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A NEW NUMERICAL TECHNIQUE WITH APPLICATION TO ANALYSIS OF WIRE ANTENNAS

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1. Introduction

In a numerical analysis of an electromagnetic problem, particularly for a wire antenna problem, the method of moments[1] is considered as one of the most powerful techniques and therefore has widely been used in analysis and design of various antennas. However, when the electromagnetic objects encountered are large in size with respect to the working wavelength, the computation time consumption and the computer memory requirement might become prohibitive. To circumvent this drawback, quite a few works have been done and a certain degree of success has been achieved[2]-[7].

In this paper, a new numerical technique is presented which enjoys, in comparison with the method of moments and other related numerical methods, merits of good solution accuracy and high computation efficiency for solving a large size problem.

A number of wire antennas are numerically analyzed by means of this method and a favorable coincidence between the calculated results and the results obtained by directly using the method of moments is reaped, which argues this numerical technique to be correct and effective.

2. Formulation

For a thin wire antenna of length L and radius a ($a \ll L$, and $a \ll \lambda$), the E-field equation relating the current distribution $i(l)$ along the axis of the antenna to the impressed electric field $\vec{E}^i$ on the surface of the antenna may be written in the form as

$$T(i) = E_l^i, \tag{1}$$

where E_l^i represents the component of $\vec{E}^i$, tangential to the length direction on the wire, and T an integro-differential operator given by

$$T(i) = j\omega\mu \int_L \left[i(l') + \frac{1}{k^2}\frac{di(l')}{dl'}\frac{\partial}{\partial l} \right] \frac{\exp(-jk \mid \bar{r} - \bar{r}' \mid)}{4\pi \mid \bar{r} - \bar{r}' \mid} dl'. \tag{2}$$

With the help of a typical moment method analysis, the operator equation(1) is converted to be a matrix equation as

$$[Z]\bar{I} = \vec{V}, \tag{3}$$

where $[Z]$ is the impedance matrix with its elements denoted by $z_{mn} = < w_m, Tv_n >$, $\vec{V}=[v_1, v_2, \cdots, v_N]^T$ the voltage vector with its m-th element $v_m = < w_m, E_l^i >$, and $\bar{I}=[I_1, I_2, \cdots, I_N]^T$

the current vector to be determined, which is defined by $i(l) = \vec{U}^T\vec{I}$ with the basis vector $\vec{U}=[u_1, u_2, \cdots, u_N]^T$ chosen usually to be the piecewise pulse functions for the sake of simplicity.

First of all, an approximate solution to this moment matrix equation, symbolized by $\vec{I}^{(0)}$ and referred to as the zeroth order solution to (3), is determined. If the wire antenna to be analyzed is simply structured, say, a linear antenna, then the sinusoidal current distribution $i^{(0)}(l)$, termed the zeroth order solution to (1), would be a good one for determining the vector $\vec{I}^{(0)}$. Otherwise, some calculation must be made for this purpose. To do so, the conjugate gradient method(CGM)[8][9] is employed and only a small number of steps is needed for get such an approximate solution, because the iteration converges monotonically and the convergence rate will slow down after a relatively small number of iterations.

And then, the matrix equation relating the error vector $\vec{E}=\vec{I} - \vec{I}^{(0)}$ to the residual vector $\vec{R}=\vec{V} - [Z]\vec{I}^{(0)}$ is considered, which should have a form similar to (3).

$$[Z]\vec{E} = \vec{R}, \tag{4}$$

where $\vec{I}$ is the exact solution to (3) and $\vec{I}^{(0)}$ the zeroth order solution mentioned above.

This equation might be solved in a space with a dimension lower than that of the original problem, since the error vector $\vec{E}$ has a slower spatial variation than the vector $\vec{I}$ itself.

