DYADIC GREEN'S FUNCTION IN ISOTROPIC STRATIFIED MEDIUM*

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ABSTRACT

Dyadic Green's functions in multi-layered isotropic medium has been analyzed in this paper. Three kinds of different methods for obtaining the coefficients of dyadic Green's function in multi-layered medium are given. By means of the methods, several examples are considered, the results are same as those obtained proviously.

1. INTRODUCTION

The analysis of the electromagnetic wave propagation in stratified medium is of great interest and importance in radio propagation in forests, underwater communications, space communications, geophysical explorations, agricultural applications, etc.. A compact and general analysis may be obtained by using dyadic Green's functions. In this methods, the eigenfunction expansion of the dyadic Green's functions is used (see references [1-3]).

To the authors' knowledge, there has been no general formulae for solving the coefficients of dyadic Green's functions in multi-layered medium. In this paper, for the coefficients of scattered dyadic Green's function, three methods are given here: 1. BOUNDARY CONDITION METHOD; 2. RECURRENCE MATRIX EQUATION METHOD; 3. RAY TRAIL METHOD.

2. DGF IN MULTI-LAYERED MEDIUM

Assumming i, j refer to the layer where the observation point and the source position are located, respectively. The geometry that will be considered here is shown in Fig.1.

The method of scattering superposition is used to obtain dyadic Green's function

$$\bar{\bar{G}}_e^{(ji)}(\bar{R},\bar{R}') = \bar{\bar{G}}_{es}^{(ji)}(\bar{R},\bar{R}') + \bar{\bar{G}}_o(\bar{R},\bar{R}')\,\delta_j^i \tag{1}$$

where δ is the Kronecker delta (equal to one for $i = j$ and equal to zero for $i \neq j$).

The dyadic Green's function in free space is given by

$$\bar{\bar{G}}_o(\bar{R},\bar{R}') = -(1/k_o^2)\hat{Z}\hat{Z}(\bar{R}-\bar{R}') + [j/(4\pi)]\int_0^\infty [1/(h_i\lambda)]\,d\lambda\sum_{n=o}^{\infty}(2-\delta_o)\begin{cases}\bar{M}(h_i)\bar{M}'(-h_i)+\bar{N}(h_i)\bar{N}'(-h_i) & Z >= Z' \\ \bar{M}(-h_i)\bar{M}'(h_i)+\bar{N}(-h_i)\bar{N}'(h_i) & Z =< Z'\end{cases} \tag{2}$$

where δ is the Kronecker symbol. $\bar{M}(h_i)$ and $\bar{N}(h_i)$ are the cylindrical vector wave functions (see refernce [1]). the subscripts e and o indicate even and odd functions, respectively, n, λ, and h are the eigenvalues associated with the ϕ, r, and z coordinates, respectively.

For simplicity, the subscripts of the vector wave functions will be omitted. The prime on these vector wave functions is used to indicate that they are expressed in terms of the coordinates (r', ϕ', z').

The scattering dyadic Green's function is given by

$$\bar{\bar{G}}_{es}^{(ji)}(\bar{R},\bar{R}') = [j/(4\pi)]\int_0^\infty [1/(\lambda h_i)]\,d\lambda\sum_{n=o}(2-\delta_o)\{(1-\delta_j^N)\bar{M}(h_j)[(1-\delta_i^1)a_{ij}\bar{M}'(-h_i)+(1-\delta_i^N)b_{ij}\bar{M}'(h_i)]$$

$$+(1-\delta_j^N)\bar{N}(h_j)[(1-\delta_i^1)c_{ij}\bar{N}'(-h_i)+(1-\delta_i^N)d_{ij}\bar{N}'(h_i)] + (1-\delta_j^1)\bar{M}(-h_j)[(1-\delta_i^1)e_{ij}\bar{M}'(-h_i)$$

$$+(1-\delta_{iN})f_j \bar{M}'(h_i)] + (1-\delta_{j1})\bar{N}(-h_j)[(1-\delta_{i1})g_j \bar{N}'(-h_i)+(1-\delta_{iN})i_j \bar{N}'(h_i)]\} \tag{3}$$

where, a_j, b_j, g_j, ..., i_j are the coefficients of scattered DGF to be solved.

