Lagrange multiplier setting for l_p-CS based DOA estimation

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Abstract - **This paper considers the problem on setting an appropriate value of Lagrange multiplier for lp Compressive Sensing(CS) based DOA estimation. The proposed method utilizes the receiver noise energy information, and updates the Lagrange multiplier at each iteration step. The proposed method successfully estimates DOA peaks without losing desired peaks and suppress fictitious peaks.**

Index Terms **—Compressive Sensing, Sparse, DOA.**

1. Introduction

Compressive Sensing(CS) based DOA has advantages of applicability to highly correlated signals, and not requiring the estimation of the number of imcoming signals. However, it is known that the peformance of the l_p -CS is highly sensitive to the value of the Lagrange multipler. In [1], L-curve[2] based method is proposed. However, this approach suffers from the fact that it requires heavy λ search, and the critical problem is that the l_p function is not convex and therefore [1] concluded that L-curve method is not applicable. [3] has proposed SNR based method. This method provides the range of λ search, based upon the *a priori* measured SNR. However this approach still requires λ search in the given range. Furthermore, it is not suitable for practical use because SNR changes under the real time monitoring situation where an unknown number of signals with different power are received anytime.

This paper proposes a new adaptive method based on receiver noise energy information. The proposed method does not require λ search, and only require the receiver noise information, which can be measured *a priori*.

2. l_p-CS DOA[1]

Denote the number of incoming signals by J (j =1,…, J), the number of antenna array elements by I (i =1,..., I), and the number of snapshots by K (k =1,…, K). The measurement model is given by the following equation.

$$\mathbf{Y} = \mathbf{AS} + \mathbf{N} \tag{1}$$

$\mathbf{Y} \in C^{I\times K}$ is the measurement matrix, $\mathbf{A} \in C^{I\times J}$ contains steering vector of j th incoming signals in its j th column, $\mathbf{S} \in C^{J\times K}$ is the incoming signal matrix, and $\mathbf{N} \in C^{I\times K}$ is the receiver noise matrix. The l_p -CS solves the following optimization problem.

$$J_o(\mathbf{X}) = \|\mathbf{Y} - \mathbf{A}_s\mathbf{X}\|_f^2 + \lambda\|\mathbf{x}\|_p^p \tag{2}$$

$$\min_{\mathbf{X}} J_o(\mathbf{X}) \tag{3}$$

The first term in (2) is called DOA term and the second term is sparse constraint term. f is the Frobenius norm, $\mathbf{A}_s \in C^{J\times N_\phi}$ is the dictionary matrix whose l th column contains the l th steering vector dictionary corresponding to the l th DOA ϕ_l (l =1,..., N_ϕ), and N_ϕ is the total number of the dictionary. $\mathbf{X} \in C^{N_\phi\times K}$ is the sparse matrix. $\mathbf{X}$ has nonzero row components where there are incoming signals and zero row components elsewhere. λ is the Lagrange multiplier and p indicate the p norm. $\mathbf{x}$ is computed by the following equations.

$$\mathbf{x} = \left[x_1, \cdots, x_{N_\phi}\right]^T \tag{4}$$

$$x_l = \left\|\left[\mathbf{X}_l(t_1), \cdots, \mathbf{X}_l(t_K)\right]\right\|_2, \quad l = 1, ..., N_\phi \tag{5}$$

The iterative method is proposed to solve the above optimization problem[1]. We omitt the detail here and show only the flow in Figure 1.

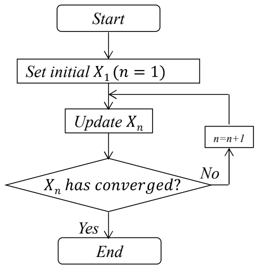

Figure 1 Flow of the ordinary l_p -CS method.

3. The proposed λ setting method for l_p-CS DOA

The proposed method is based upon the following important property on the DOA term of (2). The DOA term satisfies the following if the DOA is estimated properly.

$$\|\mathbf{Y} - \mathbf{A_s X}\|_f^2 = \|\mathbf{N}\|_f^2 \tag{6}$$

We utilize this property to adjust λ. Given the average receiver noise energy, we propose an adaptive method to adjust λ so that the DOA term converges to the receiver noise energy. Figure 2 shows the schematic picture of λ and DOA term. Figure 3 shows the flow of the propose method. In order for the DOA term to converge to the receiver noise energy, we propose the following α filter.

$$\lambda_{n+1} = \lambda_n - \alpha(\|\mathbf{Y} - \mathbf{A_s X}_n\|_f^2 - E) \tag{7}$$

where, α is a filter gain, and E is the average receiver noise energy. E can be measured *a priori*.

