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IONOSPHERIC LARGE-SCALE MOTIONS TRANSITION INTO CHAOTIC FLOWS

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Abstract

In the given work the large scale ionospheric flows, caused by spatial inhomogeneous geomagnetic field, and is studied. The closed system of nonlinear Lorentz type equations is obtained. On the basis of numerical calculations of these equations transition to chaos and formation of order-disorder is observed. Characteristics of the strange attractors are defined.

The ionosphere can be represented as a medium where the motion dynamics depends not only on the internal structure, but also on external influences. Electromagnetic disturbances (which produce essential disturbances of a geomagnetic field) manifest themselves in the form of forced oscillations which emerge when the ionosphere is subjected to the action from above during magnetic storms or from below during earthquakes. Convective motion under the action of a magnetic field, i.e. magnetic convection can be regarded as an important mechanism that determines the internal dynamics of the ionosphere. In that case, possible convective motion is completely explained by the interaction of the ionospheric plasma with the Earth's inhomogeneous magnetic field and its dynamics will depend on the disturbance of the geomagnetic field. Therefore motions of the ionospheric medium are determined by internal and external factors, their interaction and interdependence. This feedback may result in the emergence of self-sustained oscillations and spatial dissipative structures. For an adequate description of such processes connected with the emergence of structures in the ionospheric medium it is necessary to investigate the nonlinear effects taking into account the dispersion and dissipation factors.

Such problems of the dynamics of large-scale and low-frequency ionospheric disturbances of electromagnetic character are solved using equations of magnetic hydrodynamics of the ionosphere [1], where the latitudinal gradient of the geomagnetic field and the angular velocity of the Earth's rotation are taken into account. Using the generalized Ohm's law, we can write equations of magnetic hydrodynamics for large-scale ionospheric processes as follows:

$$\frac{d\,V}{dt} = -\frac{1}{\rho}\mathring{\nabla} p + \mathring{g} + \mathring{V} \times 2\mathring{\Omega}_0 + \frac{1}{\mu_0 \rho}\mathring{\nabla} \times \mathring{B} \times \mathring{B} - \Lambda \mathring{V} \tag{1}$$

$$\frac{\partial B}{\partial t} = \mathring{\nabla} \times \mathring{V} \times \mathring{B} - \frac{\alpha}{\mu_0}\mathring{\nabla} \times \mathring{\nabla} \times \mathring{B} \times \mathring{B} + \frac{1}{\mu_0 \rho v_i}\mathring{\nabla} \times \mathring{\nabla} \times \mathring{B} \times \mathring{B} \times \mathring{B}; \tag{2}$$

$$\mathrm{div} V = 0\,, \tag{3}$$

$$\mathrm{div} B = 0\,, \tag{4}$$

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where V is the particle motion velocity, α is Hall's parameter, P, ρ are the gas pressure and density, respectively; g is the free fall acceleration, Ω_0 is the angular velocity of the Earth's rotation, $B = B_0 + b$ is the magnetic field inductance, B_0 is the stationary geomagnetic field, b is the disturbance of his field, ∇ is the del operator, $d/dt = \partial/\partial t + (V\nabla)$, v_i is the collision frequency of particles, Λ is the constant friction coefficient of atmospheric layers. The last two equations are obtained from the assumption of the incompressibility of the ionosphere, which can be made in the case of planetary-scale disturbances, and the solenoidal condition of the magnetic field. The last term of the first equation is the Rayleigh friction.

According to experimental data [2], at altitudes over 100—150 km the ratio of the characteristic vertical velocity of disturbance motion to the horizontal velocity is small and therefore large-scale motions in the ionosphere are mostly quasihorizontal. Besides, in the lower E-region of the ionosphere, the velocity of large-scale motion of ions coincides with the velocity of neutral particles, i.e. a complete entrainment of ions by neutrals takes place and the ionosphere moves like a one-component fluid under the action of the magnetic field.

From equations (1) and (2) it is easy to obtain an equation for the vortex $\Omega = \mathrm{rot}V$ and for the magnetic field disturbance b:

$$\frac{d}{dt}\mathrm{rot}\mathring{V} = \mathrm{rot}[\mathring{V}(\mathrm{rot}\mathring{V} + 2\mathring{\Omega}_o)] + \frac{1}{\mu_0\rho}\mathrm{rot}[\mathrm{rot}\mathring{b}, \mathring{B}] + \Lambda\mathrm{rot}\mathring{V} \qquad (5)$$

$$\frac{\partial b}{\partial t} = (\mathring{B}_0\mathring{\nabla})\mathring{V} - (\mathring{V}\mathring{\nabla})\mathring{B}_0 - \frac{\alpha}{\mu_0}\mathring{\nabla}\times\mathring{\nabla}\times\mathring{b}\times\mathring{B} + \frac{1}{\mu_0\rho v_i}\mathring{\nabla}\times\mathring{\nabla}\times\mathring{b}\times\mathring{B}_0\times\mathring{B}_0, \qquad (6)$$

where the conditions $\nabla \times B_0 = 0$, $\nabla \cdot B_0 = 0$ and $\nabla b = 0$. are used.