Let us choose a set of partially overlapping piecewise functions, say, parabola functions,

$$p_m(l) = \begin{cases} 1 - [2(l - L_m)/\Delta L]^2, & L_m - \Delta L/2 \le l \le L_m + \Delta L/2, \\ 0, & \text{elsewhere} \end{cases} \tag{5}$$

where ΔL indicates the length of all intervals and L_m the midpoint of the m-th interval on which the m-th parabola function is defined . $m = 1, 2, \cdots, M$ and each pair of adjacent parabolas is overlapped in a length of $\Delta L/2$. It is conceptually understood that these partially overlapped piecewise functions are better in quality than the pulse functions in the sense that a linear combination of a smaller number of such functions would approach the current distribution equally well in comparison with what the pulses have done. Namely, ΔL can be taken to be larger than Δl, and let us, say, set $\Delta L = 6\Delta l$, here Δl is the length of all subsections on which the piecewise pulse functions are defined. It is easily found that the integers N and M should satisfy $N = 3M + 2$, and the $M \times N$ basis transformation matrix $[Q]$ relating the old vector $\vec{U}=[u_1, u_2, \cdots, u_N]^T$ to the new one $\vec{P}=[p_1, p_2, \cdots, p_M]^T$ should have elements q_{mn} as

$$q_{mn} = \begin{cases} 1 - [2(l_n - L_m)/\Delta L]^2, & n = 3(m - 1) + k, \\ 0, & \text{else}, \end{cases} \tag{6}$$

$$m = 1, 2, \cdots, M;\ n = 1, 2, \cdots, N;\ k = 1, 2, 3, 4, 5.$$

where l_n indicates the midpoint of the n-th subsection on which the n-th pulse function is defined.

If the current distribution $i(l)$ on the wire antenna were expanded again in terms of the new functions as $i(l) = \vec{P}^T\vec{I}'$, then the old current vector $\vec{I}$ would be related to its counterpart $\vec{I}'$ by

$$\vec{I} = [Q]^T\vec{I}'. \tag{7}$$

Substitution of (7) into (3) yields

$$[Z][Q]^T \vec{I}' = \vec{V}. \tag{8}$$

It is easily found that the relationships for error and residual vectors should have the similar forms to (7) and (8). That is

$$\vec{E} = [Q]^T \vec{E}', \tag{9}$$
$$[Z][Q]^T \vec{E}' = \vec{R}, \tag{10}$$

where $\vec{E}'=\vec{I}' - \vec{I}'^{(0)}$, and $\vec{I}'$ and $\vec{I}'^{(0)}$ are related, respectively, to $\vec{I}$ and $\vec{I}^{(0)}$ by (7).

This is an overdetermined equation for which the exact solution does not exist and the least-square approximate solution might be found again by using the conjugate gradient method.

Once the least-square solution $\vec{E}'$ to (10) is acquired, the solution $\vec{I}^{(1)}=\vec{I}^{(0)} + [Q]^T \vec{E}'$ to the matrix equation (3), and $i^{(1)}(l)=\vec{U}^T \vec{I}^{(1)}$ to the operator equation (1) are obtained, which might be referred to as the first order solution.

The second and higher order solutions can be gained in the same way if needed. The higher the solution order is pursued, the more accurate the solution will be, and naturally the more the computational cost would be paid. A trade-off between the solution accuracy requirement and the computation expense should be properly made. In our observation, the solution of the zeroth order is accurate enough for most practical purposes, and the first order solution sometimes might be necessary for some problems in which extraordinary accuracy is required. As for second and higher order solutions, they might not be employed very often from a practical point of view.

3. Examples

To test this new method, a number of wire antennas have been analyzed by this method and the calculated results are compared with those achieved by using the method of moments. Due to the limitation of space in this paper, only the results for analyzing a dipole antenna are shown and the numerical results for other examples would possibly be presented in the symposium.

The linear antenna under consideration is of length $L = 0.5\lambda_0$ and radius $a = 0.003369\lambda_0$, λ_0 represents the central wavelength. The calculated and specified zeroth and first order results for current distribution, radiation pattern, input resistance and input reactance are individually compared with the method-of-moments results in Figs. 1-3. The nice agreement between those results have well manifested in justification of this new numerical technique.

