ON VLF WAVE ATTENUATION IN THE ANISOTROPIC EARTH-IONOSPHERE WAVEGUIDE NEAR CUT-OFF FREQUENCIES

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It is known that under the conditions of night ionosphere and during solar eclipses low-frequency signals generated in the Earth-ionosphere waveguide by lightning discharges are registered at large distances by a receiver as a sum of quasisinusoidal oscillations with monotonously increasing periods tending to periods which correspond to the cut-off frequencies of waveguide modes. Such signals are known as tweeks. A necessary condition for the tweek manifestation is the magnetization of the electrons of low ionospheric plasma at the effective frequencies of the reflection of waves with frequencies forming the tweeks: $\omega_{Be} \gg 1$, where ω_{Be} is gyrofrequency, ν is effective frequency of the cocollisions of electrons.

Ohtsu [1] was the first to calculate the attenuation coefficients of first modes near the cut-off frequencies for an isotropic Earth-ionosphere waveguide. Barr [2] investigated wave propagation characteristics in a frequency range of 100 Hz - 10 kHz for a waveguide model with an inhomogeneous ionosphere under daytime conditions taking into account a vertically directed geomagnetic field. As to typical night ionospheric conditions, the computation of the attenuation coefficients of normal waves of an anisotropic waveguide in a frequency range of 1,5 - 10 kHz was done by Yamashita [3] . What was used was a plane waveguide model with a sharply restricted homogeneous ionosphere, an ideally conducting Earth and a vertical geomagnetic field. Yamashita showed that in a frequency domain essential for tweeks the attenuation coefficients of normal quasi-TE waves (QTE) are appreciably lower than those of normal quasi-TM waves (QTM) and have deep minima near the cut-off frequencies $\alpha^{(n)}_{min} <$ 1 dB/Mm. Their values increase monotonously with the increase of a mode number n . This secures the QTE wave propagation to considerable distances from the source, and, due to a strong dispersion near the cut-off frequences, the pulsed signals acquire a tweek form observed. In a recent work [5] Yedemskii et all. the results of [3] are recived analytically. The purpose of the present communication is to obtain analytical expressions of the attenuation coefficients of the normal Earth-ionosphere waveguide modes for the model [3] with due regard to a finite Earth's conductivity. The latter turns to be rather essential for interpreting the tweek characteristics.

The equation for the eigen values of the normal modes of an anisotropic waveguide with a sharply restricted homogeneous ionosphere has a form [4]

$$(e^{-2i\kappa_0 Ch} - {}_{\parallel}R_{\parallel} R_g)(e^{-2i\kappa_0 Ch} - {}_{\perp}R_{\perp} R_g^h) - {}_{\parallel}R_{\perp}\,{}_{\perp}R_{\parallel} R_g R_g^h = 0 \qquad (1)$$

(the dependence of fields on time $e^{-i\omega t}$), the refraction indices of waves from ionosphere ${}_{\alpha}R_{\beta}$ and from the Earth's surface R_g , R_g^h are given by the expressions

$$\begin{aligned}
{}_{\parallel}R_{\parallel} &= [(\mu_o+\mu_e)(C^2-C_oC_e)+(\mu_o\mu_e-1)(C_o+C_e)C]/\mathcal{D}\\
{}_{\perp}R_{\perp} &= [(\mu_o+\mu_e)(C^2-C_oC_e)-(\mu_o\mu_e-1)(C_o+C_e)C]/\mathcal{D}\\
{}_{\parallel}R_{\perp} &= -2iC(\mu_oC_o-\mu_eC_e)/\mathcal{D}\\
{}_{\perp}R_{\parallel} &= -2iC(\mu_oC_e-\mu_eC_o)/\mathcal{D}\\
R_g &= (\mu_gC-C_g)/(\mu_gC+C_g)\\
R_g^h &= (C-\mu_gC_g)/(C+\mu_gC_g)
\end{aligned} \qquad (2)$$

where $\mathcal{D} = (\mu_o+\mu_e)(C^2+C_oC_e)+(\mu_o\mu_e+1)(C_o+C_e)C$,

$C^2+S^2 = 1$, C and S are cosinus and sinus of a complex angle of incidence of a wave, $S = \mu_o S_o = \mu_e S_e = \mu_g S_g$, $\kappa_o = \omega/c$ is wave number in vacuum, h is waveguide height. The refraction indices of an ordinary μ_o and an extraordinary μ_e wave for the ionosphere are

$$\mu^2_{\substack{o\\e}} = 1 - \frac{\omega_{pe}^2}{\omega(\omega+i\nu\pm\omega_{Be})} , \qquad (3)$$

the refraction index for the Earth

$$\mu_g^2 = i\frac{\sigma_g}{\omega\varepsilon_o} . \qquad (4)$$

Here ω_{pe} is plasma frequency, σ_g is the Earth's conductivity, and ε_o is dielectric constant.

