A SIMPLE FORM FOR A POTENTIAL INTEGRAL

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1. Introduction

A lot of methods have been developed and used to evaluate potential integrals which are met frequently in the solution of electromagnetic scattering or radiation from dipoles. Numerical and approximate ways of calculating these integrals were presented by Preis , Harrington [3]. Others, like Overfelt [4] introduced exact closed-form expressions. However, these turn out to be rather lengthy ones. In the following, the potential integral for a sinusoidal distribution is represented by a very simple and quite accurate approximation.

2. The Potential Integral

The following potential integral will be considered :

$$I = \int_{-\pi}^{\pi} |\sin(x)| \cdot G(x; D_x, D_y)\, dx \qquad (1)$$

where

$$G(x; D_x, D_y) = \exp\{-j\,[(D_x - x)^2 + D_y^2]^{0.5}\} \,/\, [(D_x - x)^2 + D_y^2]^{0.5}$$

It may be shown that the mutual impedance between two dipoles in an echelon configuration is given by

$$Z_{12} = 30 j \cdot I \qquad (2)$$

where $D_x = k_o d_x$, $D_y = k_o d_y$, $\theta = \operatorname{atan}(D_y / D_x)$.

It has been previously shown by the author [1] that there exist very simple approximations for evaluating the mutual impedance (2) for the special cases of parallel and collinear dipoles ($d_x=0$ or $d_y=0$). The question now remains of presenting some generalization of these expressions for the general case ($d_x, d_y \neq 0$). A few trial and error experiments were needed for this purpose. Finally, the following terms were found adequate for calculating the mutual impedance between echelon dipoles :

$$R_{12} \sim 30\,(R_1 + R_2)$$
$$X_{12} \sim X_1 + X_2 \qquad (3)$$

where

$$R_1 = 4 \sin(c\, s_1) / (\sin(c)\, a\, s_1)$$

$$R_2 = \sin(\theta)\,(\pi^2 - 6)\,(\cos(s_1)/s_1^2 - \sin(s_1)/s_1^3)$$

$$s = (1 + D_x^2 + D_y^2 - 3 \cdot D_x)^{0.5} , \quad \theta = \operatorname{atan}(D_y / D_x)$$
$$c = \sin^{1.8}(\theta) + \cos^{1.8}(\theta) , \quad a = \sin(\theta) + 6 \cos^2(\theta)$$

and

$$X_1 = 120 \cos(s_2) / s_2 \cdot \sin(\theta)$$
$$X_2 = 20 \cos(s_3) / s_3 \cdot \cos^2(\theta)$$

$$s_2 = (4 + D_x^2 + D_y^2)^{0.5} , \quad s_3 = (1 + D_x^2 + D_y^2 - 3.5\, D_x)^{0.5}$$

It is now evident from (2) that the real and imaginary parts of the potential integral is given approximately by :

$$\int_{-\pi}^{\pi} |\sin(x)| \cdot G(x;\, D_x, D_y)\, dx \sim -1/30 \cdot j \cdot (R_{12} + j \cdot X_{12}) \qquad (4)$$

3. Numerical Results

The values of the mutual impedances for different direction angles 'θ' are shown in figures 2,3. These should be compared to the exact values in figures 4,5 which were derived by running a program written by Hansen et al.[2]. Results are seen to be satisfactory for distances larger than about 0.7λ. A maximal error of 3 ohms is observed. Additional 'cut and try' effort or a systematic estimation of parameters may further improve the accuracy.

4. Conclusions

The potential integral plays an important role in the solution of many problems in Electromagnetics. Often, the designer would prefer to use explicit, simple and quite accurate formulas instead of performing numerical integration or using lengthy expressions. This lends insight into the nature of dependence of the integral upon the parameters that are involved.

