Target Direction Estimation by MIMO Radar Using Root-MUSIC with Minimum Redundancy Array

Masatada Hokiguchi, Nobuyoshi Kikuma, and Kunio Sakakibara
Dept. of Electrical and Mechanical Engineering
Nagoya Institute of Technology, Nagoya 466-8555, Japan
Email: kikuma@m.ieice.org

Abstract – **Multiple-Input Multiple-Output (MIMO) radar which applies MIMO communication technology in the radar has attracted attention as a kind of high-resolution radar. In this paper, target direction is estimated by applying the MUSIC method to the minimum redundancy MIMO radar which has been used as the adaptive beamforming array. Furthermore, the Root-MUSIC method which has been limited to the uniform array is applied to the minimum redundancy array to reduce the computational load in the direction estimation.**

Index Terms — **MIMO radar, Minimum redundancy array, MUSIC, Root-MUSIC, Thinned array, DOA estimation.**

1. Introduction

Recently, in order to implement the automated car driving and develop car systems for avoiding dangerous collision or crash, urgent development of high-performance radars is expected. As one of the expected radars, the Multiple-Input Multiple-Output (MIMO) radar which applies the MIMO communication technology in the radar has attracted attention because of its high resolution compared to the conventional radars. Also, the MIMO radar using the minimum redundancy array is known for its higher-resolution performance [1]. The minimum redundancy MIMO radar has been used mainly as the adaptive beamforming array so far. In this paper, we use the minimum redundancy MIMO radar for the target direction estimation with the MUSIC (MUltiple SIgnal Classification) method [2]. Furthermore, we attempt to apply the Root-MUSIC method which is normally limited to the uniform array. As a result, we can expect to reduce the computational load in the minimum redundancy MIMO radar.

2. Minimum Redundancy MIMO Radar and Virtual Array

The MIMO radar transmits M signals which are orthogonal to each other from M-element transmitting antennas. In the reception, the MIMO radar receives signals reflected by the targets by N-element receiving antennas. Fig. 1 shows the MIMO radar model for $M = 3$ and $N = 4$. If the target is assumed to be sufficiently far in the θ direction, the mth matched filter output signal of the nth receiving antenna can be represented as shown in (1) in the case of unit amplitude.

$$S_{m,n} = \exp\left\{-j\frac{2\pi}{\lambda}(x_{T,m} + x_{R,n})\sin\theta\right\} \tag{1}$$

where $x_{T,m}$ and $x_{R,n}$ are positions of the transmitting and receiving antenna elements, respectively. As found from (1), the matched filter output signals are equal to the signals received by the elements placed in the positions of $x_{T,m}+x_{R,n}$. The array of these virtual elements is referred to as the virtual array. In addition, the minimum redundancy array can be used for the virtual array [1]. The most famous example of 4-element minimum redundancy array is shown in Fig. 2. By using this type of array in the MIMO radar, we can construct a larger aperture array with fewer elements and achieve the direction estimation with high resolution. If the MUSIC method is used in the minimum redundancy MIMO radar, we can attain still higher resolution and accuracy in target estimation.

3. Use of Root-MUSIC Method in Minimum Redundancy Array

The Root-MUSIC method is conventionally applicable only to the uniform linear array. However, by introducing the thinning matrix $\boldsymbol{T}$, the Root-MUSIC can be used in the minimum redundancy array which is regarded as the thinned array. The thinning matrix is obtained by deleting rows corresponding to thinned positions from the identity matrix. The thinned mode vector $\boldsymbol{a}'(\theta)$ is generated by multiplying the thinning matrix to the conventional mode vector $\boldsymbol{a}(\theta)$, thereby performing the same calculation as the conventional Root-MUSIC method. In the example of the 4-element minimum redundancy array of Fig. 2, the conventional mode vector $\boldsymbol{a}(\theta)$, the thinning matrix $\boldsymbol{T}$ and the thinned mode vector $\boldsymbol{a}'(\theta)$ are described as follows.

$$\boldsymbol{a}(\theta) = [1, z, z^2, z^3, z^4, z^5, z^6]^T \tag{2}$$

$$z = \exp\left(-j\frac{2\pi}{\lambda}d\sin\theta\right) \tag{3}$$

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

$$\boldsymbol{a}' = \boldsymbol{T}\boldsymbol{a}(\theta) \tag{5}$$

$$= [1, z, z^4, z^6]^T \tag{6}$$

where d is the minimum element spacing of the minimum redundancy array. Then, by finding θ which satisfies the following equation, we can obtain the estimates of the target directions.

$$\boldsymbol{a}'^H(\theta)\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{a}'(\theta) = \boldsymbol{a}^H(\theta)\boldsymbol{T}^T(\theta)\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{T}\boldsymbol{a}(\theta) = 0 \qquad (7)$$

where $\boldsymbol{E}_N$ is the matrix consisting of eigenvectors in the noise subspace of the virtual array correlation matrix. The Root-MUSIC method does not need to perform the peak search of spectrum, and we can obtain the directions of arrival of targets from direct numerical calculation. Therefore, we can reduce the computational load associated with the target direction estimation.

