A Simple Absorbing Boundary Algorithm for the FDTD Method with Arbitrary Incidence Angle

Jasmin E. Roy and Dennis H. Choi
Faculty of Engineering, University of Regina
Regina, Saskatchewan, Canada, S4S 0A2

Introduction

The purpose of this absorbing boundary consists in simulating a boundary that permits the electromagnetic waves to propagate through it with a minimum of reflection so as to limit the size of the computational domain required for characterizing open structures.

Recently, an algorithm [3] similar in nature to the one proposed in this paper was used for simulating the propagation of the quasi-TEM wave in a microstrip structure with the wave impinging at normal incidence onto the absorbing boundary. The amount of reflection was reported to be of the order of 3% to 5%. The use of a super-absorbing boundary [4] was said to further reduce the amount of reflection, although no specific figure was given.

This paper rationalizes the concept of the absorbing boundary in terms of phase and group velocities, improves on the algorithm with a resulting decrease of two orders of magnitude in the amount of reflection at the tuning frequency, and generalizes the algorithm to encompass the case of arbitrary incidence angle. Yet, the algorithm remains very simple and very local, requiring the knowledge of the field values at points located only one space increment away from the absorbing boundary.

Development

This development stems from the realization that the velocity at which a signal propagates on a cartesian grid, in terms of phase velocity for the signal's carrier and of group velocity for the signal's envelope, is different from the velocity $V_n = \Delta l/\Delta t$ at which a numerical disturbance propagates on the same grid. V_n is coined hereafter the numerical velocity. Δl is the spatial increment and Δt is the temporal increment.

For low frequencies propagating in an homogeneous medium, both phase and numerical velocities are related simply by a constant factor $S = C/V_n$ where C is the intrinsic physical propagation velocity for the given medium.

For stability of the 3-D FDTD algorithm , $S \leq (1/\sqrt{3})$.

In order to ensure that the absorbing boundary is transparent, the continuity of the phase velocity for the carrier, and of the group velocity for the envelope must be ensured between the points of the interior computational domain and the points of the absorbing boundary. Indeed, if one visualizes a modulated signal propagating at some incidence angle through a perfectly absorbing boundary, one realizes that the knowledge of the field values at the points lying on the absorbing boundary depends not only on the knowledge of the propagation direction and of the field values at the points behind and perpendicular to the wavefront at some earlier time, but depends also on the knowledge of the carrier's variation, its velocity V_p, the envelope's shape and its velocity V_g. This knowledge requires in general the knowledge of the field values within a large portion (if not all) of the computational domain.

For small enough Δl and Δt values however, the change in the carrier's variation and the change in the envelope's shape become minimal for two closely neighboring grid points lying on the progagation path. Thus both the carrier's variation and the envelope's shape can be taken simply as being constant while the signal propagates from one grid point to the other grid point. Hence for small enough Δl and Δt values, the field values at the absorbing boundary can be closely predicted from the sole knowledge of the propagation direction and of the field values at some earlier time at the points located behind and perpendicular to the wavefront. Since the phase velocity is generally larger than the group velocity, the phase velocity represents therefore the limiting parameter for such prediction to remain accurate.

Take for instance a plane wave impinging at normal incidence onto a plane absorbing boundary lying at the face X=0 of a 3-D rectangular computational domain. Take also the field values obtained by the interior computational scheme, the FDTD method with Yee's lattice [1][2], for the points lying next to the plane absorbing boundary, that is at plane $X=\Delta l$. On each of these two planes lie three of the six electromagnetic field components, namely here the two electric field components Ey and Ez parallel to these planes and the magnetic field component Hx perpendicular to these planes. This absorbing boundary algorithm consists simply in transferring after a delay of $(1/S)\Delta t$ the values of the two field components parallel to the absorbing boundary, from the grid points lying at plane $X=\Delta l$ to the grid points lying at plane X=0. The field values for the normal component Hx is computed directly with the FDTD scheme, and thus is not subject to such transferal scheme (unless the 'super-absorbing boundary algorithm' were also employed, which is not the case herein).

Since updating a field value takes up $1\Delta t$, the algorithm must then introduce a delay of (1/S)-1 additional time steps. Naturally, this algorithm works best only for integer values of (1/S) since delays can be readily implemented only in integer number as a result of the time discretization. But this restriction is not serious since the choice of S can be adjusted, so long as the stability criterion ($S \leq (1/\sqrt{3})$ is satisfied.

