EXTERIOR MOMENT METHOD ANALYSIS OF CONDUCTING SCATTERERS BY USING THE INTERIOR GREEN'S FUNCTION AND THE METHOD OF LEAST SQUARE — 2-D AND 3-D ANALYSIS OF SLOT ANTENNAS LOCATED ON SURFACES OF PERFECTLY CONDUCTING SCATTERERS —

K.Sawaya, S.Yatabe, Y.Fujino, and S.Adachi
Department of Electrical Engineering,
Faculty of Engineering, Tohoku University, Sendai 980, Japan

There are many problems such as a conducting cylinder with finite length where the Green's function in the exterior region can not be obtained whereas the Green's function in the interior region can be easily obtained. When the Galerkin-moment method is adopted to obtain the surface current of such a conducting scatterer, the procedure to calculate the impedance matrix is very complicated[1]. Recently we have proposed a simple method to obtain the mutual admittance matrix by using Green's function for the interior region[2]. In this method it is necessary to assume some sources in the interior region and the number of the assumed sources was equal to that of the test functions representing the surface current of the scatterer.

In the new method using the interior Green's function, the assumed sources can be arbitrary in principle. However, it has been pointed out that there are the cases where the arbitrary sources do not yield an acceptable result. In this report, we present a procedure to overcome this problem using the method of least square (MLS). Numerical results of the analysis of a two-dimensional magnetic strip antenna located on the exterior surface of a perfectly conducting rectangular cylinder and a slot antenna on a perfectly conducting cube are also presented.

The method to obtain the mutual admittance matrix by using the interior Green's function is based on the fact that the matrix equations appearing in the moment method,

$$[V]=[\ Z\ ][I], \quad [I]=[\ Y\ ][V] \tag{1}$$

can be used for both the exterior problem and the interior problem. When the Green's function in the interior region is known, [I] can be obtained and the admittance matrix will be evaluated from the known quantities [V] and [I] by solving some interior problems as samples. The procedure mentioned above is summarized as follows.

(i)Assume a suitable set of sources $\mathbf{K}_1,\mathbf{K}_2,..\mathbf{K}_L$ in the interior region.

(ii)Calculate the incident electric fields $\mathbf{E}_1^{inc},\mathbf{E}_2^{inc},..\ \mathbf{E}_L^{inc}$ excited by the sources $\mathbf{K}_1,\ \mathbf{K}_2,..\mathbf{K}_L$ by using the free space Green's function and take the inner products defined by

$$V_{kl}=\int_S \mathbf{f}_k(\mathbf{r})\cdot\mathbf{E}_l^{inc}(\mathbf{r})\,ds\ ,\ k=1,2,..K,\ l=1,2,..L \tag{2}$$

where $\mathbf{f}_k$ is the test function and K is the number of the test functions. Eq.(2) yields the voltage row vector in eq.(1).

(iii)Calculate the surface currents $\mathbf{J}_1,\mathbf{J}_2,..\mathbf{J}_L$ induced by the sources $\mathbf{K}_1,\mathbf{K}_2,..\mathbf{K}_L$ by using the interior Green's function and take the inner products

$$I_{kl}=\int_S \mathbf{f}_k(\mathbf{r})\cdot \mathbf{J}_l(\mathbf{r})\,ds\ ,\ k=1,2,..K,\ l=1,2,..L \tag{3}$$

which yields the current row vector in eq.(1).

(iv) Substitute V_{kl} and I_{kl} into eq.(1).

$$\begin{bmatrix} I_{1l} \\ I_{2l} \\ \vdots \\ I_{Kl} \end{bmatrix} = \begin{bmatrix} & & \\ & Y & \\ & & \end{bmatrix} \begin{bmatrix} V_{1l} \\ V_{2l} \\ \vdots \\ V_{Kl} \end{bmatrix}\ ,\quad l=1,2,...L \tag{4}$$

When we choose as K=L, i.e., the number of assumed sources is equal to that of the test functions, the unknown admittance matrix can be calculated by

$$[\ Y\ ]=\begin{bmatrix} I_{11}\cdots\cdots I_{1L} \\ \vdots \qquad \vdots \\ I_{L1}\cdots\cdots I_{LL} \end{bmatrix}\begin{bmatrix} V_{11}\cdots\cdots V_{1L} \\ \vdots \qquad \vdots \\ V_{L1}\cdots\cdots V_{LL} \end{bmatrix}^{-1} \tag{5}$$