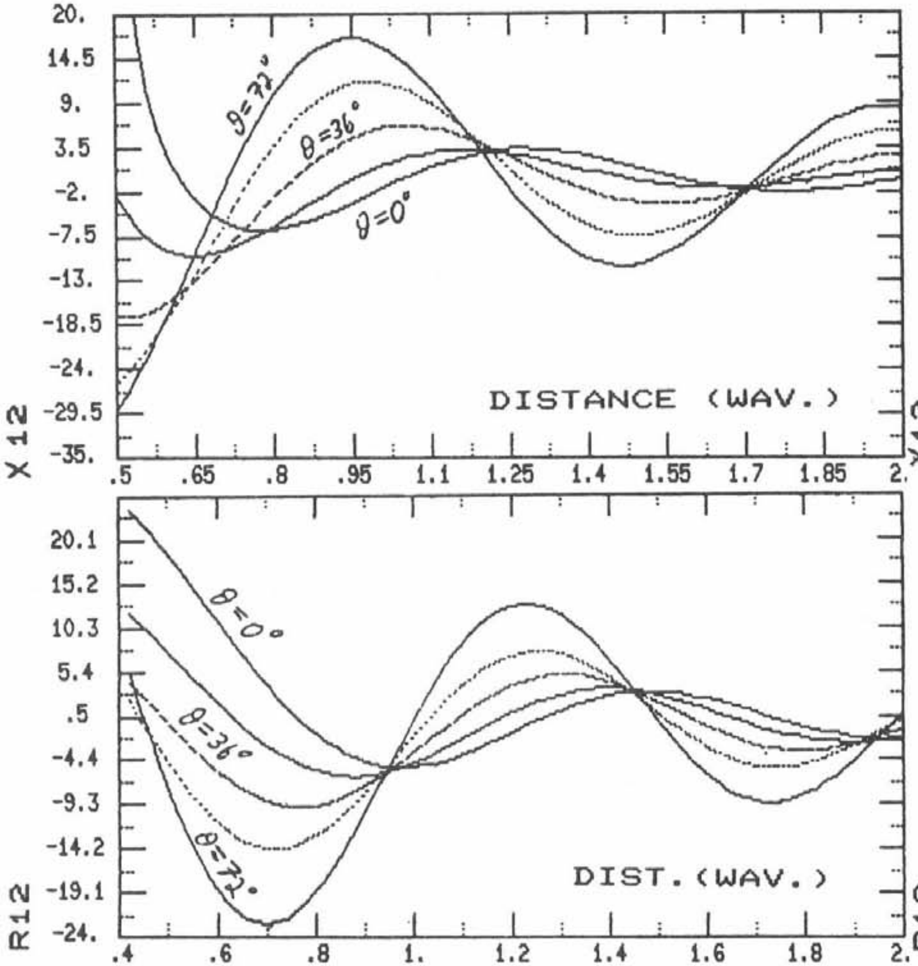
5. References

(1) A.E. Gera : "Simple Expressions for Mutual Impedances"' Proc. IEE, Vol. 135, Pt. H, No. 6, Dec. 1988, pp. 395-399.

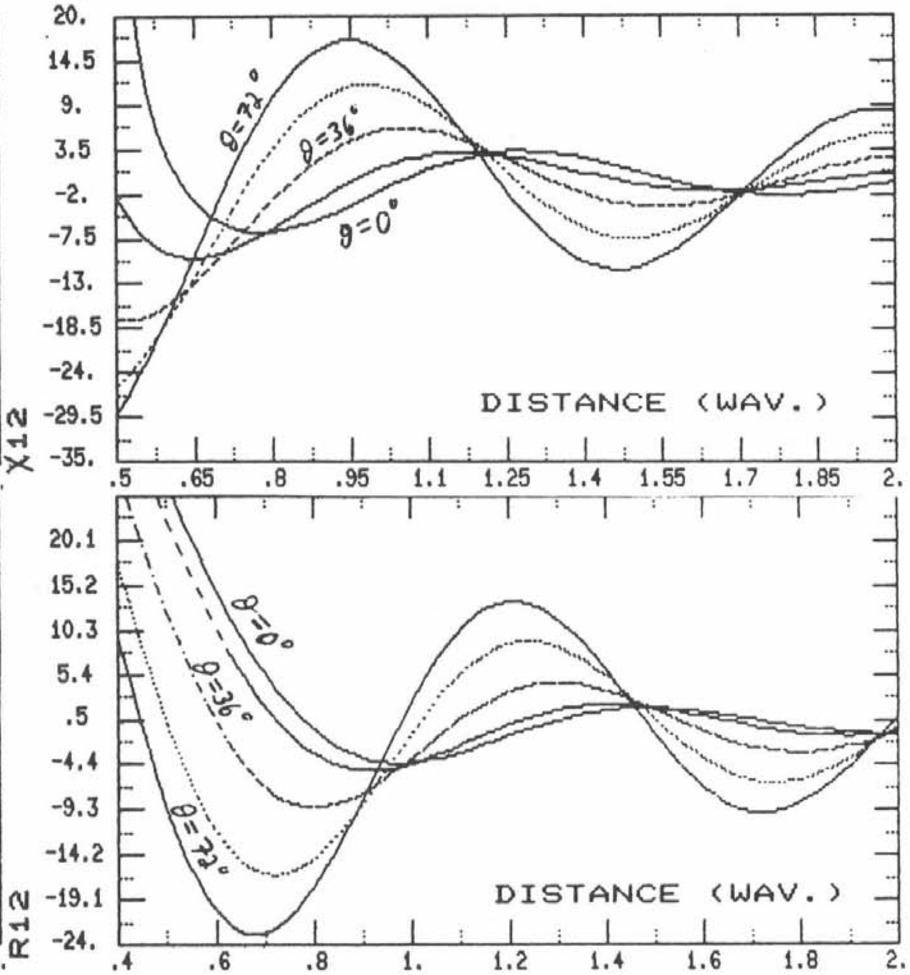
(2) P.C. Hansen, G. Brunner : "Dipole Mutual Impedance for Design of Slot Array", Mic. Jour., Vol. 22, No. 12, Dec. 1979, pp. 54-56.

(3) R.F. Harrington : "Lectures on Computational Methods in Electromagnetics", The Scee Press, 1981, pp. 12-14.

(4) P.I. Overfelt : "An Exact Method of Integration for Vector Potentials of Thin Dipole Antennas", IEEE Trans. Ant. Prop., Vol. AP-35. No. 4, April 1987, pp. 442-444.



Figures 2,3 : Using (3) for calculating the mutual impedance versus distance for different direction angles.



Figures 4,5 : Exact calculation of the mutual impedance versus distance for different direction angles.