Bit Error Rate for Satellite Communications in Ka-band under Atmospheric Turbulence Predicted from Radiosonde Data in Japan

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1. Introduction

It is known that turbulence in the earth's atmosphere affects propagation characteristics of an electromagnetic wave [1]. The effects degrade the performance on satellite communications, particularly in Ku-band at low elevation angles [2] or the above carrier frequencies. In order to consider the effects of atmospheric turbulence in the design of such satellite communication systems appropriately, some models to predict scintillation due to atmospheric turbulence have been developed for applications up to around 14 GHz in the carrier frequency [3]. However, since a carrier frequency becomes higher according to the increase in the required channel capacity of a satellite link, a method to analyze the effects of atmospheric turbulence is needed for applications at the higher carrier frequencies.

We study the effects of atmospheric turbulence on bit error rate (BER) for satellite communications in such high frequencies by the theoretical analysis of the moments of wave fields given on the basis of a multiple scattering method. We have analyzed BER derived from the average received power which is obtained by the second moment of a Gaussian wave beam for satellite communications in Ka-band at low elevation angles [4].

The strength of atmospheric turbulence, which is represented by the refractive index structure constant, is needed in the analysis of the moments of wave fields. However, since the structure constant has dependence of region, season and time as well as that of height, the suitable profile has to be applied for a location of the ground station. Some statistical methods have been reported concerning the estimation of the structure constant from radiosonde data in Europe and the United States [5]–[7]. On the other hand, we do not know any studies for the estimation of the structure constant suitable around Japan.

In this paper, we predict the vertical profiles of the structure constant from radiosonde data measured in Fukuoka, Japan by applying the above statistical method, and then analyze BER for satellite communications in Ka-band at low elevation angles using the predicted profiles of the structure constant.

2. Formulation

Fig. 1 shows the propagation model. We examine BER for QPSK modulation in downlink derived from the average received power [4]:

$$\mathrm{PE_P} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{S_P \cdot \frac{T_b}{k_B} \cdot \mathrm{EIRP} \cdot \frac{1}{(2kz_L)^2} \cdot \mathrm{G/T}}\right), \tag{1}$$

where T_b, k_B, EIRP and G/T denote bit time, Boltzmann's constant, the required effective isotropic radiated power of a transmitter and the required G/T of a receiver, respectively. The wave number for free space is denoted by $k = 2\pi f/c$, where f and c are the carrier frequency and velocity of light,

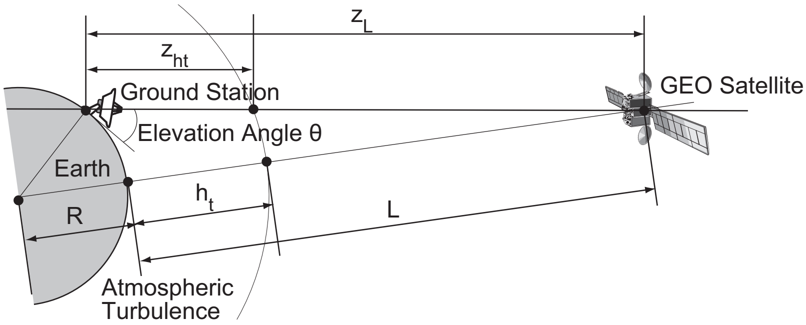

Figure 1: Earth - satellite propagation model where $R = 6,378$ km, $L = 35,786$ km and $h_t = 20$ km.

respectively. It is noted that G/T can be described with an aperture radius of a receiving antenna a_e and the noise power density of a receiver N_0 as follows.

$$\mathrm{G/T} = \frac{k_B}{N_0} \cdot \frac{(ka_e)^2}{2} \tag{2}$$

In (1), S_P represents the average received power normalized by the received power in free space:

$$S_P = \frac{\iint_{-\infty}^{\infty} d\boldsymbol{r}_+ \iint_{-\infty}^{\infty} d\boldsymbol{r}_- M_{11}(\boldsymbol{r}_+, \boldsymbol{r}_-, z_L) \exp\left(-\frac{2r_+^2}{a_e^2} - \frac{r_-^2}{2a_e^2}\right)}{\iint_{-\infty}^{\infty} d\boldsymbol{r}_+ \iint_{-\infty}^{\infty} d\boldsymbol{r}_- M_{11}^{\mathrm{in}}(\boldsymbol{r}_+, \boldsymbol{r}_-, z_L) \exp\left(-\frac{2r_+^2}{a_e^2} - \frac{r_-^2}{2a_e^2}\right)}, \tag{3}$$

