Estimation of Received Signals at Arbitrary Remote Locations based on Estimation of Arriving Waves by Compressed Sensing

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Abstract **– Propagation characteristics have locality in a multipath fading environment. The locality is utilized for various techniques using radio wave. Recently, on the other hand, new techniques to break the locality have been studied. In the techniques, received signals at arbitrary locations are estimated from remote receiving points. We present in this paper a new estimation technique of received signals utilizing Compressed Sensing (CS). In the technique, we estimate directions of arrival and each complex amplitude of arriving waves by CS, and calculate a received signal at a remote location based on the estimated results. We present the technique and evaluate the estimation performance through computer simulations.**

Index Terms **— Compressed sensing, Estimation of received signal, Multipath environment.**

1. Introduction

Propagation characteristics of radio waves are local in a multipath fading environment. Various techniques based on the characteristics have been developed. An example is found in wireless physical layer security [1]. The technique utilizes the propagation characteristics of wireless channels between legitimate two users in order to transmit messages securely. In the case that we are remote from the users, we cannot eavesdrop on the messages because we do not know the legitimate propagation characteristics.

On the other hand, new techniques to break the locality have been studied in recent years. If we can estimate the received signals at the legitimate destinations, the possibility of eavesdropping in the physical layer security increases. Thus the estimation techniques of received signals at remote locations can be applied to various radio techniques.

In this paper, we regard complex amplitude at a remote location as the received signal. In the previous study [2], we estimate the complex amplitude at the remote location based on the directions of arrival (DOAs) and each complex amplitude of the arriving waves. We estimate the DOAs by the Multiple Signal Classification (MUSIC) method, and the complex amplitude by the least squares method. In this paper, we estimate DOAs and complex amplitude simultaneously by Compressed Sensing (CS). CS is a technique to obtain a solution from an underdetermined linear system [3, 4]. Based on the DOAs and each complex amplitude of arriving waves, we calculate a received signal at a remote location. We evaluate the estimation performance via computer simulations.

2. Estimation of Received Signal at an Arbitrary Remote Location by Compressed Sensing

CS is a technique to provide an optimum solution of an underdetermined system taking advantage of the prior knowledge that the true solution is sparse**.** We consider the linear system having M equations and N unknowns as

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{n} \qquad (1)$$

where $\boldsymbol{A} \in \mathbb{C}^{M\times N}$ is a sensing matrix, $\boldsymbol{n} \in \mathbb{C}^{M}$ is a noise vector, $\boldsymbol{x} \in \mathbb{C}^{N}$ is an unknown vector, and $\boldsymbol{y} \in \mathbb{C}^{M}$ is a measurement vector. In the case of $M<N$, we cannot obtain the true solution generally. However, in the case that the true solution is a sparse vector, in other words, the number of the non-zero elements of $\boldsymbol{x}$ is less than that of the equations, we can calculate the optimum solution. This is the basic idea to obtain the optimum solution by CS. CS is utilized for various fields including communications [5, 6].

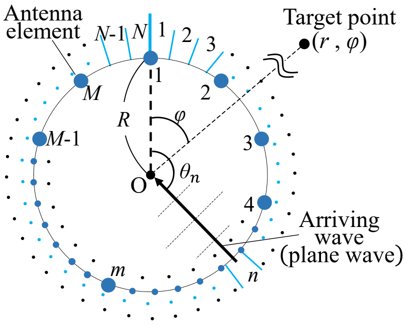

Figure 1. Assumed environment and estimation model.

Figure 1 shows the assumed environment and the estimation system used in the estimation of the received signals. Assuming a two-dimensional space, an M-elements circular array is placed, and the radius of the element arrangement is R. In the polar coordinate, the origin of the coordinate is set at the center of the circle. The position of the target point where the received signal is estimated is denoted (r, φ). The whole angular range of the estimation, -180°~180° in this case, is divided into N small angular bins. θ_n is the angle of the n-th angular bin. x_n is the n-th element of the vector $\boldsymbol{x}$. It corresponds to the complex amplitude of the arriving wave contained within the n-th angular bin. y_m is the m-th element of the vector $\boldsymbol{y}$. It is the received signal at the m-th antenna element. We can calculate the optimum solution $\hat{\boldsymbol{x}}$ by CS, setting the estimation system as the number of the angular division N is much larger than that of the arriving waves.

