Effect of Redundancy of Element Placement on DOA Estimation with Circular Array

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Abstract – **The high-resolution DOA estimation with an array antenna is utilized as an important technique for source localization in mobile communication or radio monitoring. This paper deals with MUSIC with circular array which can perform DOA estimation in all directions of azimuth. Uniform circular array (UCA) and minimum redundancy circular array (MRCA) are compared in DOA estimation and the effectiveness of MRCA is shown through computer simulation.**

Index Terms — **Circular array, Element placement, DOA estimation, MUSIC, Minimum redundancy.**

1. Introduction

In the mobile communication or radio monitoring, the high-resolution direction of arrival (DOA) estimation using an array antenna is an important technique for source localization [1]. As one of the methods, MUSIC (MUltiple SIgnal Classification) [1] is known for its high resolution and high accuracy. In addition, when circular arrays (CA) are used for MUSIC, we can perform DOA estimation in all directions of azimuth. In this paper, we investigate the effect of element placement of the circular array in DOA estimation. Especially, we focus on the redundancy of array arrangement. Via computer simulation, we show the DOA estimation performance with the minimum redundancy circular array (MRCA).

2. Circular Array and Signal Model

Fig. 1 shows a uniform circular array (UCA) in which K elements are equally spaced on the circumference. Here, all elements are isotropic, and the radius of the circle is r, and the number of incoming waves is L. The azimuth angles of arrival of incoming waves are represented as φ_l $(l = 1, 2, \cdots, L)$ as shown in Fig. 1. At time instant t, the array input vector $\boldsymbol{x}(t)$ is expressed by the following equations.

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \quad \in \mathbb{C}^{K\times 1} \tag{1}$$

$$\boldsymbol{A} = [\boldsymbol{a}(\varphi_1), \boldsymbol{a}(\varphi_2), \cdots, \boldsymbol{a}(\varphi_L)] \quad \in \mathbb{C}^{K\times L} \tag{2}$$

$$\boldsymbol{a}(\varphi_l) = \left[e^{j\mu_{1l}}, e^{j\mu_{2l}}, \cdots, e^{j\mu_{Kl}}\right]^T \tag{3}$$

$$\mu_{kl} = \frac{2\pi}{\lambda} r \cos\left\{\varphi_l - \frac{2\pi}{K}(k-1)\right\} \quad (k = 1, 2, \cdots, K) \tag{4}$$

where $\boldsymbol{A}$ is the mode matrix, $\boldsymbol{a}(\varphi_l)$ is the mode vector of the lth signal, $\boldsymbol{s}(t)$ is the signal vector consisting of L signal waveforms, and $\boldsymbol{n}(t)$ is the internal noise vector.

3. Minimum Redundancy Circular Array

For linear arrays, the minimum redundancy array was proposed for improving the beamforming performance [2]. Also, for circular arrays, the concept of the minimum redundancy was introduced for the adaptive array [3]. Therefore, we can expect the enhanced DOA estimation performance when the minimum redundancy circular array (MRCA) is used.

The performance of the array can be analyzed by considering all possible pairings of elements, with each pair acting as a 2-element interferometer. For the K elements, there are $K(K-1)/2 \equiv M$ possible parings but not all of these are distinct. The minimum redundancy array uses the minimum number of antenna elements to generate a certain set of pairings by minimizing duplicate pairs.

In the case of circular array, the way of selecting the element position is equivalent to the problem of "determining the set $\{0, 1, \cdots, N\}$ which consists of K integers so that the sum of two arbitrary distinct elements may be set to $\{1, 2, \cdots, M = K(K-1)/2\}$"[2].

For example, when $K = 4$ and $M = 6$, the solution of the problem is $\{0, 1, 2, 4\}$. Using this integers, a set of the position arguments of array elements $\{\gamma_k\}$ can be given by

$$\gamma_k = \left\{0, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}\right\} \tag{5}$$

The 4-element minimum redundancy array which is determined in this way is shown in Fig. 2.

4. DOA Estimation using MUSIC

In general, the MUSIC spectrum is given by the following equation [1].

$$P_{MU}(\varphi) = \frac{\boldsymbol{a}^H(\varphi)\boldsymbol{a}(\varphi)}{\boldsymbol{a}^H(\varphi)\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{a}(\varphi)} \tag{6}$$

where $\boldsymbol{E}_N$ is the matrix consisting of eigenvectors in the noise subspace of the correlation matrix $\boldsymbol{R}_{xx} = E[\boldsymbol{x}(t)\boldsymbol{x}(t)^H]$. You can obtain the DOA estimates from angles corresponding to peaks of the spectrum of (6).

5. Computer Simulation

Under the conditions of Table I, we compare the DOA estimation performance of MUSIC using UCA and MRCA. MUSIC spectrums of UCA and MRCA are shown in Fig. 3. It is found from Fig. 3 that there are many large peaks (spurious) at the angles other than DOA of the incoming waves. In contrast, such spurious peaks are reduced in the MRCA. Fig. 4 shows the Root Mean Square Errors (RMSE) of the DOA estimates when changing φ_1 with $\varphi_2 = \varphi_1 + 20°$ from $-180°$

to 180°. It is observed from Fig. 4 that RMSE with MRCA is stable over the whole angles and low on the average compared to that with UCA. It results in the improved performance of MRCA over UCA.

