Electromagnetic Scattering and Radiation From Perfectly-Conducting Cylindrical Parabolic Reflector by MEI

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1. Introduction

The analysis of electromagnetic scattering and radiation for an infinite cylindrical reflector have been established for decades. One of the most notable approaches is the Method of Moment (MoM) [1]. By MoM , the integral equation is converted into a full matrix equation system. To solve the full matrix equation, a large number of computer memory is needed in case of electrically large dimension objects and this makes MoM less efficient.

Recently, the measured equation of invariance (MEI) method was introduced by Mei et al. [2] as a mesh truncation condition for the finite difference (FD) method, which allows the termination of the mesh very close to the scatterer surface. This approach has been proved to be more robust than the absorbing or radiation boundary condition applied close to the scatterer surface and, in contrast to the hybrid FE-BEM method, the interrelation between the field points is sparse. Thus, it has the advantages of the two popular methods: the sparse matrix in the FD method and small number of unknowns in MoM.

In this paper, MEI are applied to analysis the scattering and radiation characteristics of two dimensional perfectly-conducting cylindrical parabolic reflectors. MEI solutions of induced surface current of reflectors with different dimensions have been obtained, accompanied with corresponding Physic-Optics (PO) solutions. The scattering and radiation field patterns are also obtained by integration the induced current over the reflector surface.

2. Formulation

Consider an Electromagnetic scattering when a uniform plane wave or a cylindrical wave in free space impinges on a two dimensional perfectly-conducting cylindrical parabolic reflector, as shown in Fig.1. The equation of the parabolic cylindrical reflector is

$$y^2 = 4F_c x \tag{1}$$

in which $|y| \le A$, A is the reflector aperture dimension. The reflector will become a parabolic sheet when the thickness of the reflector $w = 0$. F_c is the focus length of the parabolic cross-section.

a. The Uniform Incident Wave

It is assumed that the electric field of the incident wave has only a z-component (TM_z case) and it is not a function of z. The time-factor $e^{j\omega t}$ has been suppressed. Thus, the incident wave can be expressed as

$$\vec{E}_z^{inc}(\vec{r}) = \hat{a}_z E_0 \exp\{ik(x\cos\varphi_0 + y\sin\varphi_0)\} \tag{2}$$

where $\hat{a}_z$ is a unit vector and $\vec{r} = x\hat{a}_x + y\hat{a}_y$. φ_0 is the angle between the traveling direction $\vec{k}$ of incident wave and the $\hat{x}$-axis. Consider the total electric field $\vec{E}$ at $\vec{r}$ in space, there are

$$\vec{E}(\vec{r}) = \vec{E}_z^{inc}(\vec{r}) + \vec{E}_z^{sc}(\vec{r}) \tag{3a}$$

and

$$(\nabla_t + k^2)\vec{E}_z^{sc}(\vec{r}) = 0 \tag{3b}$$

$\nabla_t = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$ is the transverse Laplace operator. $\vec{r}'$ is the position vector of the current filament on the reflector surface. The unknown $\vec{E}^{sc}(\vec{r})$ is the scattering field and is generated by the z-directed induced current density $\vec{J}_z(\vec{r}')$ radiating in unbounded free space. $\vec{E}^{sc}(\vec{r})$ has only a z-component and is not a function of z. The induced current density $\vec{J}_z(\vec{r}')$ on the reflector surface is

$$\vec{J}_z(\vec{r}') = -\frac{1}{j\eta_0 k}\frac{\partial E}{\partial n} \tag{4}$$

η_0 is the intrinsic impedance of free space. To numerically calculate scattering field and induced current distribution, the space around the scatterer is discretized into a wraparound mesh and the equation (3b) is written into a linear matrix equation. For the points on the reflector surface, the boundary condition (B. C.) is applied, that is

$$E_z^{inc}(\vec{r}') + E_z^{sc}(\vec{r}') = 0 \tag{5}$$

For the points between the scatterer surface and the terminating layer, the wave eqn.(3b) is employed and is discretized into standard 5-point FD equation. MEI is applied for the points on the terminating layer. In this paper, a six-point MEI is used, as shown in Fig.2. The MEI for the fields at these six points is

$$\sum_{i=1}^{6} c_i E_i^{sc} = 0 \tag{6}$$

where c_i $(i = 1,2,\ldots 6)$ is the MEI coefficient. According the three postulates of [2], the MEI coefficients are found from eqn.(6), in which E_i^{sc} is replaced by the measuring function E_i^p which can be calculated from scattering field formula in the integral form of

$$\vec{E}_z^{sc}(\vec{r}) = -\frac{\eta_0}{4}\oint_c \vec{J}_z(\vec{r}')H_0^{(2)}[k(\vec{r} - \vec{r}')]d\ell' \tag{7}$$

and let $\vec{J}_z(\vec{r}')$ be a postulated current distribution on the scatterer surface. C is the cross-section boundary of the cylindrical parabolic reflector. In general $\vec{r} \notin C$, which allows us avoiding the integration over a singularity. $d\ell'$ is the arc length of the current filament on C.

