NUMERICAL ANALYSIS OF AN ARBITRARY HORIZONTAL ANTENNA ABOVE A LOSSY GROUND

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1. Introduction

Numerical analysis of an antenna above a lossy ground was first formulated by A.N.Sommerfeld in 1909[1]. Many authors analyzed its formulation by an approximation. For example. E.K.Miller introduced the Reflection Coefficient Method(RCM) and emphasized the usefulness of the method[2]. However, when the antenna height against a wave length is low, the input impedance of the antenna shows a discrepancy between the RCM and the exact solution[3].

In this paper, we introduce the integral equation of an arbitrary horizontal antenna above the ground, and analyze it using the point matching method. As an example, we calculate a current distribution and an input impedance of a curved half-wave dipole analyzed already in free space by H.Nakano[4].

2. An integral equation of an arbitrary horizontal antenna above the ground

We consider now a straight horizontal antenna above the ground. Fig.1 shows a geometry of the antenna. The antenna is sitting in parallel with X axis at the height h. In Fig.1, L is a length and a is a radius of the antenna, and ϵ_S is the relative dielectric constant and σ is the conductivity of the ground. A current distribution of the antenna can be calculated to solve the integral equation as

$$\int_L G(x|x')I(x')dx' = -E_i(x) \tag{1}$$

where the symbol $G(x|x')$ is Green's function which gives the electric field at the observation point x produced by the generator at the point x', $I(x')$ is the antenna current of the point x', $\int_L dx'$ is an integral over the antenna, and $E_i(x)$ is the tangential component of the impressed field.

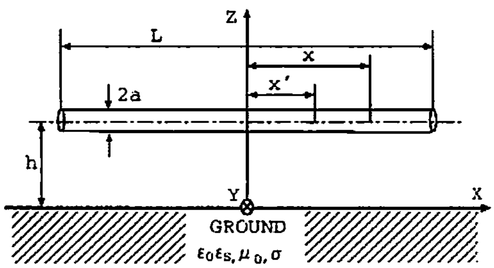

Fig.1 Geometry of the straight antenna.

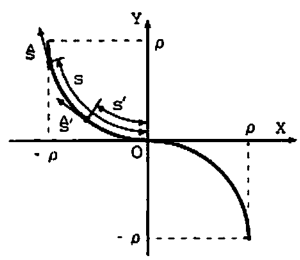

Fig.2 Geometry of the curved half-wave dipole.

Once we analyzed the horizontal antenna above the ground using a method of equivalent circuit[3], and obtained Green's function as

$$\begin{aligned} G(x|x') = &-\frac{j}{4\pi\omega\varepsilon_0}[(k_0^2+\frac{\partial^2}{\partial x^2})(\frac{e^{-jk_0\sqrt{D^2+a^2}}}{\sqrt{D^2+a^2}}-\frac{e^{-jk_0\sqrt{D^2+(z+z')^2}}}{\sqrt{D^2+(z+z')^2}} \\ &+\int_0^\infty \frac{2J_0(rD)e^{-j\sqrt{k_0^2-r^2}(z+z')}}{j(\sqrt{k_0^2-r^2}+\sqrt{k_1^2-r^2})}rdr)-\frac{\partial^2}{\partial x^2}\int_0^\infty \frac{j2e^{-j\sqrt{k_0^2-r^2}(z+z')}}{k_0^2(\sqrt{k_1^2-r^2}+\varepsilon_r\sqrt{k_0^2-r^2})} \\ &\cdot\sqrt{k_0^2-r^2}\cdot(\sqrt{k_0^2-r^2}-\sqrt{k_1^2-r^2})\cdot rJ_0(rD)dr] \end{aligned} \tag{2}$$

where $\varepsilon_r = \varepsilon_S - j\frac{\sigma}{\omega\varepsilon_0}$, the wavenumber for free space $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, $k_1 = \sqrt{\epsilon_r}k_0$, $z + z' = 2h$, and D is the distance between the observation point and the generator. In the case of the straight antenna, $D = |x - x'|$. We showed already that this Green's function is equivalent to Sommerfeld's one[5].

