WEIGHTED WAVELET-LIKE BASIS FOR ELECTROMAGNETICS'

Dr. W. Y. Tam, MIEEE
Department of Electronic Engineering
The Hong Kong Polytechnic University
Hong Kong

1. Introduction

In the last few years, wavelet analysis has drawn a great attention in both applied mathematics and many engineering disciplines because of its multiresolution property [1]. When wavelets are employed as a basis set, the solution of integral equation arising in electromagnetics can be speed up by changing the dense impedance matrices to sparse matrices [2-3]. However, problems still arise because most of the wavelets developed by mathematics community are not tailored for electromagnetic problems, which involve oscillatory kernels. For examples [4-5], the compact support orthonormal wavelets with vanishing moments were developed to solve integral equations with smooth kernels and finite number of singularities. In order to reduce the oscillation of the discretized impedance matrix, the discretization of 0.03\(\lambda\), which is much smaller than the conventional discretization size: 0.1 λ , was used in [6] to solve a 90° dihedral corner reflector under a TM-polarized plane wave incident. On the other hand, the scattering of a TM-polarized wave due to a metallic circular cylinder is studied in [2,3]. The sparseness of the impedance matrix is high even 0.1λ discretization is used because the degree of oscillation of the discretized impedance matrix for a circular cylinder is much smaller than a 90° dihedral corner reflector.

In this paper, a class of wavelet-like basis functions is introduced to solve the scattering of a TM-polarized plane wave due to a metallic strip. The basis functions have properties i, compact support with different length scale, ii. All but k basis orthonormal to k rows of the impedance matrix. The scattering problem is formulated using the electric field integral equation (EFIE) for the unknown surface current distribution. The method of moments with rectangular pulse basis functions and point matching is applied to discretize the integral equation into a matrix equation. The dense impedance matrix is transformed to a sparse matrix using the proposed wavelet-like basis functions. For the example considered, if the discretization size is large, the percentage of nonzero elements is significantly reduced when comparing with the results obtained using the wavelet-like basis with vanishing moments.

2. Formulation

A metallic strip illuminated by a TM-polarized plane wave is shown in Fig.1. The z-directed surface current distribution $J_z(r')$ on the strip is related to the incident field by an electric field integral equation (EFIE)

^{*} The work is supported by The Hong Kong Polytechnic University under grant 350/367.

$$\frac{d\mu}{d} \int_{L} J_{z}(r') H_{o}^{(2)}(k_{o}|r-r'|) dr' = E_{z}^{i} \qquad r \in L$$
 (1)

where L represents the surface of the strip. Expanding the surface current distribution $J_i(r')$ in rectangular pulse basis and point matching at x_i , $i=1,2,\ldots,N$, the resulting equation gives

where [a] is a column matrix for the coefficient of the current basis and [Z] is the dense impedance matrix.

The matrix equation (2) is then transformed into wavelet domain [Z'][a'] = [V'] where $[Z'] = [U][Z][U]^T$, [a'] = [U][a] and [V'] = [U][V] and the rows of [U] are wavelet-like basis functions.

3. Weighted wavelet-like basis

In this paper, the wavelet-like basis is constructed based on the multiresolution decomposition of the weighted Haar basis function. For a discrete set of points $\{x_1, x_2, \ldots, x_n\}$ where $n = 2^m k$, k is the order of approximation and m is a positive integer. A k-dimensional vector space V_0 spanned by Haar functions weighted with functions $W_k(x_k)$ is defined,

$$V_0 = \operatorname{span} \left\{ \left\langle W_j(x_1), W_j(x_2), \dots, W_j(x_n) \right\rangle \middle| j = 1, 2, \dots, k \right\}$$
 (3)

We define another 2k-dimensional vector space V_{-1} which is spanned by $W_j(x_i)$, $j=1,2,\ldots,k$ on $\{x_1,x_2,\ldots,x_{n/2}\}$ and $\{x_{n/2+1},x_2,\ldots,x_n\}$. The decomposition process is repeated for m times until we get the vector space V_{-m} which is the entire n-dimensional vector space. As the vector space V_{-i} is a subspace of vector space V_{-i-1} , i.e.,

$$V_{\alpha} \subset V_{-1} \subset \cdots \subset V_{-m} \tag{4}$$

an orthogonal compliment T_{-i} of V_{-i} is defined such that $V_{-i} = T_{-i+1} \oplus V_{1-i}$. Therefore, the entire vector space V_{-m} can be decomposed as follow:

In reference [5], an algorithm based on the Gram-Schmidt orthogonalization process is developed to construct the basis of the vector space T_{-i} , $i=0,1,\ldots,m-1$. This set of basis functions is named as wavelet-like basis functions as they are i. compact support with different length scale and ii. All but k basis functions are orthonormal to the weighting function W_j . In previous literature [2,4-5], polynomials were chosen as the weighting functions so that the basis functions have vanishing moments. In this paper, rows of the impedance matrix are used as the weighting functions.