Taking into account that under the conditions of night ionosphere at the frequencies essential for the VLF wave reflection $\omega_{Be} \gg \nu$, $|\mu_g| \gg 1$, we carry out an expansion in (2)-(3) over parameters $\nu/\omega_{Be} \ll 1$, $\omega/\omega_{Be} \ll 1$, $\mu^{-1/2} = (\omega\,\omega_{Be})^{1/2}/\omega_{pe} \ll 1$ and $|\mu_g| \gg 1$, keeping in the expressions for the reflection coefficients, the terms of the orders $\mu^{-3/2}$, $(\nu/\omega_{Be})\mu^{-1}$, $|\mu_g|^{-1}$.

The solutions of the equation (1) can be presented as

$$C_n = \frac{\pi n}{\kappa_o h} - i\frac{\delta_n}{\kappa_o h} , \quad n = 1, 2, \dots , \qquad (5)$$

having $|\delta_n| \ll \pi$. From (1) for δ_n there goes the equation

$$\delta_n^2 - \frac{e^{-i\frac{\pi}{4}}}{(2\mu)^{1/2}} \frac{p_n^2+1}{p_n}\delta_n - \frac{i}{\mu} = f(\delta_n^3, \delta_n^4, \frac{\nu}{\omega_{Be}}, \delta_g) \qquad (6)$$

solved via the method of subsequent approximations (here $p_n = \pi n/(\kappa_o h)$, $|f| \ll \mu^{-1}$). Out of the solution of this equation in the second approximation over small parameter $|\delta_n|/\pi \ll 1$

for the propagation constants $S_n = (1 - C_n^2)^{1/2}$ near the cut-off frequencies of modes $|k_0 h - \pi n| = |\varepsilon| \ll \pi$ we find the expression

$$S_n^{QTM} \simeq \left(\frac{2}{\pi n}\right)^{1/2} \left[\varepsilon - \frac{3\varepsilon^2}{2\pi n} + \frac{i}{\mu^{1/2}}\left(1 - \frac{2\varepsilon}{\pi n}\right) + \frac{1}{2\pi n \mu} - \frac{i}{6\mu^{3/2}} + \frac{i}{\mu_g} + \frac{1}{2\mu^{1/2}}\frac{\nu}{\omega_{Be}} + \frac{1}{2\pi^2 n^2 \mu^{1/2}}\left(\varepsilon + \frac{i}{\mu^{1/2}}\right)^2\right]^{1/2} \quad (7)$$

$$S_n^{QTE} \simeq \left(\frac{2}{\pi n}\right)^{1/2} \left[\varepsilon - \frac{3\varepsilon^2}{2\pi n} + \frac{1}{\mu^{1/2}}\left(1 - \frac{2\varepsilon}{\pi n}\right) - \frac{1}{2\pi n \mu} + \frac{1}{6\mu^{3/2}} + \frac{i}{\mu_g} + \frac{i}{2\mu^{1/2}}\frac{\nu}{\omega_{Be}} + \frac{i}{2\pi^2 n^2 \mu^{1/2}}\left(\varepsilon + \frac{1}{\mu^{1/2}}\right)^2\right]^{1/2} \quad (8)$$

The real part of the S_n constant determines phase velocity of the n-th, mode V_{ph_n}, the imaginary one – attenuation coefficient α_n : $V_{ph_n} = c/\mathrm{Re}\, S_n$, $\alpha_n = 8{,}68\ 10^3 k_0 \mathrm{Im}\, S_n$ (in dB/Mm). According to (7), near the cut-off frequency the attenuation coefficient of the n-th QTM mode (right-handed polarized [5]) decreases monotonically with the increasing frequency (see [3]), and has a weak dependence on the Earth's conductivity σ_g, varying only by several percent upon the σ_g change in the characteristic limits $10^{-4} \div 4$ S/m. As to the **description** of the tweek formation, the QTE modes present a greater interest. The attenuation coefficient of the n-th QTE mode (left-handed polarized [5]) for $0 < \varepsilon \ll \pi$ is given by the expression

$$\alpha_n^{QTE} \simeq 2{,}17 \cdot 10^3 \frac{(2\pi n)^{1/2}}{h \mu^{1/2} (\varepsilon + \mu^{-1/2})^{1/2}} \left[\frac{\nu}{\omega_{Be}} + \frac{(2\mu)^{1/2}}{|\mu_g|} + \frac{(\varepsilon + \mu^{-1/2})^2}{\pi^2 n^2}\right] \quad (9)$$

assuming for

$$f_n = \frac{cn}{2h}\left\{1 - \left(\frac{c\,\omega_{Be}}{\pi n\, \omega_{pe}^2 h}\right)^{1/2} + \left[\frac{\nu}{3\,\omega_{Be}} + \frac{\omega_{pe}}{3}\left(\frac{2\varepsilon_0}{\omega_{Be}\sigma_g}\right)^{1/2}\right]^{1/2}\right\} \quad (10)$$

the minimum value (for $\sigma_g = \infty$ see [5])

$$\min \alpha_n^{QTE} = 8{,}68 \cdot 10^3 \frac{(2\pi n c\, \omega_{Be})^{1/2}}{\omega_{pe}\, h^{3/2}} \left[\frac{\nu}{3\omega_{Be}} + \frac{\omega_{pe}}{3}\left(\frac{2\varepsilon_0}{\omega_{Be}\sigma_g}\right)^{1/2}\right]^{3/4}. \quad (11)$$

