Equivalent Steering Vector for ESPAR Antennas and Its Derivation by Using Structural Parameters of Vector Effective Length

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1. Introduction

An Electronically Steerable Parasitic Array Radiator (ESPAR) antenna is constructed with a feed antenna and parasitic antennas loaded with variable reactors [1][2]. Parasitic antennas are excited by mutual coupling. By controlling the reactance value, the excited current of each element antenna can be varied. Accordingly, the radiation pattern can be varied by reactance control. The radiation pattern can be calculated by analysis, such as using a moment method, FD-TD, HFSS, or ICT [3]. However, the analysis must be repeated every time the reactance values are changed, since both the antenna structure and the variable reactors are analyzed as a whole. By expressing an antenna structure as a set of structural parameters, a radiation pattern can be expressed as a function of variable reactance values. Such a function is built based on a mathematical model. Once a mathematical model is constructed, the variable directivity can be calculated easily by only changing the reactance values. The next section presents an equivalent weight vector (EWV) model [2] and its modified model by introducing the vector effective length [4], which have already been proposed. In section 3, an equivalent steering vector (ESV) model is proposed as a precise mathematical model. In section 4, the relationship between an equivalent steering vector and a vector effective length is derived in the case of a dipole array. This paper is concluded in section 5.

2. ESPAR Antenna and Equivalent Weight Vector Model

A port is defined at the part of an element antenna where a variable reactor or a feed circuit is joined to it. The current that flows at a port is called a port current in this paper. The port current i_m can be calculated by the following equation.

$$[i_m]=([Z_{mn}]+[Zx])^{-1}[vs] \quad (1)$$
$$[Zx]=\mathrm{diag}[Zs,\mathrm{j}x_1,\ldots,\mathrm{j}x_M]$$
$$[vs]=[vs,0,\ldots,0].$$

Here, Z_{mn} is impedance between the port of the m-th element and the port of the n-th element. Z_{mn} depends only on the design of the antenna structure. Such a parameter is called a structural parameter. $[Zx]$ is a diagonal matrix composed of reactance values x_m. vs and Zs are open circuit voltage and impedance of the feed circuit, respectively. Using the port currents as weights of each element antenna, a radiation pattern $\boldsymbol{E}(\theta,\phi)$ is calculated by the following equation.

$$\boldsymbol{E}(\theta,\phi)=\boldsymbol{Eel}(\theta,\phi)\sum_{m=0}^{M} a_m(\theta,\phi)i_m \quad (2)$$
$$a_m(\theta,\phi)=\exp(\mathrm{jk}\boldsymbol{\rho}_m\cdot\hat{\boldsymbol{r}}). \quad (3)$$

Here, $\boldsymbol{Eel}(\theta,\phi)$ is an element pattern. M indicates the number of parasitic antennas. The element number m=0 indicates the feed element. $\hat{\boldsymbol{r}}$ and k indicate the unit vector to the (θ,ϕ) direction and the wave number, respectively. The procedure used to calculate a radiation pattern using eq. (1) and (2) is called an equivalent weight vector (EWV) model [2].

In the EWV model, the current that flows on element antennas, excepting the ports, is not considered for radiation patterns. A current distribution on an element antenna can be expressed as a vector effective length (VEL). Therefore, an equivalent weight vector model can be modified by also considering VEL $\boldsymbol{le}_m(\theta,\phi)$ as current weights, as in eq. (4).

$$\boldsymbol{E}(\theta,\phi)=\sum_{m=0}^{M} a_m(\theta,\phi)\boldsymbol{le}_m(\theta,\phi)i_m. \quad (4)$$

In the case of an ESPAR antenna constructed with dipoles as shown in Fig. 1, a vector effective length

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$\boldsymbol{le}_m(\theta,\phi)$ can be expressed by the port current i_m and the port voltage v_m as follows [4].