3. BOUNDARY CONDITION METHOD

The boundary conditions of dyadic Green's function are

$$\bar{n} \times \bar{\bar{G}}_e^{(ji)}(\bar{R},\bar{R}') = \bar{n} \times \bar{\bar{G}}_e^{(j+1\ i)}(\bar{R},\bar{R}') \tag{4}$$

$$(1/u_j) * \bar{n} \times \nabla \times \bar{\bar{G}}_e^{(ji)}(\bar{R},\bar{R}') = (1/u_{j+1}) * \bar{n} \times \nabla \times \bar{\bar{G}}_e^{(j+1\ i)}(\bar{R},\bar{R}') \tag{5}$$

and in the mean while, above DGF satisfies wave equation and Sommerfeld radiative condition. Thus, by calculation, the equation groups satisfied by coefficients of the scattered dyadic Green's function in stratified medium are given (see reference [4]) by

$$\begin{vmatrix} a'_i \\ b_i \\ (h_i/K_i)c'_i \\ (h_i/K_i)d_i \end{vmatrix} e^{F(i)} - \begin{vmatrix} -e_i \\ -f'_i \\ (h_i/K_i)g_i \\ (h_i/K_i)i'_i \end{vmatrix} e^{-F(i)} = \begin{vmatrix} a'_{i+1} \\ b_{i+1} \\ (h_{i+1}/K_{i+1})c'_{i+1} \\ (h_{i+1}/K_{i+1})d_{i+1} \end{vmatrix} e^{F(i+1)} - \begin{vmatrix} -e_{i+1} \\ -f'_{i+1} \\ (h_{i+1}/K_{i+1})g_{i+1} \\ (h_{i+1}/K_{i+1})i'_{i+1} \end{vmatrix} e^{-F(i+1)} \tag{6}$$

$$\begin{vmatrix} h_i a'_i \\ h_i b_i \\ K_i c'_i \\ K_i d_i \end{vmatrix} \exp[F(i)] + \begin{vmatrix} -h_i e_i \\ -h_i f'_i \\ K_i g_i \\ K_i i'_i \end{vmatrix} \exp[-F(i)] = \begin{vmatrix} h_{i+1} a'_{i+1} \\ h_{i+1} b_{i+1} \\ K_{i+1} c'_{i+1} \\ K_{i+1} d_{i+1} \end{vmatrix} \exp[F(i+1)] + \begin{vmatrix} -h_{i+1} e_{i+1} \\ -h_{i+1} f'_{i+1} \\ K_{i+1} g_{i+1} \\ K_{i+1} i'_{i+1} \end{vmatrix} \exp[-F(i+1)] \tag{7}$$

$$a_N = b_N = c_N = d_N = e_1 = f_1 = g_1 = i_1 = 0, \quad (i = 1,\ 2,\ ......,\ N)$$

$$F(i) = jh_i(H_i + H_{i+1} + ... + H_{N-2}), \quad F(i+1) = jh_{i+1}(H_i + H_{i+1} + ... + H_{N-2})$$

$$[a', c', f', i'] = [a+1, c+1, f+1, i+1] \tag{8}$$

From these groups of equation, one can obtain the coefficients of scattered dyadic Green's function.

4. RECURRENCE MATRIX EQUATION METHOD

By using complex boundary condition, the recurrence matrix equation groups for the coefficients of scattered DGF may be obtained and given (see reference [5]) by

$$\bar{A}_{i+1} = (1/q_i)[\bar{\bar{C}}_i^T]^{-1} * [p_i q_i \bar{A}_i + \bar{\bar{C}}_i^R * \bar{B}_i]$$

$$\bar{B}_{i+1} = (1/p_i)[\bar{\bar{C}}_i^T]^{-1} * [p_i q_i \bar{\bar{C}}_i^R * \bar{A}_i + \bar{B}_i]$$

$$\bar{A}_N = \bar{B}_1 = 0 \tag{9}$$

where

$$p_i = \exp[j(h_i - h_{i+1})H(i,N)] \; ; \qquad q_i = \exp[j(h_i + h_{i+1})H(i,N)] \; ; \qquad H(i,N) = \sum_{j=1}^{N-i-1} H_j \tag{10}$$

where, H_j represents the depth of j-layer media.

$$\bar{A}_i = \begin{bmatrix} a'_i \\ b_i \\ c'_i \\ d_i \end{bmatrix} ; \quad \bar{B}_i = \begin{bmatrix} e_i \\ f'_i \\ g_i \\ i'_i \end{bmatrix} ; \quad \overset{=T}{C}_i = \begin{bmatrix} T_i^H & & & 0 \\ & T_i^H & & \\ & & T_i^V & \\ 0 & & & T_i^V \end{bmatrix} ; \quad \overset{=R}{C}_i = \begin{bmatrix} R_i^H & & & 0 \\ & R_i^H & & \\ & & R_i^V & \\ 0 & & & R_i^V \end{bmatrix} \tag{11}$$

