ANALYSIS OF WAVEGUIDE DISCONTINUITIES BY A MODIFIED BOUNDARY ELEMENT METHOD

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ABSTRACT: A new method for analyzing waveguide discontinuities is presented in this paper, which is based on the modified boundary element method (MBEM) and the transverse resonance method. Insert two perfect electric walls or magnetic walls at some distances L1 and L2 besides the discontinuity, then a cavity is formed. MBEM is used to determine the resonant lengthes L1 and L2 of the cavity at some frequency, and the equivalent circuit parameters of the discontinuity is then obtained by L1 and L2. Numerical results of inductive apertures in a rectangular waveguide are in good agreement with that given in reference.

I. INTRODUCTION

It is well known that many microwave and millimeter wave elements, such as filters and directional couplers etc., are constituted by waveguide discontinuities. Therefore, analysis of waveguide discontinuities is a important topic in guided wave theory. For some regular discontinuity problems, modes matching method, varational method and spectral domain method are availible. But for some irregular discontinuities, these methods are not effective. Boundary element method (BEM) is availible for analyzing boundary value problems with irregular boundaries [1], so it can easily be used to solve irregular discontinuity problems. The disadvantage of BEM is that a high matrix order is required to achieve a accurate numerical result when the boundary scale is considerablely large with respect to wavelength [2][3]. To overcome the disadvantage, the author presented a modified boundary element method (MBEM) [4][5], in which the boundary is divided into regular part and irregular part, the Green's function in boundary integral equation is forced to satisfy the homogeneous boundary conditions on the regular part, then the integration interval of the boundary integral equation is reduced to the irregular part, so boundary division is carried out only on the irregular part, that results in less computation effort and high accuracy. This paper presented a effective analysis method of waveguide discontinuities by combining MBEM with transverse resonance method. The accuracy and fast convergence of this hybrid method are verified by a numerical example of inductive apertures in a rectangular waveguide.

II. TRANSVERSE RESONANCE

A generalized inductive waveguide discontinuity is shown in Fig.1. Replace the two planes at distances L1 and L2 besides the discontinuity by two perfect electric walls (or magnetic walls), then a closed cavity is constructed. Its equivalent circuit is shown in Fig.2. The Z-parameters can be determined by following

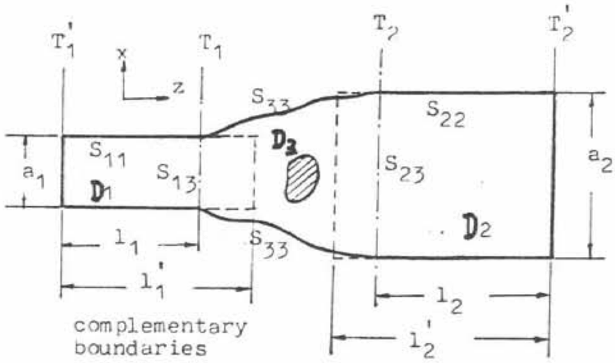

Fig.1 **A generalized inductive waveguide discontinuity**

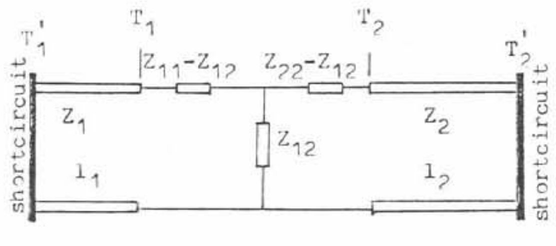

Fig.2 **Equivalent circuit of the cavity**

three steps in terms of the resonant conditions of the cavity.

a) let $L2 = \lambda g/4$, then $Z11 = -jtg(\beta_1 L1)$, $-\lambda g/4 < L1 < \lambda g/4$

b) let $L1 = \lambda g/4$, then $Z22 = -jtg(\beta_2 L2)$, $-\lambda g/4 < L2 < \lambda g/4$

c) let $L2 \in (-\lambda g/4, \lambda g/4)$ be given , then

$$(Z12)^2 = (Z11 + j\ Z1\ tg\beta_1 L1)(Z22 + j\ Z2\ tg\beta_2 L2)$$

where Z1 and Z2 are characteristic impedances of the regular waveguides at two sides respectively. $\beta 1$ and $\beta 2$ are propagation constants of left and right side waveguides respectively.