Substituting eq.(2) into (1), we obtain

$$\frac{1}{j\omega\varepsilon_0}\int_L [k_0^2\Psi_H - \frac{\partial^2}{\partial x\partial x'}\Psi_S]I(x')dx' = -E_i(x) \tag{3}$$

where

$$\Psi_H = \frac{e^{-jk_0\sqrt{D^2+a^2}}}{4\pi\sqrt{D^2+a^2}} - \frac{e^{-jk_0\sqrt{D^2+(z+z')^2}}}{4\pi\sqrt{D^2+(z+z')^2}} + \int_0^\infty \frac{2J_0(rD)e^{-j\sqrt{k_0^2-r^2}(z+z')}}{j4\pi(\sqrt{k_0^2-r^2}+\sqrt{k_1^2-r^2})} rdr$$

$$\Psi_V = \int_0^\infty \frac{j2e^{-j\sqrt{k_0^2-r^2}(z+z')}}{4\pi k_0^2(\sqrt{k_1^2-r^2}+\varepsilon_r\sqrt{k_0^2-r^2})}\cdot\sqrt{k_0^2-r^2}\cdot(\sqrt{k_0^2-r^2}-\sqrt{k_1^2-r^2})\cdot rJ_0(rD)dr$$

$$\Psi_S = \Psi_H - \Psi_V.$$

On the other hand, the tangential component of the electric field E_x is given using a well-known equation as

$$E_x = -\frac{\partial\phi_x}{\partial x} - j\omega A_x \tag{4}$$

$$A_x = \mu\int_L I(x')G'(x|x')dx' \tag{5}$$

$$\phi_x = -\frac{1}{j\omega\varepsilon_0}\int_L \frac{dI(x')}{dx'}G''(x|x')dx' \tag{6}$$

where A_x is the tangential component of the vector potential and ϕ_x is the scalar potential.
Substituting eq.(5),(6) into (4), we obtain

$$\frac{1}{j\omega\varepsilon_0}\int_L [k_0^2G'(x|x') - \frac{\partial^2}{\partial x\partial x'}G''(x|x')]I(x')dx' = -E_i(x). \tag{7}$$

Comparing eq.(3) and (7), we find out that $G'(x|x')$ and $G''(x|x')$ shall coincide with Ψ_H and Ψ_S respectively.

Using these relations, we analyze an arbitrary horizontal antenna above the ground. As an example, Fig.2 shows a geometry on the X-Y plane of the curved half-wave dipole. Let us suppose that s is the arc length measured from the feed gap, and $\hat{s}$ is the unit tangent vector at s. Then we put that the coordinates of s and s' are (x, y) and (x', y') respectively. Therefore we must use $D = \sqrt{(x-x')^2+(y-y')^2}$ in the case of the arbitrary horizontal antenna. The tangential component of the electric field E_s is given as

$$E_s = -\frac{\partial\phi_s}{\partial s} - j\omega A_s \tag{8}$$

$$A_s = \mu\hat{s}\cdot\int_L I(s')\hat{s'}\Psi_H ds' \tag{9}$$

$$\phi_s = -\frac{1}{j\omega\varepsilon_0}\int_L \frac{dI(s')}{ds'}\Psi_S ds' \tag{10}$$

where A_s is the tangential component of the vector potential and ϕ_s is the scalar potential.
Substituting eq.(9),(10) into (8), we obtain

$$\frac{1}{j\omega\varepsilon_0}\int_L [k_0^2\Psi_H\hat{s}\cdot\hat{s'} - \frac{\partial^2}{\partial s\partial s'}\Psi_S]I(s')ds' = -E_i(s). \tag{11}$$

This equation is the Pocklington's equation of an arbitrary horizontal antenna above the ground.