Thus for an incompressible fluid flow in the x, y-plane we have $V_z = 0$, (z is the vertical axis, the x-axis is directed along the parallel to the east, the y-axis is directed along the meridian to the north). It is assumed that the geomagnetic field has only the vertical component $B_z = -B_p \cos\theta$, where $\breve{B}_p$ is the magnetic field of the pole y, θ is the latitude. We introduce the current function $V = [e_z, \nabla\Psi]$, where e_z is the unit vector along the vertical line. In that case, equations (5), (6) can be reduced to the form

$$\frac{\partial\Delta\Psi}{\partial t} = -\beta\frac{\partial\Psi}{\partial} - C_H\frac{\partial b}{\partial x} - \Lambda\Delta\Psi + J(\Psi, \Delta\Psi), \qquad (7)$$

$$\frac{\partial b}{\partial t} = \beta_H\frac{\partial\Psi}{\partial x} - C_H\frac{\partial b}{\partial x} + c'\frac{\partial^2 b}{\partial y^2} + J(\Psi, b), \qquad (8)$$

where b is the z-component of the magnetic field disturbance, Δ is the two-dimensional Laplace operator, $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$, J is the Jacobian, $J(a,b) = \partial a/\partial x \cdot \partial b/\partial y - \partial b/\partial x \cdot \partial a/\partial y$.

As is known, for the disturabnce of self-sustained oscillations, the instability must be either global or absolute. The instability develops in systems where there exists a feedback between the parameters, which, in unbounded media, takes place at the expense of excitation of contrary waves. Such waves moving in opposite directions may arise under the interaction of waves in inhomogeneous media and in the case of over-reflection. In the ionospheric medium self-sustained oscillations may arise since there is a feedback there. In addition to this phenomenon, it is interesting to investigate instability convections in the ionosphere and the possibility of emergence of stochastic self-sustained oscillations and chaos in such a medium.

Thermal convection in the planar fluid layer heated from below was numerically investigated by Zaltsman [3]. Finite-dimensional models for the description of the laminar-turbulent approach for systems with distributed parameters are constructed by using the Galerkin procedure which gives us a system of a finite number of ordinary differential equations for disturbance amplitudes. Zaltsman showed that when the number of modes increases, in the Galerkin approximation near the stability threshold only the first three modes give the largest contribution. The other modes quickly damp and therefore are not essential for qualitative analysis of motion. This is obvious if we take into account the fact that viscosity hinders the existence of small-scale disturbances. From the very beginning Lorenz [4] represented solutions of equations of two-dimensional hydrodynamics in the form of three terms of a Fourier series. Following Lorenz, we represent solutions of equations (7), (8) in the form $\Psi(x,y,t) = X(t)\sin K_x x \sin K_y y$

$$b(x,y,t) = Y(t)\cos K_x x \sin K_y y - Z(t)\sin 2K_y y$$

where K_x, K_y are horizontal wave numbers. If we substitute the above representations of Ψ and b into (7) and (8) and equalize the coefficients of equal modes, then we obtain the well-known Lorenz sastem for the function X(t), Y(t), Z(t):

$$\frac{dx}{dt} = \sigma y - \sigma x$$
$$\frac{dy}{dt} = rx - xz - y \qquad (9)$$
$$\frac{dz}{dt} = xy - dz$$

where

$$\sigma = \frac{\Lambda}{c'B_{0z}^2(\alpha^2+\beta^2)}, \quad r = \frac{\beta_H C_H \alpha^2}{c'B_{oz}^2 \Lambda(\alpha^2+\beta^2)^2},$$
$$d = \frac{4}{1+a^2}; \qquad a = \frac{\alpha}{\beta} = \frac{\pi/a}{\pi/h} = \frac{h}{a},$$
$$r = \frac{R}{R_c}, \qquad R_c = \frac{\pi^2}{a^2}(1+a^2)^2, \qquad R = \frac{C_н B_н}{c'B_{oz}^2\Lambda}h^2$$

r is normalized time.

The Rayleigh number defines the region of the existence of nontrivial solutions with a growth increment greater than zero and shows the relation of lifting force (i.e. force that creates instability) to dissipative forces.

The Lorenz model of Zaltsman's system of equations is useful in qualitative analysis of the character of loss of the stability of stationary states in nonlinear systems with distributed parameters, d gives the geometry of the problem, σ is the Prandl number, $0 \le r \le \infty$ is the controlling parameter of the system, $r = R_a / R_{акк}$ is the normed Rayleigh number. The Lorenz model describes a dissipative system for $r > r_c = 24.74$ as well, motion becomes chaotic (there emerges a peculiar attractor), where $r_c = \sigma(\sigma + h + 3)/(\sigma - b - 1)$. In our case we are interested in motions at large values of r. It is proved that the first narrow periodicity window in solutions emerges for $r = 48.03$ and the second large window for $r = 148.4$, i.e. there occurs a change of chaotic and regular motions. Lorenz equations cannot claim to the description of turbulence, since they do not give the picture of structural changes but give only the evolution of the process in time.

The solution of the convection problem obtained by Rayleigh is a simple solution of a spectral boundary value problem with artificial boundary conditions which allow one to take into account the principal peculiarities of the problem. This approach gives vast possibilities for the investigation of processes of spontaneous emergence of ordered structures. In Rayleigh's solution, spatial and time effects are splitted and therefore it becomes possible to study spatial and time motions separately [5].

I. Prigozhin called stationary Benard cells dissipative structures. Such structures are formed due to the energy exchange between the matter and the environment in unbalance conditions. They get self-organized and the behavior of particles in such structures is a cooperative effort.

The fundamental characteristic feature of the convection emergence process is the existence of a threshold, i.e. of a critical value of the parameter of the system, above which there emerge ordered structures with organized motions.

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