In [2] we have assumed two types of systematically chosen sources in the interior region and have obtained acceptable results from eq.(5) for the case of two-dimensional magnetic strip source on the exterior surface of the perfectly conducting rectangular cylinder as shown in Fig.1. However, it has been pointed out that a set of arbitrarily assumed sources can not yield an acceptable result. Therefore, we choose as L>K and introduce the MLS to calculate the admittance matrix, i.e.,

$$[\ Y\ ]=\begin{bmatrix} I_{11}\cdots\cdots I_{1L} \\ \vdots \qquad \vdots \\ I_{K1}\cdots\cdots I_{KL} \end{bmatrix}\begin{bmatrix} V_{11}\cdots\cdots V_{K1} \\ \vdots \qquad \vdots \\ V_{1L}\cdots\cdots V_{KL} \end{bmatrix}\left[\begin{bmatrix} V_{11}\cdots\cdots V_{1L} \\ \vdots \qquad \vdots \\ V_{K1}\cdots\cdots V_{KL} \end{bmatrix}\begin{bmatrix} V_{11}\cdots\cdots V_{K1} \\ \vdots \qquad \vdots \\ V_{1L}\cdots\cdots V_{KL} \end{bmatrix}\right]^{-1} \tag{6}$$

In order to verify the validity of the procedure using the MLS, we consider the same problem as shown in Fig.1. The test functions of the surface current on the four surfaces are taken as

$$\begin{aligned} \mathbf{f}_m^{\#i}(x)&=\cos(m\pi x/a)\mathbf{x},\ i=1,2,\ m=0,1,..M \\ \mathbf{f}_m^{\#i}(y)&=\cos(m\pi y/b)\mathbf{y},\ i=3,4,\ m=0,1,..M \end{aligned} \tag{7}$$

As the assumed source in the interior region, we take two types of magnetic line sources directed to z axis as illustrated in Fig.2:

Type 1: Magnetic line sources located randomly in the interior space,

Type 2: Magnetic line sources located randomly on the interior surfaces.

The conventional Galerkin-moment method analysis has been also performed with taking $\mathbf{f}_m^{\#i}$ given by eq.(7) as the expansion and test functions. Fig.3 shows the input admittance as a function of frequency. Although a slight discrepancy is observed, the agreements of the three solutions are satisfactory. It is also noted that the spurious resonance is observed in the results of the conventional moment method. On the other hand such an anomaly does not appear in the present analysis. Thus the advantage and the validity of the present method are confirmed.

Next we consider a slot antenna having sinusoidal electric field distribution and located on a perfectly conducting cube as shown in Fig.4. The test functions of the surface current on the six surfaces are taken as

$$\mathbf{f}_{xmn}^{\#i}(x,y)=\cos(m\pi x/a)\sin(n\pi y/b)\mathbf{x},\quad \mathbf{f}_{ymn}^{\#i}(x,y)=\sin(m\pi x/a)\cos(n\pi y/b)\mathbf{y},\quad i=1,2,$$

$$\mathbf{f}_{ynp}^{\#i}(y,z)=\cos(n\pi y/b)\sin(p\pi z/c)\mathbf{y},\quad \mathbf{f}_{znp}^{\#i}(y,z)=\sin(n\pi y/b)\cos(p\pi z/c)\mathbf{z},\quad i=3,4,$$

$$\mathbf{f}_{zpm}^{\#i}(z,x)=\cos(p\pi z/c)\sin(m\pi x/a)\mathbf{z},\quad \mathbf{f}_{xpm}^{\#i}(z,x)=\sin(p\pi z/c)\cos(m\pi x/a)\mathbf{x},\quad i=5,6,$$

$$m,n,p=0,1,..M \qquad (8)$$

In this case, we assume a set of magnetic point sources located at the nodal points of the mesh as in Fig.4. The mesh size is NxNxN and x-, y-, and z-directed sources are assumed at each nodal points, i.e., $L=3xN^3$. The MLS is again used.

Fig.6 shows the input admittance as a function of frequency comparing with the results of the conventional Galerkin-moment method and the experiment. The agreement between the three results is fairly good confirming the validity of the present analysis. Since the conventional Galerkin-moment method requires the double surface integral to calculate the mutual impedances between test functions, the run time of the computer is formidable even though complicated modifications of the double surface integral into single, double, and triple integrals (depending on the surfaces to evaluate the mutual impedance) have been introduced[1]. On the other hand, the calculation involved in the present method is very simple. Double integral is required to evaluate V_{kl} in step (ii) but the effort is not intractable. The procedure to calculate I_{kl} in step (iii) does not involve any integrals and summations. Although the MLS is used, the run time of the computer for the present analysis is about one tenth of that for the conventional Galerkin-moment method. Thus, the advantage of the present analysis is confirmed numerically.

A new method to obtain the mutual admittance matrix by using the interior Green's function of the scatterer and the MLS has been proposed. For the two dimensional case, the accurate results have been obtained by introducing the MLS even when arbitrary sources are assumed in the interior region. In the three dimensional problem, the accurate results have been again observed and the great advantages such as a simple procedure and a short run time of the computer have been confirmed.

REFERENCES

[1]K.Sawaya, T.Kurioka, and S.Adachi,"Analysis of a slot antenna on an L-shaped conducting plane by using the Fourier series expansion and the Galerkin's method", Trans. IEICE Japan, vol. J71-B, no.12, pp.1386-1388, Nov. 1988.