where $\boldsymbol{r}_+ = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$, $\boldsymbol{r}_- = \boldsymbol{r}_1 - \boldsymbol{r}_2$, $r_\pm = |\boldsymbol{r}_\pm|$, and $M_{11}(\boldsymbol{r}_+, \boldsymbol{r}_-, z_L)$ is the second moment for a Gaussian wave beam $u(\boldsymbol{r}, z)$ which is a successively forward scattered wave in atmospheric turbulence:

$$M_{11}(\boldsymbol{r}_+, \boldsymbol{r}_-, z_L) = \langle u(\boldsymbol{r}_1, z_L) u^*(\boldsymbol{r}_2, z_L)\rangle = \frac{A}{(2\pi)^2} \iint_{-\infty}^{\infty} d\boldsymbol{\kappa}_+ \exp\left[-\frac{w^2}{8}\kappa_+^2 + \left(j\boldsymbol{r}_+ + \frac{p}{2}\boldsymbol{r}_-\right)\cdot \boldsymbol{\kappa}_+ - \frac{r_-^2}{2w_0^2} - \frac{k^2}{4}\int_0^{z_L} dz_1 \int_0^{z_1} dz_2\, D\left(\boldsymbol{r}_- - \frac{z_L - z_1}{k}\boldsymbol{\kappa}_+, z_1 - \frac{z_2}{2}, z_2\right)\right], \tag{4}$$

where $\kappa_+ = |\boldsymbol{\kappa}_+|$, $w = w_0\sqrt{1 + p^2}$, $p = 2(z_L + z_0)/kw_0^2$, and A is constant. The angular brackets denote the ensemble average, and w_0 is the minimum beam radius at $z = -z_0 = 0$ where the field amplitude falls to $1/e$ of that on the beam axis. The wave field in free space is expressed by $u_{\mathrm{in}}(\boldsymbol{r}, z)$ and $M_{11}^{\mathrm{in}}(\boldsymbol{r}_+, \boldsymbol{r}_-, z_L) = u_{\mathrm{in}}(\boldsymbol{r}_1, z_L)u_{\mathrm{in}}^*(\boldsymbol{r}_2, z_L)$. We assume that the structure function of random dielectric constant of atmosphere $D(\boldsymbol{r}_-, z_+, z_-)$ satisfies the von Karman spectrum [1]:

$$D(\boldsymbol{r}_-, z_+, z_-) = \delta(z_-) \cdot \frac{96\pi^2}{5} \cdot 0.033 C_n^2(z_+) L_0^{5/3} \cdot \left[1 - \Gamma\left(\frac{1}{6}\right)\left(\frac{r_-}{2L_0}\right)^{5/6} I_{-5/6}\left(\frac{r_-}{L_0}\right) + \Gamma\left(\frac{1}{6}\right)\left(\frac{1}{\kappa_m L_0}\right)^{5/3} \cdot \sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{1}{\kappa_m L_0}\right)^{2n} {}_1F_1\left(-n - \frac{5}{6}; 1; -\frac{\kappa_m^2 r_-^2}{4}\right)\right], \tag{5}$$

where $\kappa_m = 5.92/l_0$, and $l_0 = 1$ mm represents the inner scale of turbulence. The outer scale of turbulence is denoted by L_0. The functions $\delta(z)$, $\Gamma(z)$, $I_\nu(z)$ and ${}_1F_1(a; b; z)$ are the Dirac delta function, the gamma function, the modified Bessel function of the first kind and the confluent hypergeometric function of the first kind, respectively. The refractive index structure constant is expressed by $C_n^2(z_+)$.