In this paper, we estimate the received signal $\hat{s}$ at the target point considering the phase difference based on the distance from the origin to the target point by the following formula

$$\hat{s} = \sum_{n=1}^{N} \hat{x}_n \exp\left\{-j2\pi\frac{r}{\lambda}\cos(\theta_n-\varphi)\right\} \qquad (2)$$

where λ is the wavelength of the carrier frequency of the radio waves. In the above estimation formula, all arriving waves are assumed plane waves.

3. Evaluation of Estimation Performance

We evaluate estimation performance of received signals at an arbitrary remote location by CS via computer simulations. Table 1 summarizes the assumed values of the multipath environment and the estimation system.

Figure 2 shows the amplitude ratio of the estimated received signal to the actual at the two target points ($(r, \varphi)=(10\lambda, 10°)$ and $(100\lambda, 10°)$). In the figure, 100 independent trials are carried out varying the DOAs and the phases of the arriving waves randomly. In the case of $r=10\lambda$, the errors of the estimation are very small. On the other hand when $r=100\lambda$, the errors are large. Here we use 90% value of the distribution of the amplitude ratio as a measure of the estimation accuracy. The 90% value is a value of the amplitude ratio where ninety percent of the absolute values of the amplitude ratios of the independent trials are below the value. In the case of $r=100\lambda$, the 90% value is 2.2 dB.

Figure 3 shows the 90% value over the variation of the distance to the target points. We calculate the received signals with both the estimated DOAs and the actual. We add certain intentional angular error ψ to each actual DOA in order to consider the effect of the DOA estimation error. In the case of $\psi=0°$, the 90% values are very small. It appears that we can obtain complex amplitude of arriving waves correctly by CS. The results shown in the figure shows the errors of the estimated DOAs cause the degradation of the estimation of the received signals. It is shown that the 90% values obtained using the estimated DOAs are close to that with 0.02° intentional error. It indicates that the angular errors of the estimation are about 0.02°. Since DOA is discretely expressed with 0.01° separation in the estimation, it inherently includes the quantization error up to 0.005°. However the above result shows the DOA estimation error is larger than the quantization error. Therefore, we see that the improvement of the DOA estimation accuracy is a key in order to improve the whole estimation performance.

4. Conclusion

In this paper, we present a technique to estimate received signals at an arbitrary remote location based on estimation of arriving waves by CS. We show that the errors of the DOA estimation deteriorate the estimation. It is our future task to improve the DOA estimation accuracy to improve the whole estimation performance.

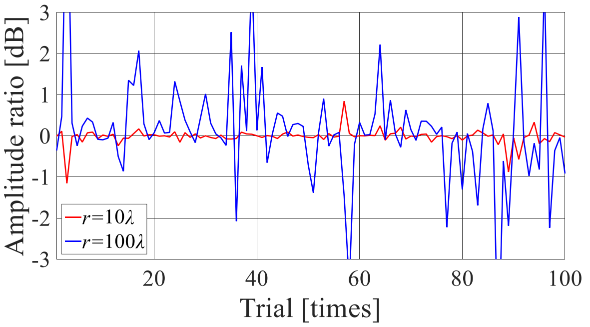

Figure 2. Example of estimation using CS.

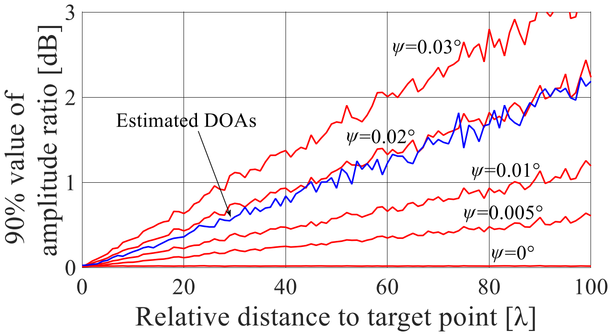

Figure 3. Influence of distance to target points.

Table 1. Values of environment and estimation system.

Multipath environment	
Number of arriving waves	3
Arriving angle	Random (-180°~180°)
Complex amplitude of arriving waves	Amplitude : Unity Phase : Random (-180°~180°)
SNR	40 dB
Estimation system	
Antenna separation	λ
Number of antennas	20
Number of snapshots	100
Bin size	0.01°

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