Another important aspect of the proposed method is that we start with relatively small λ. This is because for small λ, there are many peaks including both fictitious and desired peaks. We start with many peaks and then, gradually suppress the fictitious peaks and eventually only desired peaks remains. For this purpose, we start with small λ, and then, start using (7) so that the DOA term converges to E.

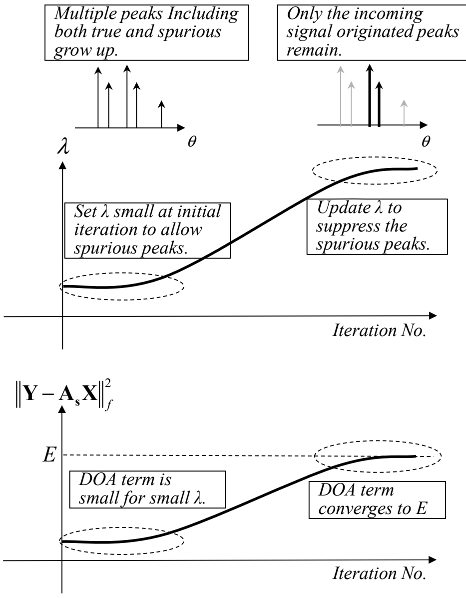

Figure 2 Schematic picture of λ and DOA term.

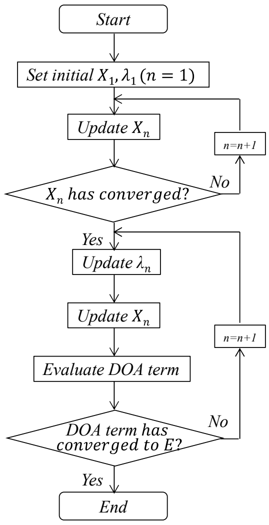

Figure 3 Flow of the proposed l_p -CS method.

4. Simulation

Figure 4 shows the array geometry and DOA of incoming signals. Assume five linearly spaced array antennas with half of wavelength apart. In this case, the beam width is about 30deg. DOAs of two incoming signals are 0deg. and 10deg. respectively. They are both CW whose normalized frequencies are both 0.2, namely completely correlated. SNRs are both 20dB. The number of snapshots is 10.

Figure 5 shows the DOA term and the received energy and the λ_n versus iteration No.. Figure 6 shows the spectrum versus iteration No.. Figure 6 shows the spectrum in dB in gray scale. From Figure 5, DOA term converges to the noise energy at the 64th iteration. Furthermore, from Figure 5, the value of λ_n has converged to about 1.5. Finally, from Figure 6, it can be seen that the before the 30th iteration, there are spurious peaks as well as desired peaks. After the 30th, spurious peaks are suppressed and only the desired peaks remain. From these result, the proposed method has performed as expected.

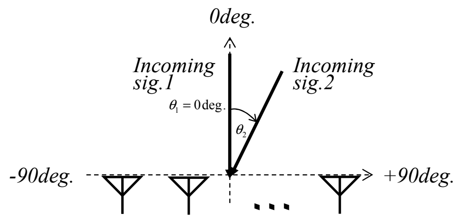

Figure 4 Array geometry and DOA of incoming signals.

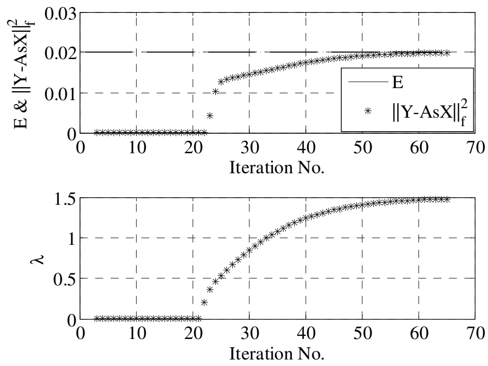

Figure 5 Upper: DOA term λ_n vs. iteration No., Lower: λ_n vs. iteration No.

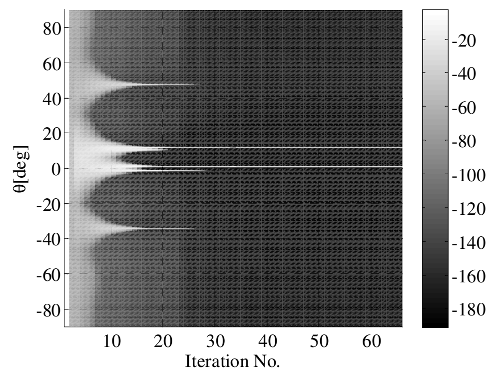

Figure 6 Spectrum vs. iteration No.

5. Conclusion

This paper proposed an adaptive method to adjust the value of the Lagrange multiplier. The simulation result shows that the proposed method adequately remains only the desired peaks and suppressed the spurious peaks. Further research direction includes the selection of filter gain for faster convergence.

References

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