Using radiosonde data which consist of pressure P (in hPa), temperature T (in K), relative humidity RH (in percent), wind speed v (in m/s) and wind direction ϕ (in deg), the average value of the structure constant at height h can be estimated as follows [7].

$$\langle C_n^2(h)\rangle = 2.8 M_0(h)^2 \langle R(h)\rangle \int_0^{\infty} dL_0\, L_0^{4/3} p_{L_0}(L_0) \int_0^{\infty} dS\ p_S(S) \int_{-\infty}^{S^2 R_{\mathrm{ic}}} dN^2\, \left(N^2\right)^2 p_{N^2}(N^2) \tag{6}$$

$$M_0(h) = -77.6 \times 10^{-6}\frac{P(h)}{gT(h)}, \quad \langle R(h)\rangle = \left[1 + 15,500\frac{q(h)}{T(h)} - \frac{15,500}{2}\frac{g \cdot \partial q(h)/\partial h}{\langle N^2(h)\rangle T(h)}\right]^2, \tag{7}$$

where $R_{ic} = 0.25$ and g are the critical Richardson number and the acceleration of gravity, respectively. The values $\langle N^2(h) \rangle = g \cdot \partial \ln \Theta(h)/\partial h$ and $q(h) = 0.622e(h)/[P(h) - 0.378e(h)]$ are the mean value of buoyancy forces and the specific humidity, respectively. Note that $\Theta(h) = T(h)\,[1,000/P(h)]^{0.2858}$ and $e(h) = e_w(h)\mathrm{RH}(h)/100$ denote the potential temperature and the vapor pressure, respectively. The $e_w(h)$ is the saturation vapor pressure [8] whose logarithm is given by

$$\log e_w(h) = 10.79574\,[1 - 273.16/T(h)] - 5.02800 \log\,[T(h)/273.16] + 0.78614 \\ + 1.50475 \times 10^{-4} \left\{1 - 10^{-8.2969[T(h)/273.16-1]}\right\} + 0.42873 \times 10^{-3} \left\{10^{-4.76955[1-273.16/T(h)]} - 1\right\}. \quad (8)$$

The probability density function (PDF) of a wind shear S is assumed to be a Rice distribution as follows.

$$p_S(S) = \frac{S}{\sigma_S^2(h)} \exp\left[-\frac{S^2 + \langle S(h) \rangle^2}{2\sigma_S^2(h)}\right] I_0\left(\frac{S \langle S(h) \rangle}{\sigma_S^2(h)}\right), \quad \langle S(h) \rangle = \sqrt{\left(\frac{\partial v \cos\phi}{\partial h}(h)\right)^2 + \left(\frac{\partial v \sin\phi}{\partial h}(h)\right)^2}, \quad (9)$$

where $\sigma_S(h) = 0.18 L_0^{-0.3} \left|\langle N^2(h) \rangle\right|^{0.25} \rho(h)^{-0.15}$, and $\rho(h) = 0.348P(h)/T(h)$ is the mean density of dry air. The PDF of a buoyancy force is assumed to be a Gaussian distribution:

$$p_{N^2}(N^2) = \frac{1}{\sqrt{2\pi}\sigma_N(h)} \exp\left[-\frac{\left(N^2 - \langle N^2(h) \rangle\right)^2}{2\sigma_N^2(h)}\right], \quad \sigma_N(h) = \sqrt{6/5}\sigma_S(h)\sqrt{\left|\langle N^2(h) \rangle\right|}. \quad (10)$$

For the outer scale of turbulence, there are some PDFs [5]–[7]. In this paper, we assume the delta function [6]:

$$p_{L_0}(L_0) = \delta(L_0 - L_{0\mathrm{eff}}), \quad L_{0\mathrm{eff}}^{4/3} = \int_{L_{0\min}}^{L_{0\max}} dL_0\, L_0^{4/3} \cdot \frac{1}{L_{0\max} - L_{0\min}}, \quad (11)$$

where $L_{0\mathrm{eff}}$ represents the effective outer scale of turbulence. It is assumed that $L_{0\min} = 3$ m and $L_{0\max} = 100$ m [7], which results in $L_{0\mathrm{eff}} = 54.2$ m. Note that $L_{0\mathrm{eff}}$ is substituted for L_0 in (5). Furthermore, the integration of $C_n^2(z_+)$, which is needed in an integration of the structure function $D(\boldsymbol{r}_-, z_+, z_-)$ in (4), is replaced by the following summation:

$$\int_0^{z_L} dz_+ C_n^2(z_+) = \sum_{i=0}^{i_{\max}} \langle C_n^2(h_i) \rangle \Delta h_i \left[1 - \left(\frac{R\cos\theta}{h_i + R}\right)^2\right]^{-1/2}, \quad (12)$$

where $\Delta h_i = h_{i+1} - h_i$ is the thickness of the horizontal slab through which $\langle C_n^2(h_i) \rangle$ is calculated.