[2]K.Sawaya, S.Yatabe, and S.Adachi,"Moment method analysis by using Green's function in the interior region -Two dimensional analysis of a magnetic source located on an exterior surface of a perfectly conducting rectangular cylinder-", 1988 IEEE AP-S Digest, vol.II, pp.902-905, June 1988.

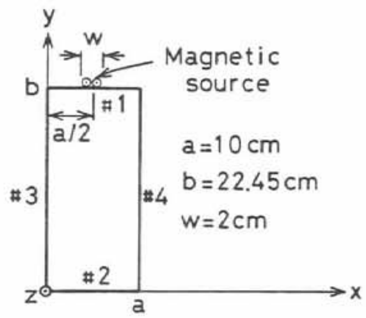


Fig.1 Two dimensional magnetic strip source located on the exterior surface of a perfectly conducting rectangular cylinder.

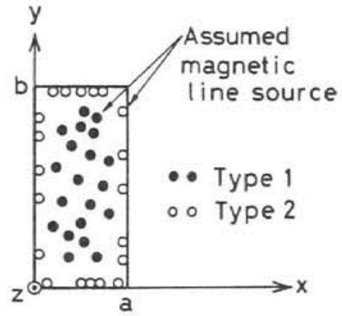


Fig.2 Assumed magnetic line sources to obtain the admittance matrix of the perfectly conducting rectangular cylinder.

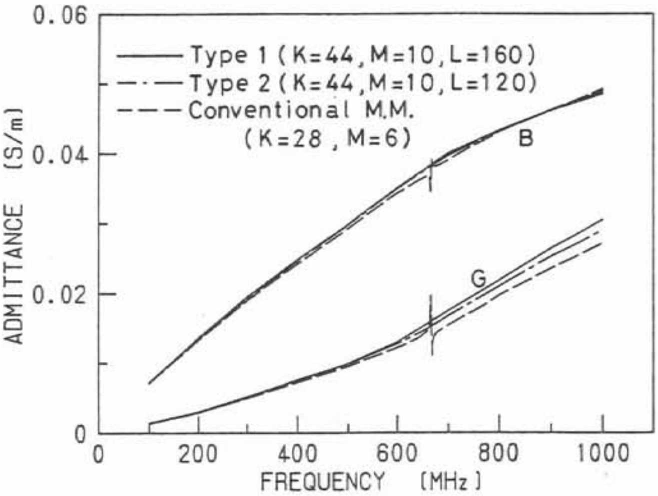


Fig.3 Input admittance of the magnetic source shown in Fig.1 as a function of frequency.

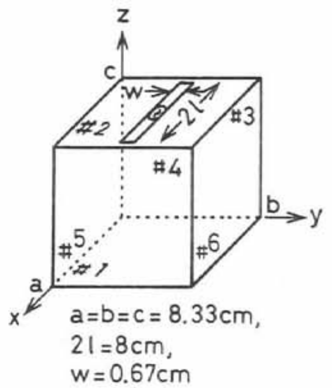
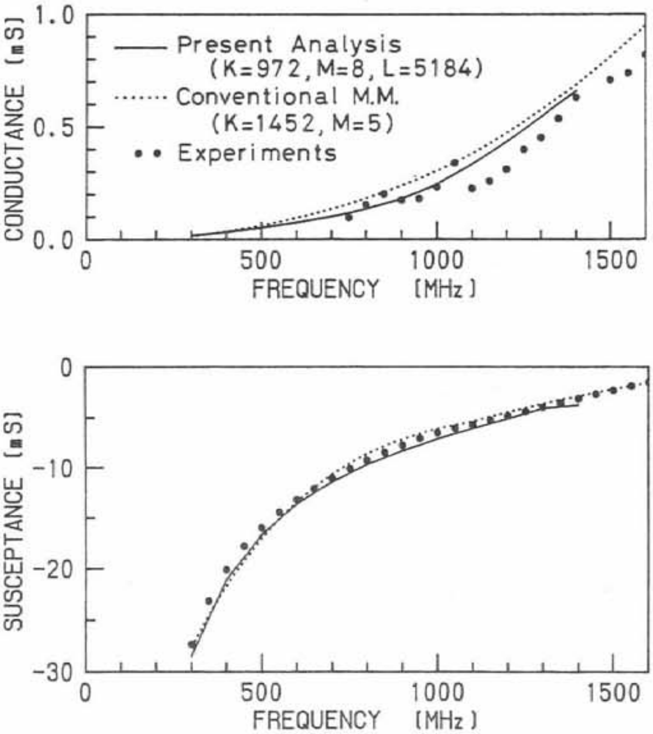


Fig.4 Slot antenna located on a surface of a perfectly conducting cube.



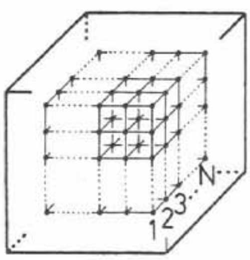


Fig.5 Positions of assumed magnetic point sources to obtain the admittance matrix of the perfectly conducting cube.

Fig.6 Input admittance of the slot antenna shown in Fig.4 as a function of frequency.