TABLE I
Simulation Conditions

Number of elements	4
Radius r	0.5λ
Number of incident waves	2
DOA [deg]	$0, 20$
Power	$1.0, 1.0$
Input SNR [dB]	20
Number of snapshots	100
Number of trials	100

6. Conclusion

In this paper, we compared the DOA estimation using MUSIC with 4-element UCA and MRCA. As a result of computer simulation, it is confirmed that the accuracy of the DOA estimation using the 4-element circular array can be improved by placing the elements in the minimum redundancy arrangement. In the future works, we will examine the performance when the number of elements is more than 4 and when there is the mutual coupling effect.

References

[1] N. Kikuma, *Adaptive antenna technology* (in Japanese), Ohmsha Inc., 2003.

[2] A. Moffet, "Minimum-redundancy linear arrays," *IEEE Trans. Antennas and Propag.*, Vol.AP-16, No.2, pp.172–175, March 1968.

[3] N. Inagaki, "Broadband power inversion adaptive array," *IECE Trans. (Japanese Edition)*, Vol.J68-B, No.4, pp.531–538, Apr. 1985.

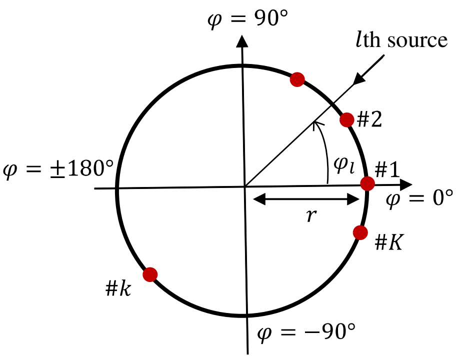

Fig. 1. Uniform circular array and incident wave

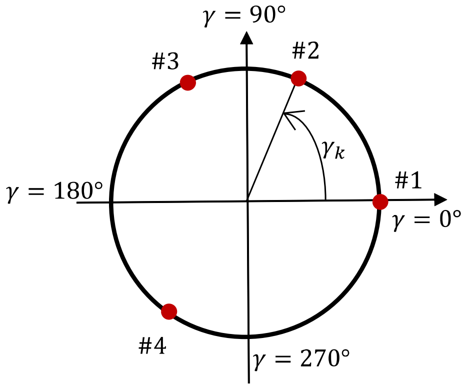

Fig. 2. Minimum redundancy circular array with 4 elements

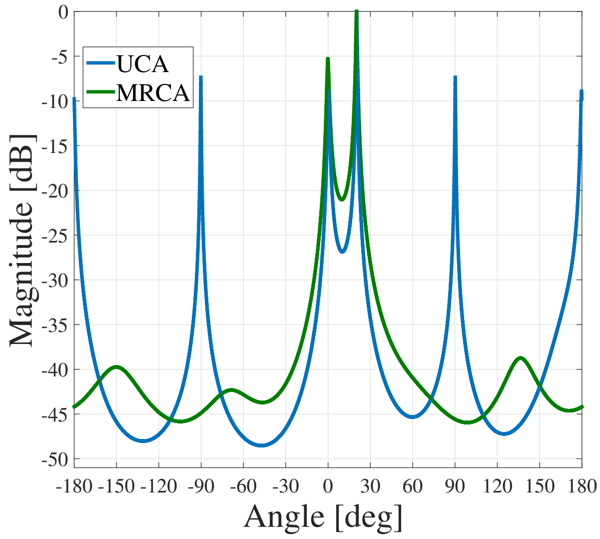

Fig. 3. MUSIC spectrums of UCA and MRCA

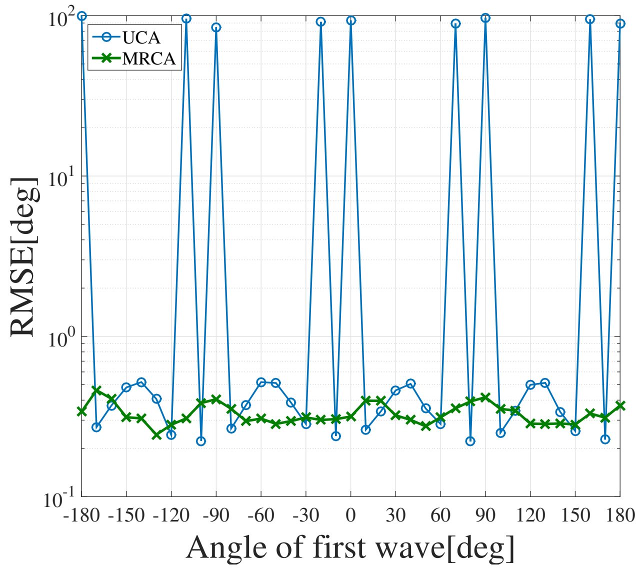

Fig. 4. RMSE of DOA estimates vs. angle of the first wave