$$\boldsymbol{le}_m(\theta,\phi)=A\, le_m^{(0)}(1-\mathrm{j}a_m v_m/i_m)\, \boldsymbol{c}(\theta) \quad (5)$$

$$A=-\mathrm{j}kZ_0 e^{-\mathrm{j}kr}/(4\pi r)$$

$$\boldsymbol{c}(\theta)\cong\hat{\boldsymbol{\theta}}\sin\theta$$

Here, $\hat{\boldsymbol{\theta}}$ and Z_0 indicate the unit vector of θ direction and the wave impedance in free space, respectively. In the case of a parasitic dipole loaded with a variable reactor of reactance value x_m, the ratio of a port current i_m and a port voltage v_m has the following relation.

$$v_m=-\mathrm{j}x_m i_m. \quad (6)$$

Inserting eq. (6) into eq. (5), we have the following equation.

$$\boldsymbol{le}_m(\theta,\phi)=le_m^{(0)}(1-\alpha_m x_m)\, \boldsymbol{c}(\theta). \quad (7)$$

Because $le_m^{(0)}$ and α_m are independent of variable reactance x_m, they are structural parameters. Accordingly, the current distribution can be easily considered in calculating the radiation pattern of an ESPAR antenna by using eq. (1), (4), (5), and (7). This calculation procedure is called the EWV-VEL model.

3. Equivalent Steering Vector Model

Although the current distribution along a dipole antenna can be considered in a vector effective length, the current that flows on the antenna body other than the dipoles, such as on the ground plane, and the effects of dielectric material, such as a radome, cannot be considered in the EWV-VEL model. This paper proposes a new mathematical model that can consider these effects. Current $\boldsymbol{i}(\boldsymbol{\rho})$ at an arbitrary point $\boldsymbol{\rho}$ on an antenna depends on port voltage v_m linearly as eq. (8).

$$\boldsymbol{i}(\boldsymbol{\rho})=\sum_{m=0}^{M} \boldsymbol{f}_m(\boldsymbol{\rho})v_m. \quad (8)$$

Since a radiation pattern can be expressed by the integral of the current, we have the following equation.

$$\begin{aligned}\boldsymbol{E}(\theta,\phi)&=A\,\hat{\boldsymbol{r}}\times\int \boldsymbol{i}(\boldsymbol{\rho})a_\rho(\theta,\phi)\mathrm{d}\boldsymbol{\rho}\times\hat{\boldsymbol{r}}\\ &=A\,\hat{\boldsymbol{r}}\times\sum_{m=0}^{M}\int \boldsymbol{f}_m(\boldsymbol{\rho})a_\rho(\theta,\phi)\mathrm{d}\boldsymbol{\rho}\, v_m\times\hat{\boldsymbol{r}}\\ &=A\sum_{m=0}^{M}\boldsymbol{u}_m^{(v)}(\theta,\phi)v_m, \end{aligned} \quad (9)$$

$$\boldsymbol{u}_m^{(v)}(\theta,\phi)=\hat{\boldsymbol{r}}\times\int \boldsymbol{f}_m(\boldsymbol{\rho})a_\rho(\theta,\phi)\mathrm{d}\boldsymbol{\rho}\times\hat{\boldsymbol{r}}. \quad (10)$$

Because $\boldsymbol{f}_m(\boldsymbol{\rho})$ is independent of port voltage v_m, $\boldsymbol{u}_m^{(v)}(\theta,\phi)$ is a structural parameter. As impedance Z_{mn} and admittance Y_{mn} are defined as,

$$v_m=\sum_{m=0}^{M} Z_{mn}i_n, \qquad i_m=\sum_{m=0}^{M} Y_{mn}v_n, \quad (11)$$

eq. (9) can be rewritten as eq. (12).

$$\boldsymbol{E}(\theta,\phi)=A\sum_{m=0}^{M}\boldsymbol{u}_m^{(i)}(\theta,\phi)i_m, \quad (12)$$

$$\boldsymbol{u}_m^{(i)}(\theta,\phi)=\hat{\boldsymbol{r}}\times\sum_{m=0}^{M}\boldsymbol{u}_n^{(v)}(\theta,\phi)Z_{mn}\times\hat{\boldsymbol{r}}. \quad (13)$$

