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ON THE FORWARD SCATTER ALIGNMENT AND THE BACK SCATTER ALIGNMENT CONVENTIONS OF BI-STATIC RADAR POLARIMETRY

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Abstract: The proper formulation of the electromagnetic wave scattering problem from coherent point targets in the general bi-static case depends on the introduction of appropriate polarimetric coordinate systems. This contribution points out deficiencies of often used 'Forward Scattering Alignment' (FSA) and the 'Back Scatter Alignment' (FSA) conventions and attempts to modify these concepts by explicitly introducing the time reversal operation.

Coordinate systems are used to realize and concretize operator-valued field relations formulated in abstract vector spaces in terms of vectors and matrices in finite-dimensional complex linear vector spaces. Great care must be exercised in choosing the correct coordinate systems and conventions in such a way that the formulations allow a direct application of well-established mathematical theorems to the physical world of the various scattering scenarios. This applies in particular to the formulation of scatter problems using the conventional FSA and the BSA conventions in general bi-static radar polarimetry. This suggestion is also connected with the attempt to recombine the traditional formulations via the radar equation and via the voltage equation, considered to be the two independent cornerstones of radar polarimetry, on an equal cohesive footing.

Introduction. Let us consider an antenna with antenna vectors $\vec{h}_1$ located at position 1 and transmitting an electric field that in the far field at position 2 is given by $\vec{E}_1$ and is received there by an antenna with antenna vector $\vec{h}_2$. This antenna on the other hand emits an electric field that at position 1 is given by $\vec{E}_2$ and is received there by the antenna with antenna vector $\vec{h}_1$. Lorentz' reciprocity theorem implies the basic relationship, see Mott [1]

$$(\vec{h}_1,\vec{E}_2) \equiv \vec{h}_1 \cdot \vec{E}_2 \equiv \vec{h}_1^T \vec{E}_2 = \vec{h}_2^T \vec{E}_1 \equiv \vec{h}_2 \cdot \vec{E}_1 \equiv (\vec{h}_2,\vec{E}_1)$$

where each one of the pairs $\vec{h}_1$, $\vec{E}_2$ and $\vec{h}_2$, $\vec{E}_1$ are given in its own linear coordinate system. The expressions are bilinear forms (not scalar products) and describe induced voltages. These forms are therefore often called voltage equations. They are quite different from the standard unitary scalar products $<\vec{p},\vec{q}> \equiv \vec{p}^{T*}\vec{q}$ encountered in polarimetry and used for instance to define orthonormality and orthogonality.

Time reversal. The connection between the bilinear forms and the standard scalar product is based on the realization that the bilinear forms involve two vectors defined for wave propagation in opposite directions whereas the unitary scalar product involves only polarization vectors that

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correspond to waves propagating in one and the same direction. Applying the concept of time reversal a bilinear form like $(\vec{h}_1, \vec{E}_2)$ should be written more precisely as

$$(\vec{h}_1, \vec{E}_2) \equiv \vec{h}_1^T \vec{E}_2 = \vec{h}_1^{T**} \vec{E}_2 = (\vec{h}_1^*)^{T*} \vec{E}_2 \equiv < \vec{h}_1^*, \vec{E}_2 > .$$

The formal conjugation $\vec{h}_1^*$ is due to the time reversal operation that changes the direction of propagation and (in a linear basis) produces a change of sense of rotation of the polarization ellipse by complex conjugation, see Lüneburg [2]. Suppose that the electric field $\vec{E}_1$ (produced by antenna vector $\vec{h}_1$) at position 2 has been modified by a scatterer S located at position 3 and similarly that the electric field $\vec{E}_2$ (produced by antenna vector $\vec{h}_2$ at position 1) has been modified by the same scatterer at position 3. Then in obvious notation, one obtains

$$\vec{E}_1(\vec{r}_2) = S(\vec{r}_2 \leftarrow \vec{r}_3 \leftarrow \vec{r}_1)\vec{h}_1(\vec{r}_1) \quad \text{and} \quad \vec{E}_2(\vec{r}_1) = S(\vec{r}_1 \leftarrow \vec{r}_3 \leftarrow \vec{r}_2)\vec{h}_2(\vec{r}_2)$$