The case of a plane wave impinging onto a plane absorbing boundary at an arbitrary incidence angle and with propagation velocity C (see Fig. 1) can be treated simply by considering in the plane next to the absorbing boundary the field pattern resulting from the off-normal incidence angle. The field value at point R of the absorbing boundary can be obtained either as the field value at point P delayed by $(V_n/C) = (1/S)$ time steps or as the field value at point Q delayed by $(V_n/V_p) = (1/S)/(V_p/C)$ time steps. This procedure holds true providing that the wavefront amplitude is nearly constant for both points P and Q. This latter condition is satisfied readily if the spatial increment Δl is small enough.

For high frequencies however, the dispersion of the FDTD method causes the simulated phase velocity V_p to decrease with frequency. The number of delays that must be introduced artificially must then be modified accordingly in order to maintain continuity in the phase velocity between the interior computational domain and the absorbing boundary (the algorithm stability is not affected by the dispersion incurred by the FDTD method since S depends on the physical phase velocity C and not on the simulated phase velocity V_p).

Defining the parameter W as $W = V_n/V_p$, the absorbing boundary algorithm for an X plane wave impinging from the right (X>i) onto the absorbing plane X=i at an arbitrary incidence angle θ becomes simply: $U^{n+1}(i,j,k) = U^{n+1-[W]}(i+1,j,k)$ where U represents the field value, [...] means Nearest Integer operator and $V_p = \zeta C/cos\theta$. The factor ζ accounts for the dispersive behavior of the FDTD method and is function of $\Delta l/\lambda$.

Conclusion

This paper has presented the development of a very simple yet very effective plane absorbing boundary for the FDTD method in three dimensions with arbitrary incidence angle. A companion paper demonstrates the efficacy of this absorbing boundary algorithm for the case of a rectangular wavequide operated in the TE_{10} mode.

References

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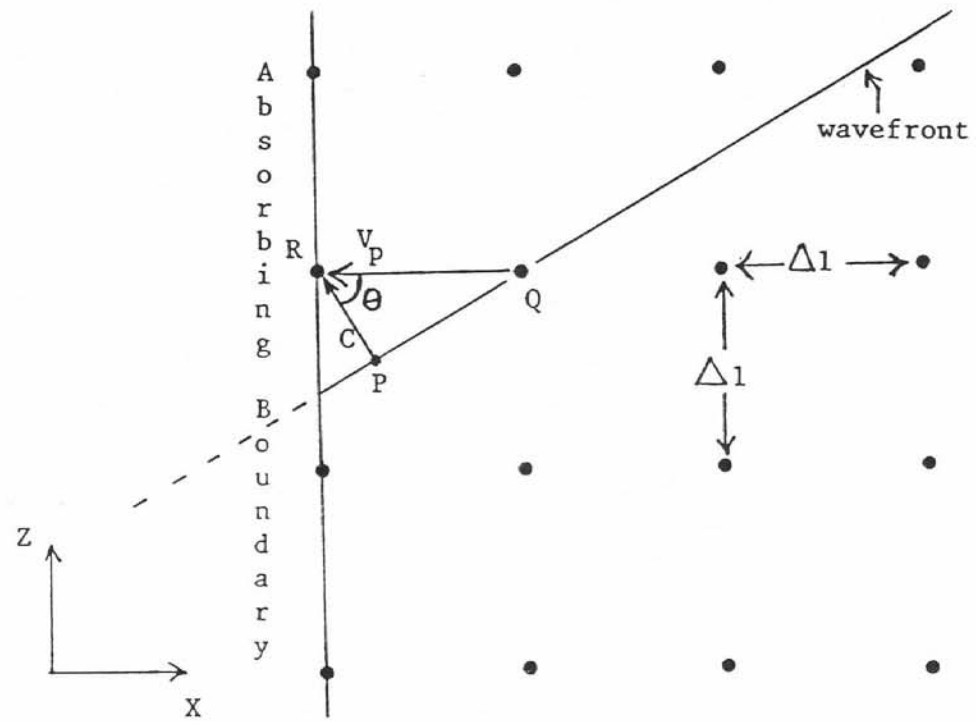

Figure 1: Treatment of an arbitrary incidence angle