It follows from (9)-(10), that the attenuation coefficients of the QTE modes, that form tweeks, increase with the increase of the mode number n, their minimum values are proportional to $n^{1/2}$. Near the cut-off frequencies the absorption of the QTE waves is appreciably determined by the losses upon the reflection not only from the ionosphere, but also from the Earth. In particular, for the characteristic values of the electronic concentration $N_e \sim 3\ 10^2 \div 3\ 10^3$ cm^{-3} and the cocollision frequency $\nu \sim (1 \div 3)\ 10^3$ s^{-1} this takes place even at $\sigma_g \sim 10^{-2}$ S/m. When the Earth's conductivities are very low ($\sigma_g < 10^{-3}$ S/m), the contribution of the latter into attenuation can be more tangible.

Fig.1 demonstrates the plots of the frequency dependence of the attenuation coefficients of the first three QTE modes for h = 90 km, $N_e = 10^3 cm^{-3}$, $\nu = 10^5$ s^{-1} and $\omega_{Be} = 7\cdot10^6$ s^{-1}. Solid lines show the results of Yamashita's computation [3] ($\sigma_g = \infty$), dots correspond to the results of the computation via the formula (9) also for $\sigma_g = \infty$, dashed lines are the results of the computation via the formula (9) for $\sigma_g = 10^{-3}$S/m. Fig.2 illustrates the variation of the attenuation coefficient minimum of the first QTE mode and the frequency corresponding to this minimum (10) upon variation of the Earth's conductivity from magnitude σ_g = 4 S/m to $\sigma_g = 10^{-3}$S/m.

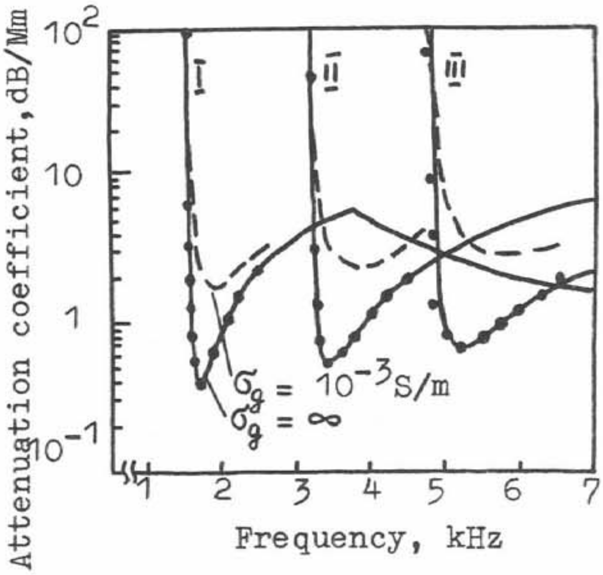

Fig. 1

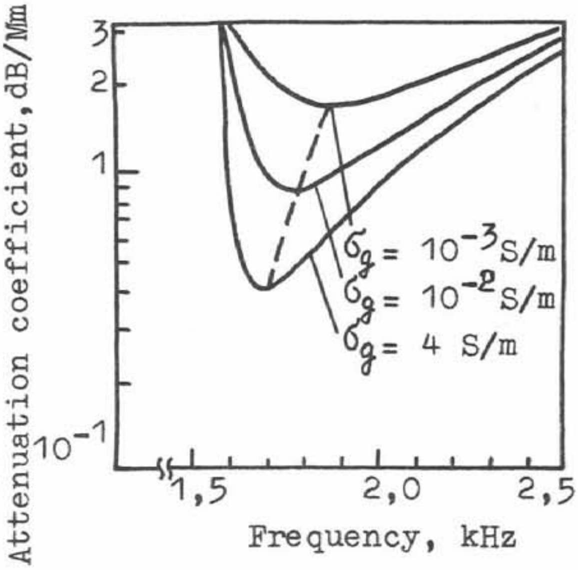

Fig. 2

Thus, the present paper shows that the approximation of an ideally conducting Earth used usually in the VLF range and well-fulfilled for the paths over the sea can be unacceptable for the paths over the ground under the night conditions of propagation in the domain of frequencies close to the cut-off frequencies of waveguide modes. Inhomogeneity of the Earth's conductivity along a propagation path can appreciably limit the possibilities of tweek diagnostics of the low ionosphere.

References

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