3. Numerical Analysis

We estimate the average vertical profile of the structure constant using radiosonde data measured at the meteorological observatory in Fukuoka, Japan at 9:00 local time. We select radiosonde data in the fine weather condition from a data set which can be obtained via the website of Japan Meteorological Agency. We calculate the structure constant every 50 m at heights by interpolating the available significant level radiosonde data. Fig. 2 shows the average vertical profiles of the structure constant predicted from radiosonde data measured from March 2011 to February 2012. The structure constant is largest around the ground when radiosonde data measured from June to August are used in the prediction. It is known that the structure constant is highly dependent on humidity for microwave. Since a period from June to August is the most humid in years, the structure constant becomes larger in this period; atmospheric turbulence becomes strong in summer.

Using these predicted structure constants, we analyze BER for Ka-band downlink satellite communications at low elevation angles, where $\theta = 5$ deg, $f = 20$ GHz and $w_0 = 1.2$ m. Fig. 3 shows the BER as a function of ka_e for fixed EIRP and G/T, where the noise power density N_0 changes in inverse proportion to the square of ka_e in (2). It is shown that BER is largest when the structure constant predicted from radiosonde data measured from June to August is used.

From results, it is found that strong turbulence in summer causes the decrease in the average received power and results in the increase in BER for Ka-band downlink satellite communications, while turbulence in other periods is weak compared with summer and then the effects of atmospheric turbulence on BER become small.

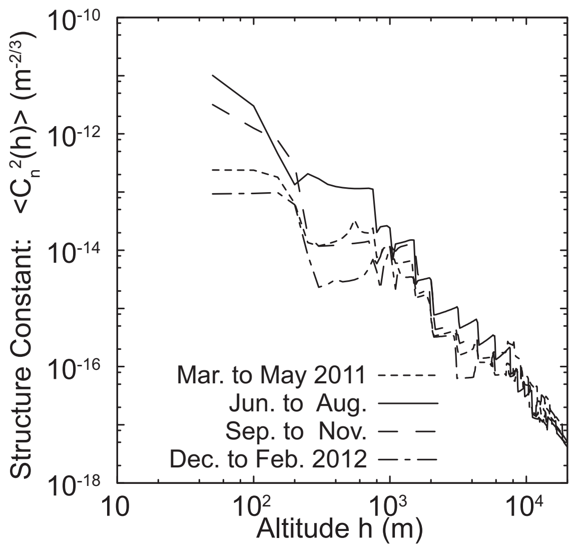

Figure 2: Refractive index structure constant predicted from radiosonde data measured at the meteorological observatory in Fukuoka (latitude: 33°35′N; longitude: 130°23′E; altitude: 18 m).

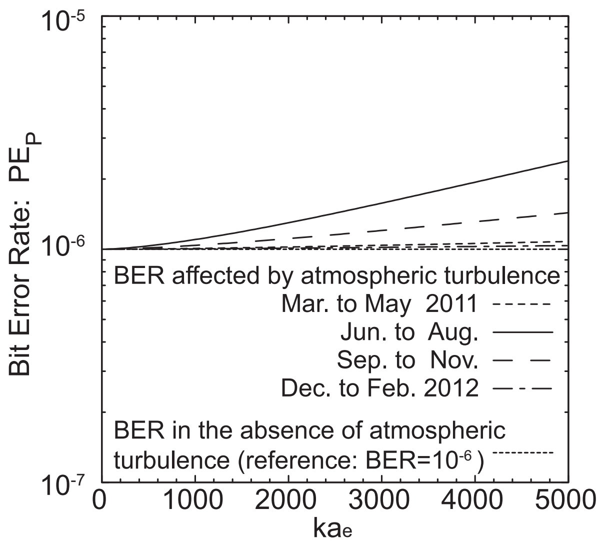

Figure 3: BER derived from the average received power for Ka-band downlink satellite communications at low elevation angles as a function of ka_e for fixed EIRP and G/T. The predicted structure constants shown in Fig. 2 are used.

4. Conclusion

We analyzed BER derived from average received power for satellite communications in Ka-band at low elevation angles under atmospheric turbulence which is predicted from radiosonde data measured in Fukuoka, Japan. From results, atmospheric turbulence becomes strong in summer and results in the increase in BER for Ka-band downlink satellite communications.

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