Obviously, if the lengthes L1 and L2 are given with respect to some frequency f, the equivalent circuit parameters Z11, Z22 and Z12 with respect to the frequency f can be obtained immediately by the equations given in a), b) and c).

If frequency f and one of L1 and L2 are given, then another of the lengthes can be calculated by applying a field analysis method to the cavity. Modes matching method and spectral domain method etc. are ineffective for the irregularity of the discontinuity region (see Fig.1). If BEM is used to solve the problem, the matrix order is obviously very high for the boundary is too long with respect to the wavelength. Since the boundaries of the waveguides at two sides are regular, less computation effort and high accuracy can be achieved if MBEM is used to solve such problems. Therefore we will use MBEM to calculate the resonant lengthes in next section.

III. DETERMINE THE RESONANT LENGTHES BY MBEM

For inductive waveguide discontinuities, only TM^y_{mno} modes ($\partial/\partial y = 0$) are existing in the cavity. Field components $Ey^{(i)}$ (i stands for region Di) satisfy following boundary value problems

$$\begin{cases} \nabla_\perp^2 Ey^{(i)} + k^2 Ey^{(i)} = 0 , & \text{in } Di \\ Ey^{(i)}\Big|_{e.w} = 0 , & i = 1,2,3 \end{cases} \quad (1)$$

where $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial z^2$, $k^2 = \omega^2 \mu \varepsilon$, and e.w stands for electric wall.

Apply Green's 2nd identity to each region, yields

$$\iint_{Di} [G^{(i)} \nabla_\perp^2 Ey^{(i)} - E^{(i)} \nabla_\perp^2 G^{(i)}] d\Omega = \oint_{Si} [G^{(i)} \partial Ey^{(i)}/\partial n - Ey^{(i)} \partial G^{(i)}/\partial n] ds \qquad i = 1 , 2 , 3. \tag{2}$$

where $G^{(i)}$ are Green's functions, n is the normal vector of the boundary and Si=Sio+Si1+Si2+Si3. Using a infinite small arc around the singular point [4], the left hand of eq.(2) can be shown to be zero. Then following boundary integral equations (BIEs) are obtained

$$\oint_{Si} [G^{(i)} \partial Ey^{(i)}/\partial n - Ey^{(i)} \partial G^{(i)}/\partial n] ds = 0 , \quad i=1,2,3. \tag{3}$$

Let Green's functions $G^{(1)}$ and $G^{(3)}$ satisfy the same boundary conditions as $Ey^{(i)}$, i.e., $G^{(1)} , G^{(3)}|_{e.w} =0$, then the BIEs eq.(3) are reduced to following equations.

$$\int_{Si3} [G^{(i)} \partial Ey^{(i)}/\partial n - Ey^{(i)} \partial G^{(i)}/\partial n] ds = 0 , \quad i=1,2 \tag{4}$$

$$\int_{Si3} [G^{(3)} \partial Ey^{(3)}/\partial n - Ey^{(3)} \partial G^{(3)}/\partial n] ds + \int_{S30} G^{(3)} \partial Ey^{(3)}/\partial n \, ds = 0 . \tag{5}$$

It can be seen the integration intervals in the BIEs are reduced to S3 , division of the boundary needs only carry out on the irregular part S3, that results in less computation effort. On the other hand, since the boundary conditions on the regular boundaries are satisfied exactly, this method must have advantage over BEM in accuracy.

Regular regions (such as rectangular or circular regions etc.) can be formed by complementing the regular part of boundary with its "imaginary complementary boundary" as shown in Fig.1. Then in the regular regions, modified Green's functions can easily be obtained and formulated as quickly converged infinite series.