The transversal coordinate systems at positions 1 and 2 are denoted as $B_1 = \{x_1, y_1\}$ and $B_2 = \{x_2, y_2\}$, respectively, and are often taken to be the same for transmission and reception. These two-dimensional coordinate systems span complexified planes perpendicular to the lines between position vectors $\vec{r}_1$ and $\vec{r}_3$ as well as $\vec{r}_2$ and $\vec{r}_3$ and are used to describe the polarization planes including the direction of rotation. We do not introduce a left- or right-handed three-dimensional coordinate system as is done conventionally where a third z - axis is parallel or anti-parallel to the direction of propagation $\vec{k}_i$ or $\vec{k}_s$. Hence,

$$(\vec{h}_2(\vec{r}_2), \vec{E}_1(\vec{r}_2)) = (\vec{h}_2(\vec{r}_2), S(\vec{r}_2 \leftarrow \vec{r}_3 \leftarrow \vec{r}_1)\vec{h}_1(\vec{r}_1)) = (\vec{h}_1(\vec{r}_1), \vec{E}_2(\vec{r}_1)) = (\vec{h}_1(\vec{r}_1), S(\vec{r}_1 \leftarrow \vec{r}_3 \leftarrow \vec{r}_2)\vec{h}_2(\vec{r}_2))$$

and comparing terms we conclude

$$S(\vec{r}_2 \leftarrow \vec{r}_3 \leftarrow \vec{r}_1) = S^T(\vec{r}_1 \leftarrow \vec{r}_3 \leftarrow \vec{r}_2) \quad \text{or} \quad {}_{B_2}S_{B_1} = {}_{B_2}S_{B_1}^T$$

Here the right index A in ${}_BS_A$ denotes the domain and the right index B the range of the operator S. From the preceding convention it follows directly that the coordinate system for the domain of the scattering operator $S(\vec{r}_2 \leftarrow \vec{r}_3 \leftarrow \vec{r}_1)$ agrees with the antenna coordinate system $\{x_1, y_1\}$ and that the coordinate system of its range agrees with the antenna coordinate system $\{x_2, y_2\}$. For the inverse scattering operator $S(\vec{r}_1 \leftarrow \vec{r}_3 \leftarrow \vec{r}_2)$ the roles of domain and range are interchanged.

In the strict monostatic radar backscattering case the position vectors $\vec{r}_1$ and $\vec{r}_2$ coincide $\vec{r}_1 = \vec{r}_2 \equiv \vec{r}_0$. It is not necessary even in this case to use coinciding coordinate systems at the common location for the domain and the range of the operator S. However, if we choose $B_1 = B_2 = B$, i.e., coinciding coordinate systems, then we obtain the particular appealing situation that

$$_B S_B = {}_B S_B^T \quad \text{or short} \quad S = S^T .$$

In the more general situation when $B_1 \neq B_2$ the general relation $_{B_2} S_{B_1} = {}_{B_2} S_{B_1}^T$ shown above is valid but of limited use since to be able to draw useful conclusions the same coordinate systems on both sides for the domain and the range of the operator must be taken.

FSA and BSA. For power optimization purposes in the general so-called bi-static scattering case we have to consider the transmit antenna with its local coordinate system B_1 and the receive antenna with its local system B_2. The domain of the scattering operator S will use the coordinate system B_1 and its range the coordinate system B_2. The direction of propagation $\vec{k}^i$ of the electromagnetic wave incident upon the target coincides with the direction of propagation of the transmit antenna, i.e., $\vec{k}^i = \vec{k}^t$. On the other hand the direction of propagation $\vec{k}^s$ of the scattered wave is opposite to the direction of propagation of the receive antenna. It should be remembered that the polarization of
an antenna is always defined as the polarization that it transmits, irrespectively if the antenna is actually used for transmission of reception. The receive antenna polarization is defined as the polarization of that electromagnetic wave that is best received by the antenna, see IEEE Standard Definitions [3]. Using the same linear coordinate systems for the scattered wave and the receiving antenna the polarization ellipse that is best received has the same geometric form or locus as the polarization ellipse emitted by the antenna but opposite sense of rotation when both ellipses are looked at from a common point of view. Taking into account the opposite directions of propagation both waves have the same sense of rotation or the same handedness. If the antenna polarization is $\vec{h}$, always defined for transmission in a linear orthonormal polarization basis, the polarization of the receive antenna or the polarization of the incident wave that is best received by the antenna is given by the complex conjugate $\vec{h}^*$.

