

## **Physical Implications of Backscattering from Non-Gaussian Correlated Randomly Rough Surfaces**

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**Abstract:** In this study we analyze non-Gaussian correlated surface backscattering behavior with the implementation of the “*Advanced IEM model*” in terms of (1) components of roughness scales, (2) frequency, and (3) rms slope. We consider an exponential-like correlation function that contains an adjustable amount of roughness components and, unlike the commonly used exponential function, possesses rms surface slopes. The correlation function under consideration offers a direct view of how backscattering changes from an exponentially correlated surface to a Gaussian correlated surface by varying parameter  $x$  and provides insight into the physical behavior and implications.

### **1. Introduction**

In this paper, we analyze backscattering properties for non-Gaussian correlated surfaces. It corresponds to surfaces with a spectrum containing an excessive amount of high frequency spectral components. Thus, it can only be used reliably in the low to moderate frequency region or equivalently in the small incident angle region in backscattering. This is because in the low frequency region scattering is dependent on the shape of the surface spectrum and not on the *rms* slope of the surface. As is well known, in the high frequency limit, scattering is proportional to the surface slope distribution. Thus, some errors may be included, in case the commonly applied exponential correlation function is used in the high frequency calculations for surfaces that possess an *rms* slope. In practical applications, it is not realistic to expect that a ground surface can always be represented as a continuous surface at all frequencies. In many cases, there are isolated pebbles, rocks, vegetation, etc. residing above a ground surface. They may be negligible at low frequencies but not at higher frequencies when their physical size is comparable to the incident wavelength. This is one of the reasons why the theoretical high frequency limit of a surface scattering model does not find many applications in a natural environment, which will be analyzed and demonstrated.

### **2. The AIEM model**

Much effort has been devoted to improving the accuracy of the IEM originally

reported by Fung et al. [1, 2]. This is mostly done by re-deriving the expression without or reducing the assumptions in the original development. One significant step made forward was the introduction of a transition function in the calculation of Fresnel reflection coefficients to take spatial dependences into account and thus remove the restrictions on the limits of surface roughness permittivity. Although the approach is kind of heuristic, it proves to perfectly work for a broad range of surface conditions. A heuristic approach is necessary since there are no analytic forms existing for an IEM version, called Advanced IEM (AIEM) [3-5], which contains many more terms compared to the original version, but remains in algebraic form for the ease of numerical implementation.

### 3. The non-Gaussian correlation function

In order to have an *rms* slope leading to some uncertainty about its validity in the high frequency region as well as the reliability of the value it converges to in frequency. A correlation function (1), that has an *rms* slope and hence possesses a second derivative at the origin. The algebraic representation of resulting backscattering model is possible. Its functional form is

$$r(r) = \exp\left[-r/L\left(1 - e^{-r/x}\right)\right] \quad (1)$$

where  $L$  is correlation length and  $L \geq x$ . Its  $n^{\text{th}}$  spectra can be analytically expressed as

$$W^{(n)}(K) = \sum_{m=0}^{\infty} \left(\frac{nL_e}{L}\right)^m \frac{L_e^2}{m!} \Gamma(m+2) {}_2F_1\left(\frac{2+m}{2}, \frac{3+m}{2}, 1, -KL_e\right) \quad (2)$$

where  $L_e = (xL)/(nx + mL)$ , and  ${}_2F_1(a, b, c; z)$  is a hypergeometric function.

To achieve a clear separation between a Gaussian shape around the origin and an exponential elsewhere, it is necessary to choose  $L \gg x$ . When  $x$  is small, the correlation length will be equal approximately to  $L$ . The transition region from exponential to Gaussian is directly proportional to the size of  $x$  which controls the actual shape of the correlation function. To illustrate these points we plot Fig.1.

## 4. Backscattering Behavior from Non-Gaussian Correlated Surfaces

### 4.1 Effects of changing $x$

We show the backscattering angular trends based on equation (1) for vv and hh polarizations as showing in Fig.2. For large  $x$  there is a significant drop over large angles of incidence for both polarizations. This is because the addition of Gaussian effects into the correlation function around the origin reduces the amount of small scale roughness, thus reducing backscattering, while the small-angle region remains relatively unaffected. As  $x$  decreases, the large-scale region of the backscattering curve begins to increase and shows a flatter response. This is because an exponential correlation represents the presence of more small scale roughness which gives rise to

backscattering in the large angular region.

#### 4.2 Effects of frequency

As frequency increases, the curves change in Fig.3 from a more isotropic scattering into one that shows a peak near vertical incidence. A given incidence frequency can only sense a range of roughness. Roughness scales too large or too small are not sensitive to the incident wave thus generating a saturation effect first in large-angle region dominated by scattering from small to medium roughness. This saturation will spread towards the small incident angle region as frequency increases further.

#### 4.3 Effects of rms slope

In 4.2, we noted that by increasing the frequency but keeping the small roughness scale, scattering cannot approach the geometric optics at large angles of incidence where small roughness scales are dominating scattering. However, for a given surface with a roughness spectrum, backscattering does trend to saturate as shown in Fig.3. There is another condition where vv tends to hh without approaching geometric optics. This is where we increase surface rms slope. An illustration of this point is shown in Fig.4, where the size of rms height has been increased from 0.15 to 0.75 cm, while the correlation length remains constant.