On the interfaces S13 and S23, Ey and $\partial Ey/\partial n$ satisfy following continuous conditions

$$Ey^{(3)} = Ey^{(1)} , \text{ on S13} ; \quad Ey^{(3)} = Ey^{(2)} , \text{ on S23} ,$$

$$\partial Ey^{(3)}/\partial n = \partial Ey^{(1)}/\partial n = Q1 , \text{ on S13} , \tag{6}$$

$$\partial Ey^{(3)}/\partial n = \partial Ey^{(2)}/\partial n = Q2 , \text{ on S23} ; \quad \partial E^{(3)}/\partial n = Q3, \text{ on S30}.$$

Substitute these conditions into eq.(4) and (5), a system of coupling BIEs with respect to $Ey^{(1)}$, $Ey^{(2)}$, Q1, Q2 and Q3 are obtained. Its discretion gives a homogeneous matrix equation

$$[A]\ \overline{c} = \overline{0} \tag{7}$$

The resonant lengthes L1 or L2 is then obtained in terms of the nonzero solution condition det[A]=0. The equivalent circuit parameters are determined immediately by L1 and L2.

This method can easily be extend to capacitive waveguide discontinuities etc.

IV. NUMERICAL RESULTS

Inductive apertures in a rectangular waveguide are analyzed by the method described above. Numerical results shown in Fig.3 are in good agreement with that given in reference [6]

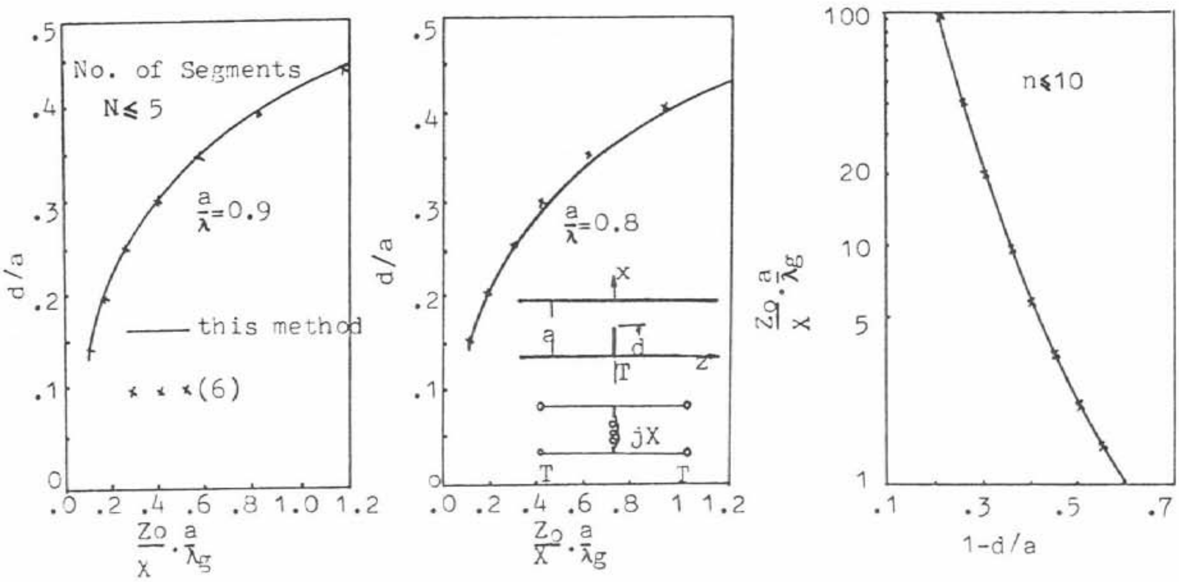

Fig.3 Numerical results for inductive apertures in a rectangular waveguide

ACKNOWLEDGEMENT

The author would like to thank Prof. Sifan Li for his support.

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