Once K.K.Mei introduced the integral equation of an arbitrary thin-wire antenna in free space[6]. In the same manner, we can introduce the integral equation from eq.(8),(9) and (10) as

$$\int_L G(s|s')I(s')ds' = C'\cos k_0 s - \frac{jV_0}{2Z_0}\sin k_0|s| \tag{12}$$

$$G(s|s') = \Psi_H \hat{s}\cdot\hat{s'} - \int_0^s [\frac{\partial \Psi_H}{\partial \xi}\dot{\xi}\cdot\hat{s'} + \frac{\partial \Psi_S}{\partial s'} + \Psi_H \frac{\partial(\dot{\xi}\cdot\hat{s'})}{\partial \xi}]\cdot \cos k_0(s-\xi)d\xi \tag{13}$$

where C' is the unknown constant. This equation is the Mei's equation of an arbitrary horizontal antenna above the ground.

3. Analysis using the point matching method

We use the point matching method to solve the current distribution of the antenna from eq.(12). We choose any N points $s_m(m = 1,2,3,\cdots,N)$ along the antenna, and stipulate that eq.(12) is satisfied at these points. Let us suppose that $I(s')$ can be represented as

$$I(s') = \begin{cases} I_n & ,s' \in \Delta s_n \\ 0 & ,otherwise \end{cases}$$

where I_n are complex coefficients to be determined. Then we obtain

$$\sum_{n=1}^{N} I_n \int_{\Delta s_n} G(s_m|s')ds' = C' \cos k_0 s_m - \frac{jV_0}{2Z_0}\sin k_0|s_m|. \tag{14}$$

As a result, we obtain the linear equations with N unknowns as

$$\sum_{n=1}^{N} Z_{nm} I_n = V_m \tag{15}$$

$$Z_{nm} = \int_{\Delta s_n} G(s_m|s')ds' \tag{16}$$

$$V_m = C' \cos k_0 s_m - \frac{jV_0}{2Z_0}\sin k_0|s_m|. \tag{17}$$

Since C' is the unknown constant, we add one equation satisfied the boundary condition to these equations, and obtain the linear equations with N+1 unknowns. We can calculate the current distribution of the antenna by solving it. If the feed gap exists in Δs_j of the antenna, the input impedance Z_{in} represents as

$$Z_{in} = \frac{V_0}{I_j} \tag{18}$$

where V_0 is the voltage of the feed gap.

$G(s_m|s')$ includes some differentials and integrals, so that it is still complicated. Therefore we transform analytically $\frac{\partial \Psi_H}{\partial s}$ and $\frac{\partial \Psi_S}{\partial s'}$ as

$$\begin{aligned}\frac{\partial \Psi_H}{\partial s} = &- \frac{e^{-jk_0 D_1(s,s')}}{4\pi D_1(s,s')}\cdot\frac{1+jk_0 D_1(s,s')}{D_1(s,s')}\cdot\frac{(x-x')+(y-y')\frac{dy}{dx}}{D_1(s,s')\sqrt{1+(\frac{dy}{dx})^2}} \\ &+ \frac{e^{-jk_0 D_2(s,s')}}{4\pi D_2(s,s')}\cdot\frac{1+jk_0 D_2(s,s')}{D_2(s,s')}\cdot\frac{(x-x')+(y-y')\frac{dy}{dx}}{D_2(s,s')\sqrt{1+(\frac{dy}{dx})^2}} \\ &- \int_0^\infty \frac{2e^{-j\sqrt{k_0^2-r^2}(z+z')}}{j4\pi(\sqrt{k_0^2-r^2}+\sqrt{k_1^2-r^2})}\cdot\frac{(x-x')+(y-y')\frac{dy}{dx}}{D\sqrt{1+(\frac{dy}{dx})^2}}\cdot r^2 J_1(rD)dr\end{aligned} \tag{19}$$

$$\begin{aligned}\frac{\partial \Psi_S}{\partial s'} = &+ \frac{e^{-jk_0 D_1(s,s')}}{4\pi D_1(s,s')}\cdot\frac{1+jk_0 D_1(s,s')}{D_1(s,s')}\cdot\frac{(x-x')+(y-y')\frac{dy'}{dx'}}{D_1(s,s')\sqrt{1+(\frac{dy'}{dx'})^2}} \\ &- \frac{e^{-jk_0 D_2(s,s')}}{4\pi D_2(s,s')}\cdot\frac{1+jk_0 D_2(s,s')}{D_2(s,s')}\cdot\frac{(x-x')+(y-y')\frac{dy'}{dx'}}{D_2(s,s')\sqrt{1+(\frac{dy'}{dx'})^2}}\end{aligned}$$

