_2}_{\alpha k_2\nu_2}N^{S_3}_{\alpha k_3\nu_3}\Big\}\end{aligned} \tag{10}$$

Expression (9) can be reduced to:

$$H^{(3)}=\sum_{s_{123}}\sum_{\nu_{123}}\int\left\{V^{s_1s_2s_3}_{\nu_1\nu_2\nu_3}\,a^{s_1*}_{k_1\nu_1}\,a^{s_2}_{k_2\nu_2}\,a^{s_3}_{k_3\nu_3}+U^{s_1s_2s_3}_{\nu_1\nu_2\nu_3}\,a^{s_1*}_{k_1\nu_1}\,a^{s_2*}_{k_2\nu_2}\,a^{s_3*}_{k_3\nu_3}\right\}d\vec{k}_{123} \qquad (11)$$

where $\sum_{s_{123}} \equiv \sum_{s_1s_2s_3}$ is analogous for ν, $d\vec{k}_{123} \equiv d\vec{k}_1 d\vec{k}_2 d\vec{k}_3$ and coeffitients $V^{s_1s_2s_3}_{\nu_1\nu_2\nu_3}(\vec{k}_1,\vec{k}_2,\vec{k}_3)$; $U^{s_1s_2s_3}_{\nu_1\nu_2\nu_3}(\vec{k}_1,\vec{k}_2,\vec{k}_3)$ are called matrix elements. Hamiltonian $H^{(4)}$ and corresponding matrix elements are determined analogously.

Particularly, from equations (8), one can get the set of equations, which describes an interaction on HF and LF wabes of arbitrary nature (they can be two surface waves, or two volume waves, also one of them is surface wave, omother is volume wave).

SUMMARY.

To conclude one can point out that hamiltonian formalism is a very effective for investigetion of nonlinear wave processes in bounded plasmalike media. Boundary conditions are obtained from a concervativity condition. The solution of linear boundary task is enough to calculate the matrix elements. They automatically posseed all necessery transform allows to leave only a number of standart hamiltonians and this considerably reduces necessery calculations. All peculiarities of the media can be forgotten after calculations of matrix elements. The general methods of investigations, developed for boundless media [3], can be applied for equations (8). A knowledge of matrix elements allows to determine rather simply the instability increments, the cors of kinetic equations, to obtain from (8) shortened equations like Blombergen, Korteveg-de-Vreez, Kadomtsev-Petviashvili, Nonlinear Shrodinger equations etc. Analogously it is possible to consider another kinds of boundaries, e.g. plasma-plasma, plasma-metal, as well as another geometries. The nonpotentiality (electromagnetic effects) and external magnetic field taking into account are possible [9].

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