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PULSE PROPAGATION IN MODEL DISPERSIVE MEDIA WITHOUT LOSSES

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A degree of signal distortions depends upon the values of amplitude and phase disturbances of the medium under investigation as well as spectral bandwidth of an impulse [1]. In current articles concerning problems of electromagnetic wave propagation quasicoherent radiopulses ($\Delta\omega \ll \omega_0$; $\Delta\omega$-spectral bandwidth, ω_0-central spectral frequency) are considered more often than other ones. At present the examination of wideband radiopulse behavior is very actual and it is proposed here to consider some distortions of a signal when it propagates through a transparent medium (without any losses) with phase dispersion only.

Frequency responses of real media are rather complicated for analysis. Disregarding definite properties of a medium and signal propagation mechanism we consider a model medium with parabolic phase function:

$$K(\omega)= \exp\left\{i\left[a\omega^2+b\omega+c\right]\right\} \tag{1}$$

where a, b, c are the parameters connected with the medium properties, and a defines the degree of its dispersion.

In spite of physical abstraction of such a model phase characteristics of many real media can be approximated over the investigated frequency interval (ω_0-0.5$\Delta\omega$, ω_0+0.5$\Delta\omega$) by a quadratic polynomial. Such a response (1) enables also to simplify analytic calculations.

Broadening the pulse can be characterized by the value:

$$\gamma= \frac{T_{out}}{T_{in}}\ , \tag{2}$$

where T_{in} is the duration of an initial pulse and T_{out} - of the pulse passing through a dispersive layer.To investigate we need to choose some magnitude of constant γ. This choice is rather arbitrary. For instance, in [2] a problem of narrowband Gauss radiopulse propagation through the medium with parabolic phase function was examined . The authors considered the case of γ=1.1 only. However, this alteration of the duration is revealed most vividly in precise measuring. To generalize and to enlarge their results several pulses with various kinds of spectrum are discussed. No limitations on the bandwidth are made and four cases are investigated:

a) γ=1.1; b) γ=1.5; c) γ=2.0; d) γ=4.0

The case c) can be used as universal criteria that defines permissible degree of distortions in communication systems, while the case d) and b) are of theoretical interest only.

The duration of a radiopulse is determined as a time interval containing a given part of signal power, that is

$$\int_{t_1}^{t_2} \left|S(t)\right|^2 dt = \eta_p \int_{-\infty}^{+\infty} \left|S(t)\right|^2 dt = \eta_p P_0 \qquad (3)$$

where P_0 is proportional to the entire pulse power, η_p is a relative part of P_0 corresponding the duration $T=t_2-t_1$.

Dispersive distortions are determined by nonlinear part of phase function $\Phi(\omega)$ in the exploring frequency interval (ω_1, ω_1). We call value

$$\beta = |\Delta\Phi - \Delta\phi_L| \qquad (4)$$

a dispersive disphase over the spectral interval. $\Delta\Phi=\Phi(\omega_2)-\Phi(\omega_1)$ is the absolute phase difference, $\Delta\phi_L$ is a linear phase increment that results only in time displacement of a signal. It should be mentioned that some ambiguity exists in calculating $\Delta\phi_L$. It is connected with the indefenition of wave group lag time for broadband pulses passing through the dispersive medium. In this case we adopt:

$$\Delta\phi_L = \Delta\omega\Phi'(\omega_1) = \Delta\omega(2a\omega_1+b) \qquad (5)$$

For an arbitrary phase characteristics of a medium the problem of determination of $\Delta\phi_L$ demands some special examination. As to the medium in question (1) we can write:

$$\beta = a(\Delta\omega)^2 \qquad (6)$$

This formula couples spectral bandwidth of a signal with the value of media dispersion a.

Consider an example. The Gauss pulse envelope and its spectrum are:

$$R_{in}(t) = A_0 \exp\left[-\frac{t^2}{2\tau_0^2}\right] \qquad (7)$$

$$F_{in}(\omega) = \frac{A_0\tau_0}{\sqrt{2\pi}} \exp -\left[\frac{\tau_0^2(\omega-\omega_0)^2}{2}\right] \qquad (8)$$

where t_0 defines duration T and spectral bandwidth $\Delta\omega$. After passing through the medium with the dispersive characteristics in form (1) the envelope will be:

$$R_{out}(t) = A_0\tau_0(\tau_0^2+4a^2)^{\frac{1}{4}} \exp\left[-\frac{(t+b+2a\omega_0)^2\tau_0^2}{2(\tau_0^4+4a^2)}\right] \qquad (9)$$

Having used formula (2) and definition (3) we found that the broadening of Gauss pulse is equal γ when

$$\frac{a}{\tau_0^2} = \frac{\sqrt{\gamma^2-1}}{2} \qquad (10)$$

The left side of the equation is proportional to the dispersive disphase β, hence formula (10) using (3) to calculate $\Delta\omega$ helps to obtain such a value β that duration increase is γ when some magnitude η_p is given. Note the value β according to (10) is the function of γ and η_p only.

