Influence of Mutual Coupling between Array Elements in Location Estimation of Radio Sources Using Near-Field DOA-Matrix Method

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Abstract – **This paper deals with the near-field DOA-Matrix method for source localization using a uniform linear array antenna. In general, the mutual coupling between array elements degrades the DOA estimation accuracy. In this paper, we investigate the influence of mutual coupling in source localization using the near-field DOA-Matrix method and the improved method.**

Index Terms — **Source localization, Near-field source, DOA-Matrix method, Mutual coupling, Dummy elements.**

1. Introduction

The near-field DOA-Matrix method has been used for source localization with a uniform linear array antenna [1]. Combined use of the spatial smoothing preprocessing (ssp) in this method was proposed to improve still further the estimation performance [2]. The improved method is called the near-field DOA-Matrix-ssp method. However, the estimation accuracy is generally deteriorated by the mutual coupling between the array elements in the actual environments. Therefore, we investigate the influence of the mutual coupling in the source localization using the near-field DOA-Matrix-ssp method.

2. Signal Model

Consider a ULA (Uniform Linear Array) having $K = 2p+1$ (p: positive integer) elements with element spacing d, which is depicted in Fig. 1. As shown in Fig. 1, the array center is at the origin and it is designated as the phase reference point. We assume that there are L signal sources, the locations of which are given by both the distances from the origin $r_{0,l}$ and DOAs θ_l ($l = 1, 2, \cdots, L$). Then, the array response vector (mode vector) of the lth signal, $\boldsymbol{a}(\theta_l, r_{0,l})$, is defined as

$$\boldsymbol{a}(\theta_l, r_{0,l}) = \left[\frac{r_{0,l}}{r_{-p,l}}\exp\left(-j\tau_{-p,l}\right), \cdots, \frac{r_{0,l}}{r_{p,l}}\exp\left(-j\tau_{p,l}\right)\right]^T \quad (1)$$

$$r_{k,l} = r_{0,l}\sqrt{1 + \left(\frac{kd}{r_{0,l}}\right)^2 - \frac{2kd\sin\theta_l}{r_{0,l}}} \quad (2)$$

$$\tau_{k,l} = \frac{2\pi}{\lambda}\left(r_{k,l} - r_{0,l}\right) \qquad (k = -p, \cdots, p) \quad (3)$$

where $r_{k,l}$ is the distance between the lth source and the kth element, $\tau_{k,l}$ is the phase lag of the lth signal at the kth element with respect to the reference point, and λ is the wavelength of signals. When the lth signal at the reference point is denoted by $s_{0,l}(t)$, the array received signal vector $\boldsymbol{x}(t)$ is expressed as

$$\begin{aligned}\boldsymbol{x}(t) &= \sum_{l=1}^{L} s_{0,l}(t)\,\boldsymbol{a}(\theta_l, r_{0,l}) + \boldsymbol{n}(t)\\ &= \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \qquad (4)\end{aligned}$$

$$\boldsymbol{A} = \left[\boldsymbol{a}(\theta_1, r_{0,1}), \cdots, \boldsymbol{a}(\theta_L, r_{0,L})\right] \quad (5)$$

$$\boldsymbol{s}(t) = \left[s_{0,1}(t), \cdots, s_{0,L}(t)\right]^T \quad (6)$$

$$\boldsymbol{n}(t) = \left[n_1(t), \cdots, n_K(t)\right]^T \quad (7)$$

where $\boldsymbol{A}$ is the mode matrix, and $\boldsymbol{s}(t)$ and $\boldsymbol{n}(t)$ are the signal vector and the internal noise vector, respectively.

In addition, we try to use the ULA which has dummy (parasitic) elements uniformly next to both end elements to equalize the mutual coupling effect in each element of the K-element array. The dummy elements are the same as the array elements, and have the same load in their terminals. In this paper, we have two dummy elements on each side, i.e. 4 dummy elements as a whole.

3. Near-field DOA-Matrix-ssp Method

If we make approximation of $\tau_{k,l}$ that exploits its second Taylor expansion, and also we assume $r_{k,l} \approx r_{0,l}$, then the received signal $x_k(t)$ at the kth element, which is the kth component of $\boldsymbol{x}(t)$, can be expressed as follows:

$$x_k(t) = \sum_{l=1}^{L} s_{0,l}(t)\,e^{j\left(\omega_l k + \phi_l k^2\right)} + n_k(t) \quad (8)$$

$$\omega_l = \frac{2\pi d\sin\theta_l}{\lambda}, \phi_l = -\frac{\pi d^2\cos^2\theta_l}{\lambda r_{0,l}} \quad (9)$$

