# ANALYSIS USING THE VIRTUAL CAVITY METHOD OF A WAVEGUIDE COUPLING JUNCTION WITH OVERLAPPING SLOTS

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## 1. Introduction

Waveguide slot antennas are commonly used in applications where low losses or high power is needed. Because they are quite expensive to manufacture, and the tolerances of the slot dimensions are quite strict, accurate analysis tools are needed for the design of such antennas.

Planar waveguide slot arrays often make use of slots to couple power from a feed waveguide to a branch waveguide. Such coupling slots have to be inclined with respect to the center line (centered inclined slots), offset some distance from the center line (longitudinal slots) or a combination of both (compound slots) in order to couple power.

Coupling slots are usually centered-inclined, and the inclination makes the analysis very difficult. The centered-inclined slot has been analysed by Rengarajan [1] and Rajeek and Chakraborty [2]. In both these analyses simplifications were made in order to solve the problem. In [1] the testing function had a Dirac delta function for the transverse distribution, and in [2] the geometry of the slot was modified.

When there are radiating slots in the branch waveguide that overlap the coupling slot longitudinally as in fig. 1, the calculation of the mutual coupling of the slots becomes difficult. This has recently been analysed by Rengarajan and Shaw [3] using a spectral domain formulation of the dyadic Green's function inside the waveguide.

In the present paper we will use the virtual cavity method introduced by Seki [4, 5]. This method makes it possible to analyse the coupler without having to make the above mentioned approximations, since this alternative expression of the dyadic Green's function has the same functional form independent of the relative positions of the source and observation points, as opposed to the conventional waveguide Green's function. Furthermore, we don't have any singular terms, as is the case when the conventional Green's function is used. A more rigorous presentation of this analysis is given in [6].

In the following section, the analysis procedure is outlined and the alternative expression of the Green's function is given, and in section 3 the results are compared with measurements.

## 2. Theory

A sketch of the coupling junction is shown in figure 1. The feed waveguide and the branch waveguide are interconnected via a coupling slot in the common broad wall. In the branch waveguide, two radiating slots are cut in the opposite broad wall to the coupling slot. All the slots are compound slots, i.e. they are both offset from the center line of the waveguide and inclined some angle. The radiating slots overlap the coupling slot in the longitudinal direction, i.e. some part of the radiating slots have the same z-coordinate as some part of the coupling slot, if the z-axis is longitudinal to the waveguide.By applying the equivalence theorem [7], we can cover the slot apertures with perfectly conducting walls and introduce magnetic currents at their locations. By doing this, we have 6 separate canonical regions, the two waveguides, the three slots and the exterior region, where we can calculate the fields produced by the magnetic currents easily using Green's functions techniques.

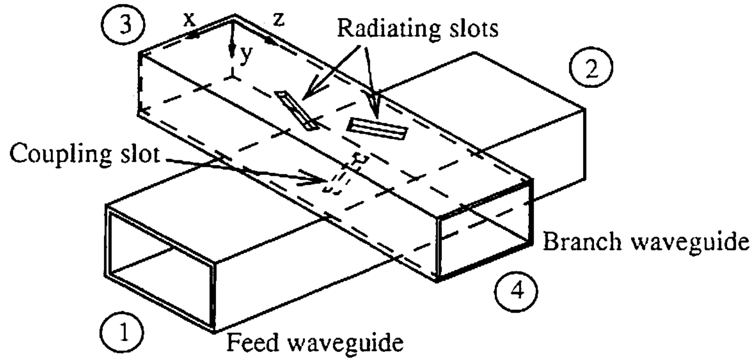


*Figure 1: Part of planar waveguide slot array, showing one short section of one branch waveguide with two compound radiating slots and truncated at ports 3 and 4, and one short section of a feed waveguide truncated at ports 1 and 2. The two waveguides are interconnected via a compound coupling slot. The origin of the coordinate system is actually at the center of the junction, but it is shown in the corner for clarity.*

For the exterior region, we use the Green's function for half free space [8, sec. 2.12]. In the slot regions, we use the conventional Green's function for a rectangular cavity [8, sec. 5.9].

In the waveguide regions, the alternative expression of the Green's function derived by Seki [4, 5] by using the virtual cavity method is used.

$$\bar{G}\left(r_o \middle| r_s\right) = \bar{G}_y^c\left(r_o \middle| r_s\right) + \tag{1a}$$

$$\frac{1}{4}\sum_u \frac{\exp(\gamma_{uz}c_1)}{\sinh(\gamma_{uz}c)}\left[H_u^+(r_o)\exp(\gamma_{uz}c_2) - H_u^-(r_o)\exp(-\gamma_{uz}c_2)\right]H_u^+(r_s)$$

$$+\frac{1}{4}\sum_u \frac{\exp(-\gamma_{uz}c_2)}{\sinh(\gamma_{uz}c)}\left[H_u^-(r_o)\exp(-\gamma_{uz}c_1) - H_u^+(r_o)\exp(\gamma_{uz}c_1)\right]H_u^-(r_s)$$

where

$$\bar{G}_y^c\left(r_o \middle| r_s\right) = \begin{cases} \frac{1}{4}\sum_u \frac{1}{\sinh(\gamma_{uy}b)}\left[H_u^+(r_o)\exp(\gamma_{uy}b) - H_u^-(r_o)\exp(-\gamma_{uy}b)\right] \\ \quad \left[H_u^-(r_s) - H_u^+(r_s)\right] & (y_s < y_o < b) \\ \frac{1}{4}\sum_u \frac{1}{\sinh(\gamma_{uy}b)}\left[H_u^-(r_o) - H_u^+(r_o)\right] \\ \quad \left[H_u^+(r_s)\exp(\gamma_{uy}b) - H_u^-(r_s)\exp(-\gamma_{uy}b)\right] & (0 < y_o < y_s) \end{cases} \tag{1b}$$