$$
+ \int_0^\infty \frac{2e^{-j\sqrt{k_0^2-r^2}(z+z')}}{j4\pi(\sqrt{k_0^2-r^2}+\sqrt{k_1^2-r^2})} \cdot \frac{(x-x')+(y-y')\frac{dy'}{dx'}}{D\sqrt{1+(\frac{dy'}{dx'})^2}} \cdot r^2 J_1(rD)dr
$$

$$
- \int_0^\infty \frac{j2e^{-j\sqrt{k_0^2-r^2}(z+z')}}{4\pi k_0^2(\sqrt{k_1^2-r^2}+\varepsilon_r\sqrt{k_0^2-r^2})} \cdot \frac{(x-x')+(y-y')\frac{dy'}{dx'}}{D\sqrt{1+(\frac{dy'}{dx'})^2}}
$$

$$
\cdot \sqrt{k_0^2-r^2} \cdot (\sqrt{k_0^2-r^2}-\sqrt{k_1^2-r^2}) \cdot r^2 J_1(rD)dr \qquad (20)
$$

where $D_1(s,s') = \sqrt{D^2+a^2}$ and $D_2(s,s') = \sqrt{D^2+(z+z')^2}$.

Putting $\frac{\partial \Psi_H}{\partial s} = \Psi'_H$ and $\frac{\partial \Psi_S}{\partial s'} = \Psi'_S$, we obtain

$$
Z_{nm} = \int_{\Delta s_n} \Psi_H \hat{s_m} \cdot \hat{s'} - \int_0^{s_m} [\Psi'_H \hat{\xi} \cdot \hat{s'} + \Psi'_S + \Psi_H \frac{\partial(\hat{\xi} \cdot \hat{s'})}{\partial \xi}] \cdot \cos k_0(s_m - \xi) d\xi ds'. \qquad (21)
$$

4. Numerical results

We apply the results of our analysis to the curved half-wave dipole. The thickness parameter of the antenna is chosen as $\Omega = 2ln(L/\rho) = 13.24$. The number of sections N is 24 and the frequency is 1GHz. When the antenna height h is large enough, we obtain the input impedance in free space, $Z_{in} = 72+j35$. This value nearly coincide with the value obtained by H.Nakano[4]. Fig.3 and Fig.4 show the current distribution of the antenna and the input impedance against the antenna height respectively. In Fig.4, we can observe that the impedance against the antenna height swings in the center of free space's one except for the case which h is low. This tendency is nearly the same with the case of the straight dipole.

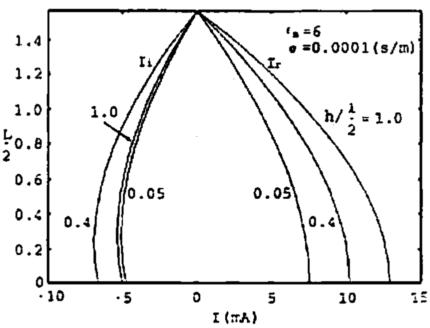

Fig.3 The current distribution of the curved half-wave dipole.

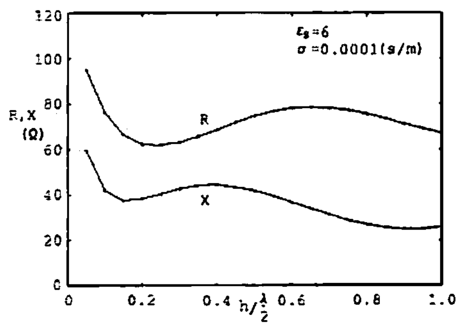

Fig.4 The input impedance of the curved half-wave dipole.

5. Conclusion

We introduced the integral equation of an arbitrary horizontal antenna above a lossy ground. As an example, we calculated the current distribution and the input impedance of the curved half-wave dipole.

6. References

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