Finally, using the FD coefficients, MEI coefficients and B.C., equation can be written out for each unknown field $E^{sc}{}_{ij} = E_z^{sc}(\vec{r}_{ij})$ at point (i, j), the resulting system of the whole scattering problem is a matrix equation

$$[A][E_{ij}^{sc}] = [s] \tag{8}$$

It is very advantageous that $[A]$ is a sparse matrix. The vector [s] contains the source terms which are usually zero except for those points on the scatterer surface. By solving (8), we can directly obtain the scattered field. The induced surface current density can therefore be calculated from (4). With induced current distribution, the scattered field at an arbitrary point in the space can be calculated from (7). The analysis of the TE_z case that the incident wave has only a magnetic field component in the z-direction is a similar process.

b. Radiation Analysis by MEI

Consider a line source located at the focus of the cylindrical reflector, and let field radiated by the line source be:

$$E_z = \frac{\exp(-jk\rho)}{\sqrt{\rho}}\begin{cases}\cos^2\theta_s & \theta_s \ge 90^0 \\ 0 & \theta_s < 90^0\end{cases} \tag{9}$$

MEI is used to calculate the induced current on the reflector surface. The radiation pattern is then calculated by integral the current contribution over the reflector contour.

3. Result

By using the MEI, number of the FD mesh layer can no longer need to be as large as to meet the radiation condition at the infinity, but just three or more layers above the scatterer. Further more the whole problem is inferred to a sparse matrix equation, not a full matrix equation. Therefore computer memories are saved. The solutions of scattering of perfect-conducting cylindrical parabolic structures with different aperture dimension and different focus length have been given out by the MEI method in this paper.

The fields at the mesh point are solved from eqn.(8) and the induced current densities are obtained from (4). Fig.3a shows the induced current distributions (TM_z case) of a reflector with dimension of $A = 50\lambda$, $Fc / A = 0.5$, $w = 0.01\lambda$ by MEI and PO. Fig.3b shows the scattered field patterns of two main planes that are perpendicular to each other and cross through the focus position. Fig.4a shows the induced current distribution of reflector with dimension of $A = 10\lambda$, $Fc / A = 0.5$, $w = 0.01\lambda$ by MEI and PO. The field of the line source is given by (9). Fig.4b shows the radiation field patterns of x-y plane by MEI and GTD [5]. Good agreement of these results can be observed.

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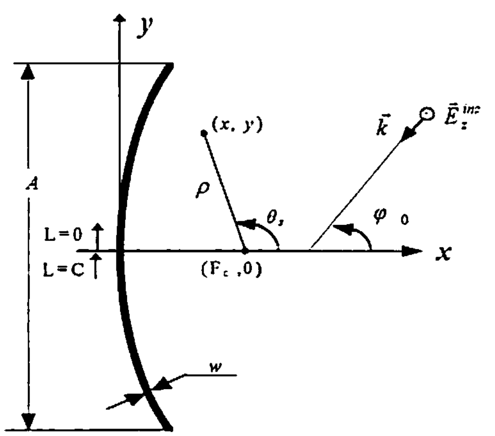