This situation can be described in quite general terms by the concept of time reversal $\mathcal{T}$. This anti-linear operator is involutory $\mathcal{T}^2 = I$, converts any trajectory into its motion-reversed counterpart and takes the complex conjugate of any field component in a linear basis, see Lüneburg [2]. The reason for the application of the time reversal is the fact that polarization characteristics, in particular optimization problems, for incoming and outgoing (incident and scattered) electromagnetic waves can be compared if only if one and the same polarization space is involved. Time reversal $\mathcal{T}$ transforms the states of polarization for waves propagating in opposite directions into states of polarization corresponding to waves propagating in only one direction. In this sense we distinguish between the Forward Scattering Alignment (FSA) convention as being fixed to the incident or scattered wave (wave oriented coordinate system) and the Backscatter Alignment (BSA) convention as being fixed to the receiving antenna, see Ulaby and Elachi [4]. The notation FSA versus BSA is misleading since both concepts can be applied to the general situation of bi-static scattering including strict forward and back-scattering but it must always be kept in mind that forward scattering and backscattering are distinct physical phenomena. If the range of the scatter operator and the domain of the receiving antenna coincide then for a linear polarization basis we have for the scattered field $\vec{E}^s$ the general time reversal relation between the modified FSA and BSA convention

$$\vec{E}^s |_{BSA} = \vec{E}^{s*} |_{FSA}$$

provided that the same linear coordinate system is used.

It must be pointed out that this new proposed modification of the BSA and FSA conventions is different from the traditional definition as for instance given by Mott [1] that considers the difference between BSA and FSA as being a result of using either the same linear coordinate systems for the range of the scattering operator and the receiving antenna (but no complex conjugation) or using different coordinate systems adapting always a right-handed wave oriented coordinate system. The proposed modifications use only 2-dimensional coordinate systems and avoid the application of 3-dimensional systems with its intimately related question of right- or left-handedness.

Having set the scheme for the coordinate systems used and taking care of waves propagating in opposite directions the actual calculation of optimal power transfer makes use of the concept of equivalence classes. We explain this for backscattering. The Sinclair scatter matrix that is symmetric in the BSA convention is a complex 2×2 matrix of the general form in the linear $\{x, y\}$- basis

$$S = \begin{bmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{bmatrix} = S^T \quad \text{due to} \quad S_{xy} = S_{yx}.$$

Going over to a different polarization basis by unitary consimilarity, see Lüneburg [2], we obtain the equivalence class

$$C(S) = \{U^T SU \mid S = S^T; \quad \text{for all unitary matrices } U^T U = I\}.$$

Every matrix of this equivalence class is a representation of the same scattering operator S, only in different bases. There is one member of this class that has a particularly convenient form: a diagonal form for a special matrix U such that $U^T SU = \text{diag}[\, l_1, l_2\,]$. The diagonal elements are the coneigenvalues and the columns of U the coneigenvactors. This is Takagi's theorem, see Takagi [5]. For the general bi-static scatter case a similar role is played by the general Singular Value Decomposition (SVD) theorem. All these considerations can be extended also to the incoherent scatter case using directed Stokes vectors and to the Mueller and Kennaugh matrices.

This contribution points out existing discrepancies in using the traditional coordinate systems of the BSA and FSA conventions and provides suggestions how to overcome these difficulties.

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