Because eq. (12) is a similar formulation to eq. (2) and $[a_m(\theta,\phi)]$ is called a steering vector, we call $[\boldsymbol{u}_m^{(v)}(\theta,\phi)]$ and $[\boldsymbol{u}_m^{(i)}(\theta,\phi)]$ equivalent steering vectors (ESV). $\boldsymbol{u}_m^{(v)}(\theta,\phi)$ is a radiation pattern when the m-th port is excited by a unit voltage and the other ports are shorted as shown in Fig. 2(b). $\boldsymbol{u}_m^{(i)}(\theta,\phi)$ is a radiation pattern when the m-th port is excited by a unit current and the other ports are opened as shown in Fig. 2(c). The calculation procedures using equivalent steering vectors are shown in Fig. 3(c), (d) and compared with other models. First, port currents are calculated by using eq. (1). Mutual coupling between ports can be considered in mutual impedance Z_{mn}. Then a radiation pattern is calculated by using eq. (12). All of the current that flows on the antenna body due to mutual interaction is considered in equivalent steering vectors $[\boldsymbol{u}_m^{(v)}(\theta,\phi)]$ and $[\boldsymbol{u}_m^{(i)}(\theta,\phi)]$. Accordingly, a radiation pattern can be calculated precisely. We call this calculation procedure an equivalent steering vector (ESV) model. The effects considered by the mathematical models are compared in Table 1. Mutual coupling between ports is considered in the EWV model. In addition, current distribution along dipoles excited by the mutual coupling are considered in the EWV-VEL model. Not only current distribution along the dipole but also current on the other metal parts of the antenna body and the effects of dielectric material parts are considered in the ESV model.

4. Derivation of Equivalent Steering Vector by Vector Effective Length

Although a steering vector $a_m(\theta,\phi)$ can be determined directly from the position of the element antenna's ports as in eq. (3), equivalent steering vectors $\boldsymbol{u}_m^{(v)}(\theta,\phi)$ and $\boldsymbol{u}_m^{(i)}(\theta,\phi)$ are not obvious. However, these vectors can be calculated by simulation, such as using a moment method. From the definition of ESV, it is clear that ESV has features similar to a steering vector as shown in Table 2 in the case of the ESPAR antenna shown in Fig. 1. In this section, the relation between a steering vector and ESV is discussed in the case of a dipole array antenna. In such a case, only current distribution along dipoles is considered in ESV. Therefore, an equivalent steering vector $u_m^{(v)}(\theta,\phi)$ can be expressed by a vector effective length. As eq. (4) and eq. (12) must be equal, the following relation is achieved.

$$c(\theta)\sum_{m=0}^{M} a_m(\theta,\phi)le_m(\phi)i_m = \sum_{m=0}^{M} u_m^{(i)}(\theta,\phi)i_m. \quad (14)$$

Using eq. (5) and (11), We obtain the following equations.

$$u_m^{(i)}(\theta,\phi)=\{le_m^{(0)}a_m(\theta,\phi)\text{-j}\sum_{m=0}^{M} \alpha_n le_n^{(0)}Z_{mn}a_n(\theta,\phi)\}c(\theta), \quad (15)$$

$$u_m^{(v)}(\theta,\phi)=\{\sum_{m=0}^{M} le_n^{(0)}Y_{mn}a_n(\theta,\phi)\text{-j}\alpha_m le_m^{(0)}a_m(\theta,\phi)\}c(\theta). \quad (16)$$

Equations (15) and (16) show that the equivalent steering vectors $u_m^{(i)}(\theta,\phi)$ and $u_m^{(v)}(\theta,\phi)$ can be calculated from a steering vector $a_m(\theta,\phi)$, structural parameters Z_{mn}, Y_{mn}, $le_m^{(0)}$, and α_m. Z_{mn} and Y_{mn} express the current-voltage relation between ports, and $le_m^{(0)}$ and α_m express the current distributions along dipoles. As $a_m(\theta,\phi)$ is a green function in free space, $u_m^{(i)}(\theta,\phi)$ and $u_m^{(v)}(\theta,\phi)$ correspond to the green function constrained by the antenna boundary. If the size and direction of dipoles are uniform and there is no mutual coupling between dipoles, the equivalent steering vector can be expressed as follows.