4. Computer Simulation

The computer simulation of target direction estimation is carried out under the conditions described in Table I. The minimum redundancy MIMO radar in Fig. 3 is compared to the conventional uniform MIMO radar in Fig. 4. Fig. 5 shows the obtained MUSIC spectrums. It is observed from Fig. 5 that the peaks of the spectrum by the minimum redundancy MIMO radar are sharper than those of the uniform MIMO radar. It means that angular resolution is improved by the minimum redundancy arrangement. Fig. 6 shows the RMSE of the target direction estimation results by the Root-MUSIC method (average of 100 trials) when SNR is changed. We can see lower RMSE of the minimum redundancy array, which means the improved estimation accuracy. Moreover, since the peak search is not required in the Root-MUSIC, we have smaller computational load in obtaining estimated directions. In the computer simulation, the time required for one estimation is about 0.042 seconds in the Root-MUSIC method. On the other hand, in peak search of the MUSIC spectrum with interval of 0.01 degrees, it takes about 0.32 seconds for one trial. It follows that the Root-MUSIC method reduces 87 percents of the computational load.

5. Conclusion

In this paper, we have shown that we can carry out target direction estimation by the minimum redundancy MIMO radar with the Root-MUSIC method. In the direction estimation, it was shown that the angular resolution is improved by the minimum redundancy array. Furthermore, by using the Root-MUSIC method with the thinning matrix, we can reduce computational load. In the future, we will reduce the number of snapshots (samples) of the receiving array input by introducing the transmission diversity, and also we will try to suppress spurious peaks more sufficiently in the MUSIC spectrum.

References

[1] Chun-Yang Chen and P.P.Vaidyanathan, "Minimum redundancy MIMO radars", *IEEE International Symposium on Circuits and Systems*, pp.45-48, 2008.

[2] N.Kikuma, *Adaptive antenna technology (in Japanese)*, Ohmsha, Oct. 2003.

TABLE I
Simulation Parameters

Number of transmitting elements	5
Number of receiving elements	3
Positions of uniform transmitting elements ($*\lambda/2$)	0,3,6,9,12
Positions of uniform receiving elements ($*\lambda/2$)	0,1,2
Positions of minimum redundancy transmitting elements ($*\lambda/2$)	0,6,13,40,60
Positions of minimum redundancy receiving elements ($*\lambda/2$)	0,1,3
Target direction [deg.]	0,30
Arrival signal power	1.0,1.0
Input SNR [dB]	20
Number of snapshots	10000

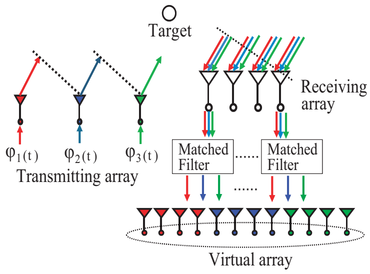

Fig. 1. MIMO radar model ($M = 3, N = 4$)

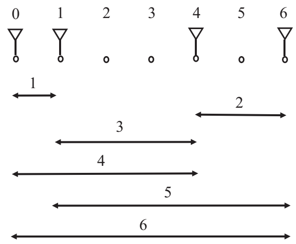

Fig. 2. 4-element minimum redundancy array

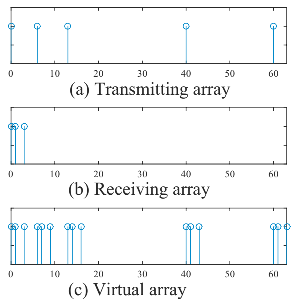

Fig. 3. Element position of minimum redundancy MIMO radar

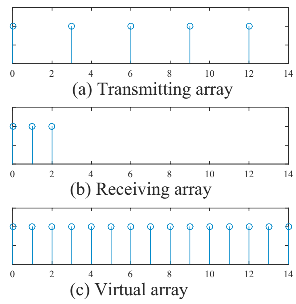

Fig. 4. Element position of conventional uniform MIMO radar

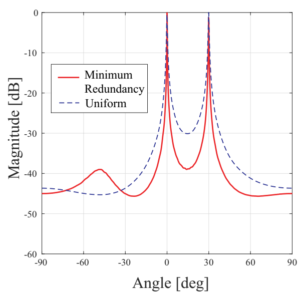

Fig. 5. MUSIC spectrums in target estimation

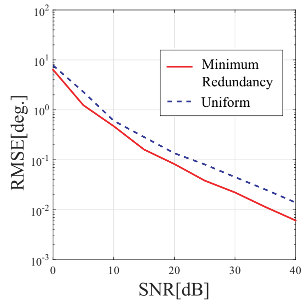

Fig. 6. RMSE by the Root -MUSIC vs. SNR