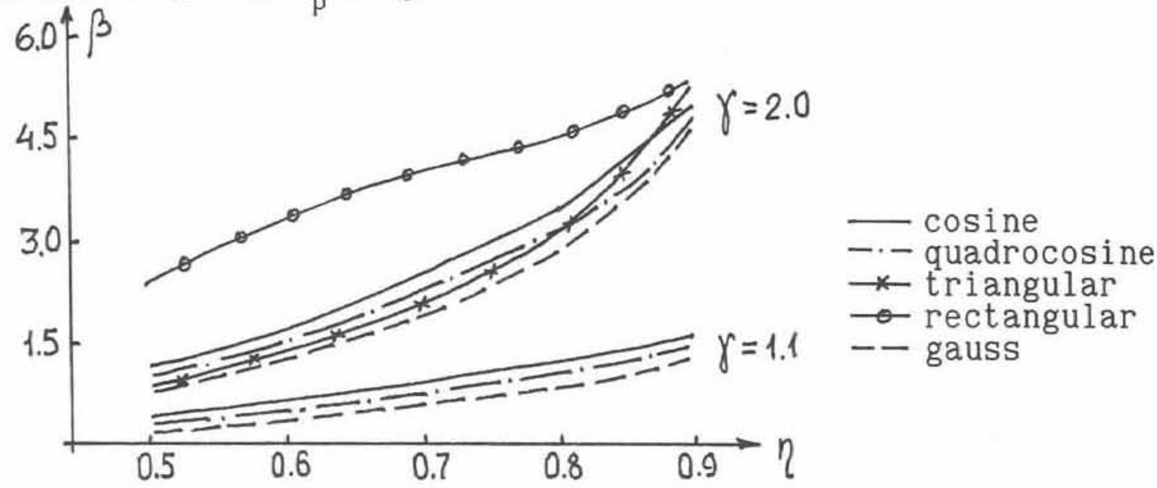

fig.1

Some analogous calculations were made for the following types of the pulse spectrum: 1) cosine; 2) quadrocosine; 3) triangular; 4) rectangular. The first step was to obtain analytic forms of envelopes after passing through medium (1). Then with the help of a computer some magnitudes of dispersive disphase β corresponding to the pulse broadening of γ were found (see figure 1 as an example). Analising these results a conclusion can be drawn: when η_p =const and γ=const the value β remains almost the same in all cases. It is impossible to examine all kinds of spectrum. However we can suppose that the smother the spectral function of a radiopulse the better agreement of its values β with the value obtain from (10).

Average magnitudes of β when η_p=0.9 are given in a table:

$\gamma(\eta_p=0.9)$	1.1	1.5	2.0	4.0
β	1.27	3.15	5.0	11.1

From the above mentioned some supposition can be made that an invariant coupling the degree of media dispersion with the spectral bandwidth of a pulse and depending on η_p and γ is determined. Such an invariant is the dispersive disphase β. When given dispersion a and η_p=const it enables to obtain such a pulse bandwidth $\Delta\omega$ that results in pulse broadening not greater than γ. Dispersive disphase turns out to be very useful in estimating distortion degree of a pulse and in choosing spectral bandwidth. To do it we have to approximate phase function of the real medium by quadratic polynomial over (ω_1, ω_2), then to use the above results.

One simple example of a parabolic ionospheric layer are represented here. Within the limits of geometric optics without absorption and when $\omega \gg \omega_m$ frequency response has the form:

$$\Phi(\omega) = -\frac{2t_o\omega_m^2}{3\omega}, \qquad (11)$$

where $t_0 = \frac{y_m}{c\cos\phi_0}$, y_m- hemithickness of a layer, ω_m- critical frequency of ionosphere, ϕ_0- the angle of a plane wave path. After approximation the formula (11) by quadratic polynomial and using (6) we determine such a relative bandwidth of a radiosignal that pulse broadening is not greater than 2:

$$\frac{\Delta}{\omega_0} = \sqrt{\frac{1}{1+4A/(\beta\omega_0)}} \qquad (12)$$

Here is $A = \frac{2t_o\omega_m^2}{3}$, $\omega_0 = 0.5(\omega_1+\omega_2)$, $\Delta = |\omega_2-\omega_0|$. The accuracy of parabolic approximation is about 2-3%. Some graphs are represented in figure 2 (β=5.0).

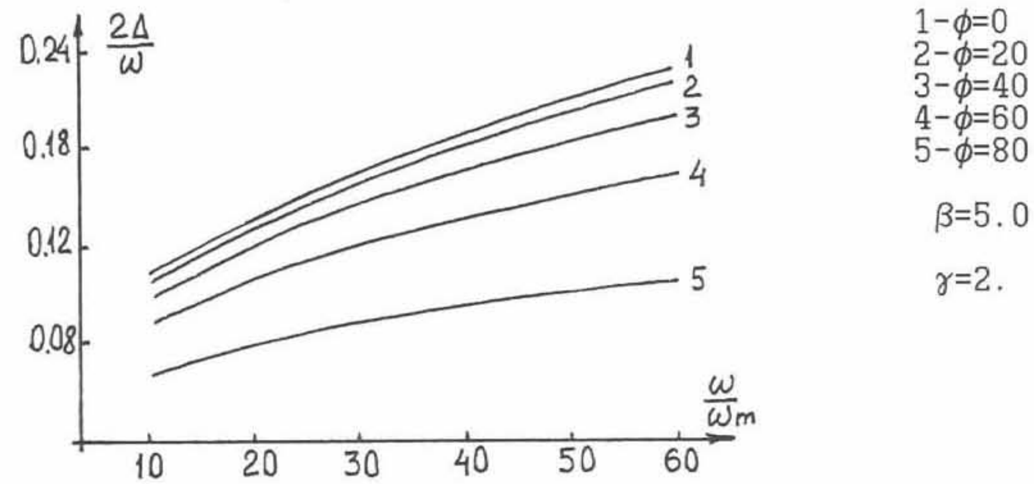

fig.2

The analysis based upon exploring model characteristics (1) describes well the behavior of radoipulses in a medium with monotonous dispersive dependence on frequency. But if the phase function is not monotonous or has some points of bend than the obtained results have limited applications.

References

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2. Lin K.H., Yeh K.C.,Soicher H. Vertical ionograms and dispersive bandwidth for an oblique path.// Radio science, 1989, 24, N4, P.519-526