and

$$\begin{bmatrix} a'_i \\ c'_i \end{bmatrix} = \begin{bmatrix} a_i \\ c_i \end{bmatrix} + L \begin{bmatrix} 1 \\ 1 \end{bmatrix} ; \qquad \begin{bmatrix} f'_i \\ i'_i \end{bmatrix} = \begin{bmatrix} f_i \\ i_i \end{bmatrix} + L' \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$L = \begin{cases} \delta_j^i & Z >= Z' \\ 0 & Z =< Z' \end{cases} ; \qquad L' = \begin{cases} 0 & Z >= Z' \\ \delta_j^i & Z =< Z' \end{cases}$$

$$T_i^H = 2h_{i+1} / (h_i + h_{i+1}) \; ; \qquad T_i^V = 2K_i K_{i+1} h_{i+1} / (K_{i+1}^2 h_i + K_i^2 h_{i+1})$$

$$R_i^H = (h_{i+1} - h_i)/(h_{i+1} + h_i) \; ; \qquad R_i^V = (K_{i+1}^2 h_i - K_i^2 h_{i+1})/(K_{i+1}^2 h_i + K_i^2 h_{i+1}) \tag{12}$$

It can be seen from above equation groups that there are 2N coefficient matrix to be solved, there are 2(N-1) equations in recurrence formulae, there are two equations about boundary condition. Therefore, 2N matrixes of coefficient to be solved corresponds to 2N coefficient matrix equations, one can obtain only one solution of coefficients to be solved.

5. RAY TRAIL METHOD

In this section, the rule for solving the coefficients of scattered DGF is presented. because scattered DGF represents the superposition of multi-reflective and transmissive waves, the coefficients of scattered DGF only depend on reflective and transmissive waves and vertical propagation factor hZ. The direct wave depends on the solution from free space DGF. The rule for solving the coefficients of scattered DGF (see reference [6]) is

(1). The reflective coefficients on the interface between layer j and layer j+1 is

The reflective coefficients:

$$R_j^H = u(i,j)(h_{j+1} - h_j)/(h_{j+1} + h_j) \quad \text{for horizontal polarization}$$

$$R_j^V = u(i,j)(K_{j+1}^2 h_j - K_j^2 h_{j+1})/(K_{j+1}^2 h_j + K_j^2 h_{j+1}) \quad \text{for vertical polarization}$$

where, $u(i,j) = 1$ for $j < i$, $u(i,j) = -1$ for $j > i$.

The transimissive coefficients:

$$T^{V,H} = 1 + R^{V,H}$$

(2). Division factor D is the divisor in the coefficients of scattered DGF. It consists of unit factor 1, R_1, R_2, ... , R_N double combination and propagation factor $h_i Z$. If the ray trail can be found in the geometrical method after combination, the combinations are made negative or else the positive will be made.

(3). The coefficients of scattered DGF consist of different combinations of reflective and transmissive coefficients (contain propagation factor $h_i Z$), then are divided by division factor. The number of combinative reflective and transmissive coefficients can't be more than the number of interfaces.

(4). In rule 3, the number and kind of transmissive coefficients are determined by the interfaces between excited source and received point.

(5). For some interfaces at which the ray can't arrive in the geometricl method, their effects must be considered when one calculates the coefficients. In the case, the effects of the interfaces which keep off the ray can be neglected, but the combined effect of the interfaces at which the ray can't arrive and their neighbouring interfaces should be considered.

(6). Considering symmetry, the coefficients of wave vectors P(h)P'(-h) and P(-h)P'(h) are same when receiving and transmitting antennas are located in same layer. Here P represents M and N, and the combination of P(h), P(-h), P'(h) and P'(-h) is determined by the combination of propagation factors hZ, -hZ, hZ', and -hZ'.

6. CONCLUSIONS

By using the method of scattering superposition of dyadic Green'n function, DGF was divided into free space part and scattered part which corresponds to direct wave, transmissive and reflective waves, respectively. By using boundary conditions and ray trail method, etc., three method for solving the coefficients of scattered dyadic Green's functions in multi-layered medium, are presented in this paper. The methods and results can be used to underwater communications, radio wave propagation in forests, space communications, etc..

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REFERENCES

[1]. Tai, C.T., Dyadic Green's Functions in Electromagnetic Theory, Intext Educational Publishers, Scranton Pa., 1971

[2]. ————, Proc. IEEE, Vol.61, PP.480-481, 1973

[3]. Cavalcante, G.P.S., D.A. Rogers, and A.J. Giardola, Radio Sci., Vol.17, PP.503-508, 1982

[4]. Li, L.W., and P.N. Jiao, Chinese Journal of Radio Sci., Vol.1, No.2, PP.10-25, 1986

[5]. ————, Chinese Journal of Radio Sci., Vol.2, No.1, PP.9-16, 1987

[6]. ————, Chinese Journal of radio Sci., to be published