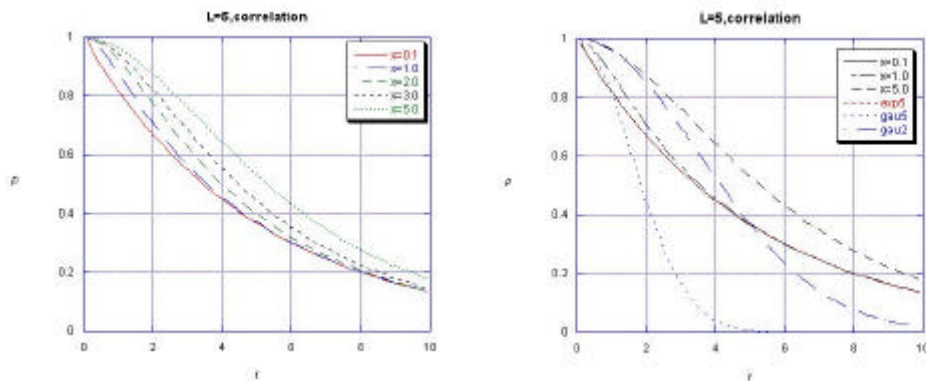


Fig. 1 (a) Changes in the shapes of the correlation due to different choices of x. (b) comparisons with relevant exponential and Gaussian functions.

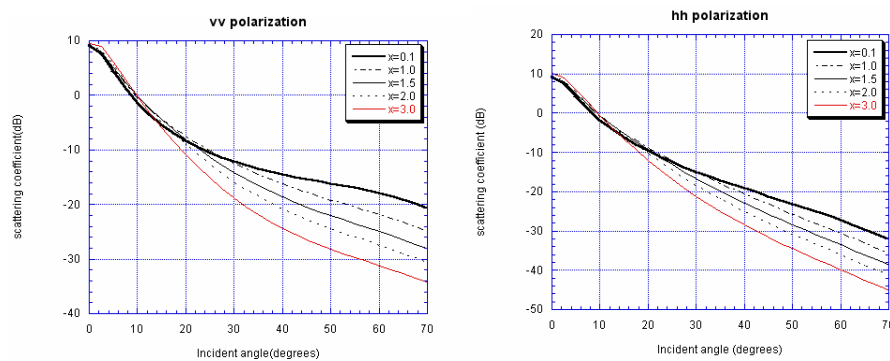


Fig.2 Polarized backscattering with rms height  $s = 0.15$  cm and  $L = 3$  cm.

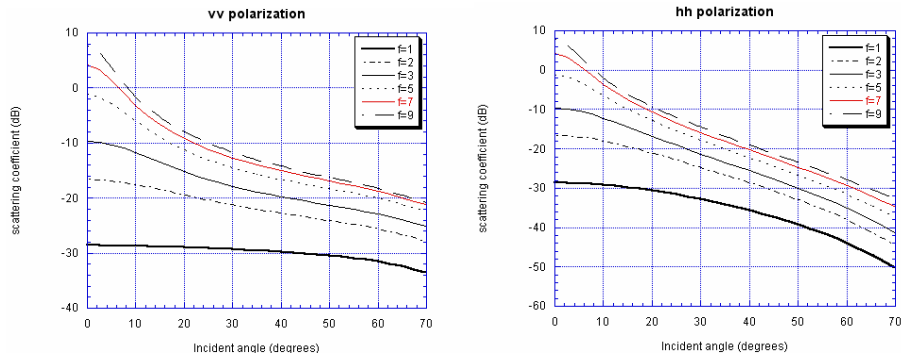


Fig.3 Polarized backscattering with  $s = 0.15$  cm and  $L = 3$  cm.

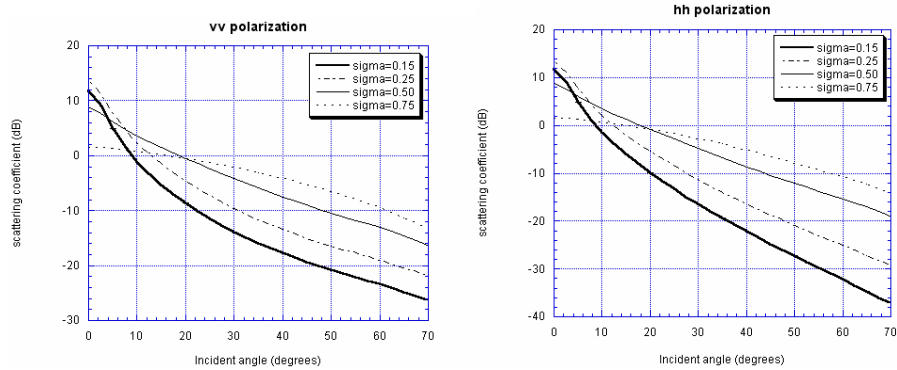


Fig.4 Polarized backscattering coefficients for  $x=1$  at  $f=10$ GHz with  $L=4$

## 5. Conclusions

In this paper, we focus on backscattering behavior from non Gaussian correlated surface with small to moderate slope. For more easy computation and without loss of our objective, we only consider like-polarized waves. We illustrated how the backscattering coefficients change with  $x$ ,  $rms$  height; correlation length and frequency that can offer direct views.

## References

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