4. Results

In this section, the weighted wavelet-like basis are employed to transform the impedance matrix for the scattering problem. The weighting functions $W_j(x_i)$ used are the rows of the impedance matrix: $W_j(x_i) = Z_{pi}$ where p = jn / 2k, j = 1,2,...,2k. The transformed impedance matrix is thresholded by zeroing elements of the impedance matrix with magnitude less than a tolerance δ times the maximum element. The percentage of sparsity (S) and percentage relative error (ϵ) are defined by

$$S = \frac{N_o}{N^2} \times 100 \% \tag{6}$$

$$\varepsilon = \frac{\left\|J_z - J_z^{\cdot}\right\|}{\left\|J_z\right\|} \times 100\% \tag{7}$$

where N_o is the zero elements after thresholding, J_z is the surface current distribution obtained with thresholding and $\|\cdot\|$ denotes the L^2 norm. The transformed impedance using weighted wavelet-like basis functions is shown in Fig. 2. In Fig. 3, the sparsity versus solution error of the problem is plotted using both weighted wavelet-like basis functions and Alpert's wavelet-like basis functions. Its shows that the performance of the Alpert's wavelet-like basis is unsatisfactory when the discretization size is large. Finally, the current distributions are computed using both un-thresholded and thresholded impedance matrix and shown in Fig. 4.

5. Conclusion

The use of the weighted wavelet-like basis to reduce the impedance matrix for a TM-polarized plane wave incident on a metallic strip is demonstrated. The sparsity of the present problem using weighted wavelet-like basis is higher than using Alpert's wavelet especially the discretization size is large.

References

- [1] S. Mallat, "A theory of multiresolution signal decomposition: the wavelet representation," *IEEE Trans. PAMI*, vol. 11, pp. 674-693, 1989.
- [2] R. L. Wagner and W. C. Chew, "A study o wavelets for the solution of electromagnetic integral equations," *IEEE Trans. Antennas Propagat.*, vol. AP-43, no. 8, pp. 802-810, Aug., 1995.

- [3] J. C. Goswami, A. K. Chan, and C. K. Chui, "On solving first-kind integral equations using wavelets on a bounded interval," *IEEE Trans. Antennas Propagat.*, vol. AP-43, no. 6, pp. 614-622, June., 1995.
- [4] G. Beylkin, R. Coifman, and V. Rokhlin, "Fast wavelet transforms and numerical algorithms I," *Comm. Pure Appl. Math.*, vol. 44, pp. 141-183, 1991.
- [5] B. Alpert, G. Beylkin, R.Coifman and V. Rokhlin, "Wavelet-like bases for the fast solution of second-kind integral equations," *SIAM J. Sci. Comp.*, vol. 14, no. 1, pp. 159-184, Jan., 1993.
- [6] H. Kim and H. Ling, "On the application of fast wavelet transform to the integral-equation solution of electromagnetic scattering problems," *Micro. Opt. Tech. Lett.*, vol 6, no.3, pp. 168-173, Mar. 1993.

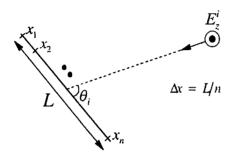


Fig.1 A TM-polarized plane wave incident on a metallic strip. n=128

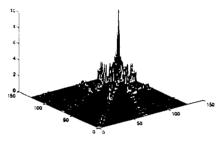


Fig.2 Wavelet-domain impedance matrix for a TM-polarized plane wave incident on a metallic strip with length 12.8 λ_o . n=128 and k=8 are used.

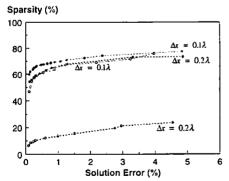


Fig.3 Sparsity for a TM-polarized plane wave incident on a metallic strip as a function of solution error. n=128, k=8, $\Delta x=0.1$ and 0.2. Sold circle: weighted wavelet, hollow circle: Alpert's wavelet.

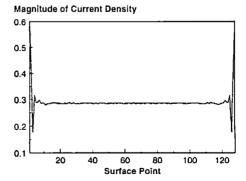


Fig.4 Magnitude of the current distribution on the metallic strip. Incident angle $\theta_i = 90^{\circ}$, n = 128, k = 8, $\Delta x = 0.1\lambda$, solution error = 1.00%, sparsity = 69.7%. Solid line: without thresholding, dotted line: with thresholding