Fig.1 Geometry of the parabolic reflector

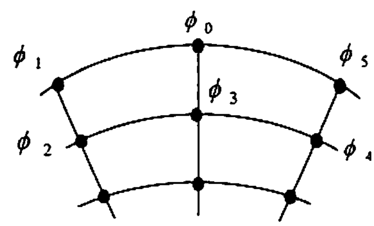

Fig.2 Mesh points of the MEI

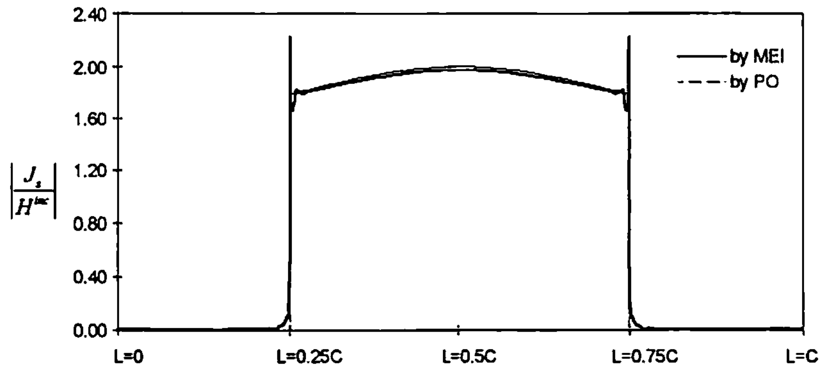

Fig.3a. The induced current density on the reflector surface.

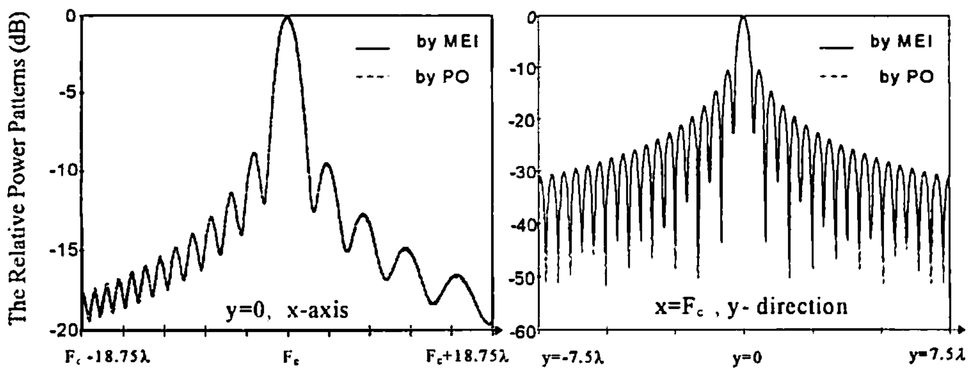

Fig.3b The scattered field patterns

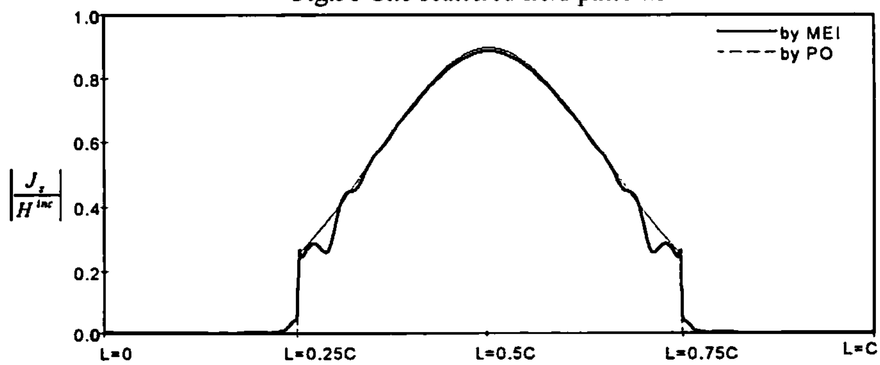

Fig4a. The Induced Current Density on the Reflector Surface

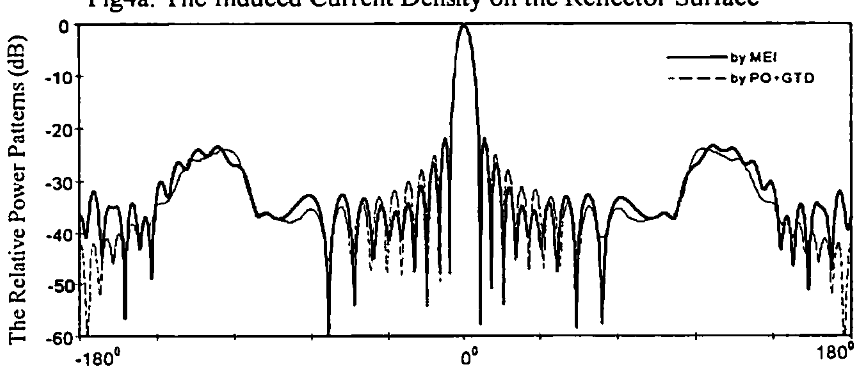

Fig4b. The Radiation Field Pattern of Parabolic Reflector