$$u_m^{(i)}(\theta,\phi)=le_m^{(0)}(1\text{-j}\alpha_m Zin_m)a_m(\theta,\phi)c(\theta), \quad (17)$$

$$u_m^{(v)}(\theta,\phi)=le_m^{(0)}(1/Zin_m\text{-j}\alpha_m)a_m(\theta,\phi)c(\theta). \quad (18)$$

Here, Zin_m is input impedance because Z_{mm} equals Zin_m when there is no mutual coupling. Equations (17) and (18) show that $u_m^{(i)}(\theta,\phi)$ and $u_m^{(v)}(\theta,\phi)$ can be substituted by $a_m(\theta,\phi)$ becuase they are proportional to $a_m(\theta,\phi)$. The relations among $u_m^{(i)}(\theta,\phi)$, $u_m^{(v)}(\theta,\phi)$ and $a_m(\theta,\phi)$ are summarized in Table 3.

In order to ascertain eq. (16), the equivalent steering vector $u_m^{(v)}(\theta,\phi)$ of the ESPAR antenna (Fig. 1) is calculated. The Horizontal pattern calculated by a moment method is indicated as solid lines in Fig. 4. The broken line shows the results calculated by using eq. (16). The structural parameters Y_{mn}, $le_m^{(0)}$, and α_m in eq. (16) are calculated by using the same moment method and shown in Table 4. The two lines in Fig. 4(a) agree well. The lines in Fig. 4(b) agree by shifting about 90 degree. This shift can be explained by the difference in the phase center definition. Consequently, eq. (16) is proved to be effective.

5. Summary

An equivalent steering vector model was proposed for calculating the radiation pattern of an ESPAR antenna. Using this mathematical model, variable directivity could be calculated easily by only changing the reactance value. The current distribution and the effects of dielectric material were considered in this model. Furthermore equations were proposed to calculate equivalent steering vectors from a steering vector, an impedance or admittance matrix, and structural parameters ordering vector effective lengths.

Acknowledgments

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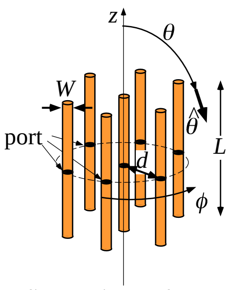
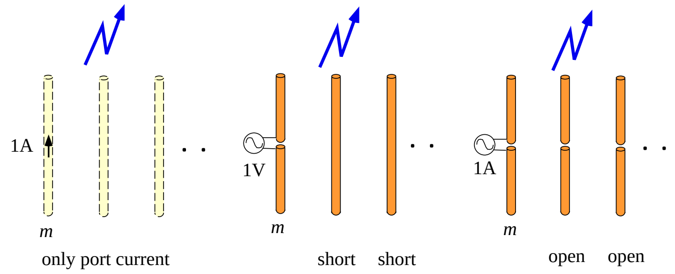
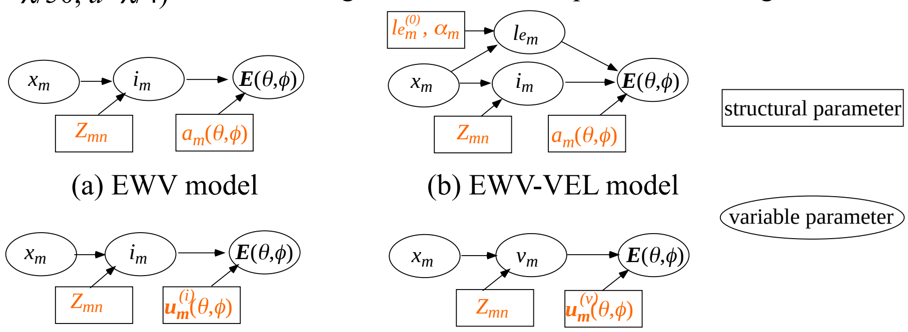