4. References

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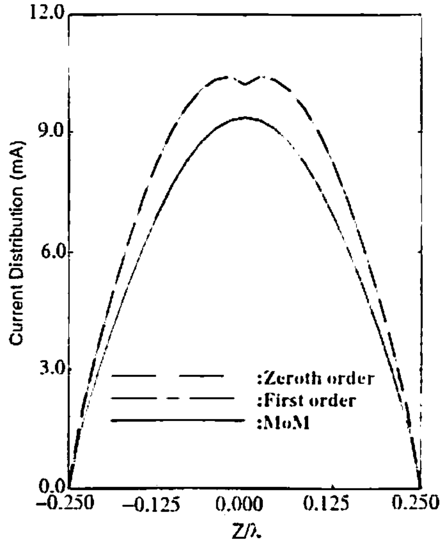

Fig. 1 Currnet Distribution

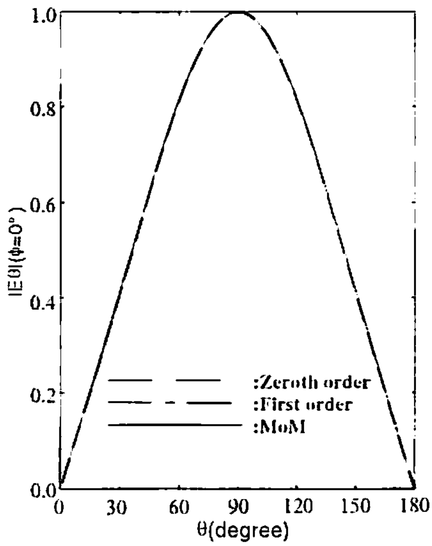

Fig. 2 E-plane E-pattern

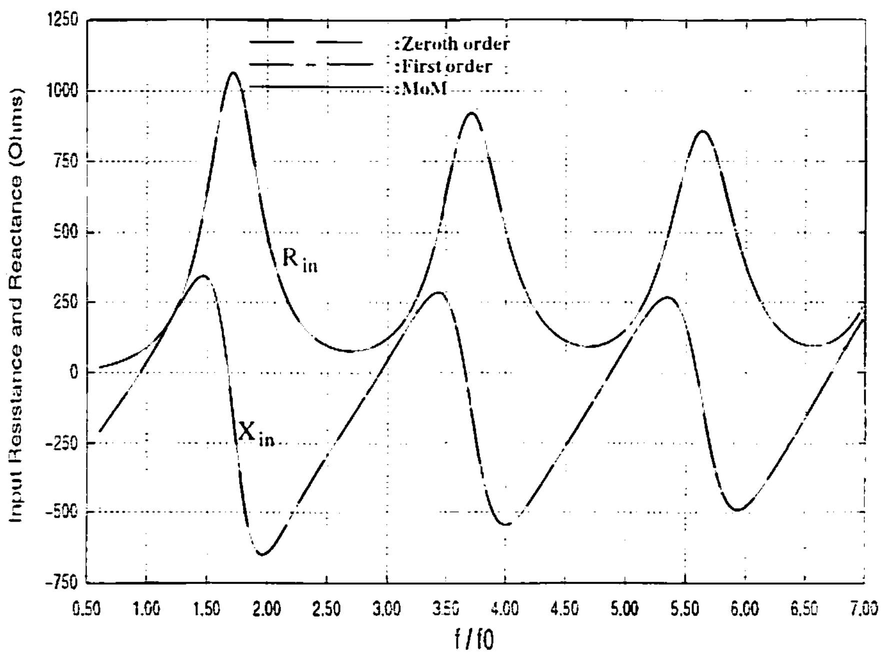

Fig. 3 Input Resistance and Input Reactance