Coordinates with indices $s$ and $o$ are source and observation points, respectively. The index $u=(n,m)$ is the double mode index, so the summations are taken over all the modes. In eq. (1a), $H_u^+$ and $H_u^-$ [8, sec. 5.6] denote the mode functions propagating in the positive and negative z-directions, respectively, while in eq. (1b) they mean mode functions propagating in the positive and negative y-directions. The complex propagation constants in the y and z directions are denoted $\gamma_{uy}$ and $\gamma_{uz}$, respectively. Furthermore, $c_1$ and $c_2$ are the z-coordinates of the virtual cavity walls, $c=c_2-c_1$ is the length of the cavity and $b$ is the waveguide height.

The advantage of this Green's function is that the same functional form of the Green's function is valid in the entire region of interest, as opposed to the conventional Green's function, where the two expressions are valid on either side longitudinal to the source point. This feature makes the integrations much easier, and they can be evaluated completely analytically also for compound slots.

The remaining problem is to determine the magnetic currents in the slot apertures, which is done by imposing the continuity condition of the fields across the aperture plane. The continuity condition of the tangential electric field results in that the magnetic currents on either side of an aperture must be equal but of opposite direction, i.e if we have $M_i$ just below the aperture, we must have $-M_i$ just above the aperture.

The continuity of the tangential magnetic field results in a set of coupled integral equations, involving integrals of the scalar product of the dyadic Green's function and the magnetic currents, the integrals to be taken over the slot aperture [6].

The integral equations are solved using the moment method. The unknown magnetic currents are expanded in entire domain sinusoidal basis functions along the slot and are assumed to be constant across. The test functions are chosen according to Galerkin's method to be the same as the basis functions.

## 3. Results

To verify the theory, an experiment model of the coupling junction was manufactured. It consists of two standard H-band waveguides WR-187 with inner dimensions 47.55 mm x 22.15 mm and with slot dimensions as in table 1.

The S-parameters of the four-port were measured using a network analyser and compared with calculated results. This is shown in figure 2. We see that the agreement between measured and calculated results is very good. Some deviation can be seen in $S_{11}$ when the reflection is small. This is caused by unideal termination of port 2 in the measurements.

*Table 1: Dimensions of the experiment model. Both waveguides are standard H-band waveguides WR-187 with inner dimensions 47.55 mm x 22.15 mm. All slot widths are 3.0 mm and all wall thicknesses are 1.62 mm. The inclination angles are counterclockwise from longitudinal in the branch waveguide. The radiating slot closest to port 4 is indexed with r and the other with r2. All numbers are in mm and degrees.*

| | | |
|---|---|---|
| coupling slot longitudinal pos. | $z_c$ | 5.0 |
| coupling slot transverse pos. | $x_c$ | -3.0 |
| coupling slot length | $l_c$ | 28.2 |
| coupling slot inclination | $\theta_c$ | 30.0 |
| first radiating slot longitudinal pos. | $z_r$ | 24.3 |
| first radiating slot transverse pos. | $x_r$ | 5.0 |
| first radiating slot length | $l_r$ | 28.2 |
| first radiating slot inclination | $\theta_r$ | -15.0 |
| second radiating slot longitudinal pos. | $z_{r2}$ | -14.3 |
| second radiating slot transverse pos. | $x_{r2}$ | -5.0 |
| second radiating slot length | $l_{r2}$ | 28.2 |
| second radiating slot inclination | $\theta_{r2}$ | 15.0 |

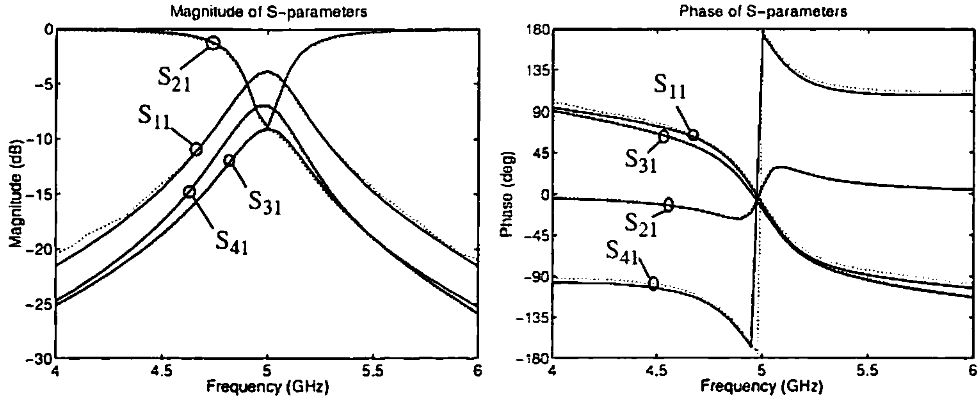


*Figure 2: S-parameters of the coupling junction. a) magnitude b) phase.*

## 4. Conclusion

A waveguide coupling junction with overlapping slots has successfully been analysed. The slots may be both inclined and offset from the center line (compound slots). We use an alternative expression of the waveguide Green's function to facilitate the analysis.

The numerical results are compared with measurements, and the agreement is very good.

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