Fig. 1 Configuration of an ESPAR antenna ($L=\lambda/2$, $W=\lambda/50$, $d=\lambda/4$)



(a) $a_m(\theta,\phi)$ (b) $\boldsymbol{u}_m^{(v)}(\theta,\phi)$ (c) $\boldsymbol{u}_m^{(i)}(\theta,\phi)$

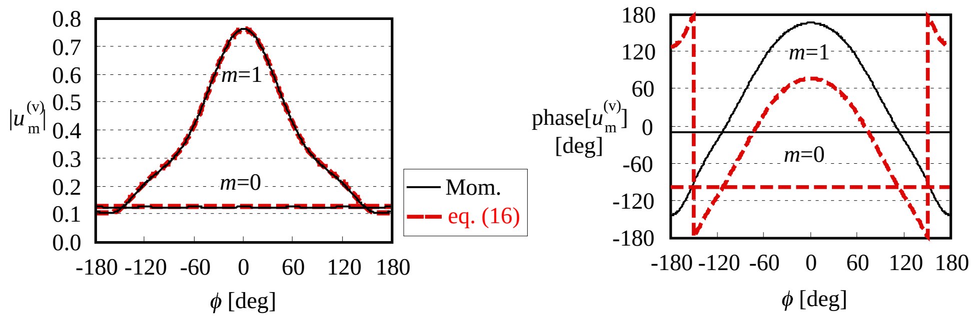
Fig. 2 Definition of equivalent steering vectors



(a) EWV model (b) EWV-VEL model

(c) ESV model ($\boldsymbol{u}_m^{(i)}(\theta,\phi)$) (d) ESV model ($\boldsymbol{u}_m^{(v)}(\theta,\phi)$)

Fig. 3 Calculation procedure



(a) Amplitude (b) Phase

Fig. 4 Horizontal pattern of equivalent steering vector $u_m^{(v)}(\pi/2,\phi)$

Table 1 Effects considered by each mathematical models

Mathematical model	EWV	EWV-VEL	ESV
Mutual coupling between ports	○	○	○
Current distribution along dipoles	×	○	○
Current on ground plane or effects of dielectric material	×	×	○

○ : considered × : not considered

Table 2 Features of SV and ESV of ESPAR antenna shown in Fig .1

Feature	Steering vector	Equivalent steering vector
Cyclic parasitic elements	$a_{m+1}(\theta,\phi)=a_m(\theta,\phi-2\pi/M)$	$u_{m+1}(\theta,\phi)=u_m(\theta,\phi-2\pi/M)$
Omnidirectional feed element	$a_0(\theta,\phi)=a_0(\theta,0)$	$u_0(\theta,\phi)\fallingdotseq u_0(\theta,0)$
Isotropic amplitude	$\lvert a_m(\theta,\phi)\rvert=1$	no

Table 3 ESV expressions by SV

	no mutual coupling	with mutual coupling
$u_m^{(i)}(\theta,\phi)$	$le_m^{(0)}\{1-j\alpha_m Zin_m\}a_m(\theta,\phi)c(\theta)$	$\{le_m^{(0)}a_m(\theta,\phi)-j\Sigma\alpha_n le_n^{(0)}Z_{mn}a_n(\theta,\phi)\}c(\theta)$
$u_m^{(v)}(\theta,\phi)$	$le_m^{(0)}\{1/Zin_m-j\alpha_m\}a_m(\theta,\phi)c(\theta)$	$\{\Sigma le_n^{(0)}Y_{mn}a_n(\theta,\phi)-j\alpha_m le_m^{(0)}a_m(\theta,\phi)\}c(\theta)$

Table 4 Structural parameters of ESPAR antenna shown in Fig .1

$m\Omega^{-1}$	Y_{00}	Y_{01}	Y_{11}	Y_{12}	Y_{13}	Y_{14}
Real	0.2459	-0.1822	3.0815	1.0010	-0.3112	-0.1007
Imaginary	-6.8699	2.5142	-4.4845	3.0136	-0.3152	-0.1541

m	0	1,...,6
$le_m^{(0)}$	0.6513	0.6503
α_m	0.00214	0.00219