Here, we define the correlations $z_{-k-1,-k}(\tau)$ and $z_{k+1,k}(\tau)$ as

$$\begin{aligned}z_{-k-1,-k}(\tau) &= E\left[x_{-k-1}(t+\tau)\,x_{-k}^*(t)\right]\\ &= \sum_{l=1}^{L} c_{sl}(\tau)\,e^{j(-\omega_l+\phi_l)}e^{2jk\phi_l} \qquad (10)\end{aligned}$$

$$\begin{aligned}z_{k+1,k}(\tau) &= E\left[x_{k+1}(t+\tau)\,x_k^*(t)\right]\\ &= \sum_{l=1}^{L} c_{sl}(\tau)\,e^{j(\omega_l+\phi_l)}e^{2jk\phi_l} \qquad (11)\end{aligned}$$

$$c_{sl}(\tau) = E\left[s_{0,l}(t+\tau)\,s_{0,l}^*(t)\right] \quad (12)$$

With those correlations, we construct two correlation vectors $z_1(\tau)$ and $z_2(\tau)$ which are given by

$$z_1(\tau) = \left[z_{p-1,p}(\tau), \cdots, z_{-p,-p+1}(\tau)\right]^T \in \mathbb{C}^{2p\times 1} \quad (13)$$

$$z_2(\tau) = \left[z_{-p+1,-p}(\tau), \cdots, z_{p,p-1}(\tau)\right]^T \in \mathbb{C}^{2p\times 1} \quad (14)$$

To the individual correlation vectors of (13) and (14), we apply the spatial smoothing preprocessing (ssp). Firstly, we extract the multiple M-dimensional subarray data ($M < 2p$) from whole $2p$-dimensional array data of the correlation vectors while shifting one component in the array data. Therefore, the number of subarray data N_s is equal to $2p - M + 1$. Secondly, using the subarray data, we create two matrices $\bar{\boldsymbol{Z}}_1(\tau)$ and $\bar{\boldsymbol{Z}}_2(\tau)$ in which the subarray data are arranged in the columns. Furthermore, for $\tau = 0, T_s, \cdots, (N-1)T_s$ (T_s : the sampling interval of $x_k(t)$, N: integer satisfying $N \geq L$), we have two matrices $\boldsymbol{Z}_1 = [\bar{\boldsymbol{Z}}_1(0), \cdots, \bar{\boldsymbol{Z}}_1((N-1)T_s)]$ and $\boldsymbol{Z}_2 = [\bar{\boldsymbol{Z}}_2(0), \cdots, \bar{\boldsymbol{Z}}_2((N-1)T_s)]$. Finally, we carry out eigendecomposition for a matrix $\boldsymbol{R} = \boldsymbol{Z}_2[\boldsymbol{Z}_1]^-$($[\cdot]^-$: pseudo-inverse). As a result, we can obtain the estimates of θ_l and $r_{0,l}$ from eigenvalues and eigenvectors of $\boldsymbol{R}$ [2]. This is the procedure of the near-field DOA-Matrix-ssp method.

4. Computer Simulation

The computer simulation of location estimation of near-field sources is carried out under the conditions shown in Table I. The antenna element is a vertical dipole with length of $\lambda/2$, which has a 50Ω load. The near-field received signals are computed by using 3-dimensional electromagnetic simulator based on the Method of Moment. We compared the performance of the near-field DOA-Matrix-ssp method using three types of array which are ULA with mutual coupling (w/ MC), ULA without mutual coupling (w/oMC), and ULA with 4 dummy elements (4dummy). The root mean square errors (RMSE) of estimates are used for evaluation of estimation accuracy. Figs. 2 and 3 show the RMSE of location estimates and stochastic CRB [3] as a function of SNR. From Fig. 2, it is found that the DOA estimation accuracy is enhanced by the influence of mutual coupling. Also, it is seen that the use of dummy elements improves estimation accuracy. On the other hand, the estimation accuracy of source distance is degraded by the influence of the mutual coupling as shown in Fig. 3. In addition, the effect of the dummy elements is not recognized.

5. Conclusion

In this paper, we examined the influence of mutual coupling between array elements in source localization using the near-field DOA-Matrix-ssp method. The computer simulation results have shown that the DOA estimation performance of the near-field DOA-Matrix-ssp method is improved unexpectedly by the mutual coupling. In future works, we will find the reason or mechanism of improvement in DOA estimation and will examine the case of multiple signal sources.

References

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TABLE I
Simulation Conditions

Number of array elements	7
Number of dummy elements	4
Element spacing	0.5λ
Number of incident signal	1 (N = 1)
DOA (θ_1)	30°
distance ($r_{0,1}$)	3λ
Antenna element	$\lambda/2$ dipole antenna
Number of snapshots	100
Input SNR	−10 ∼ 30 [dB]
Number of trials	100

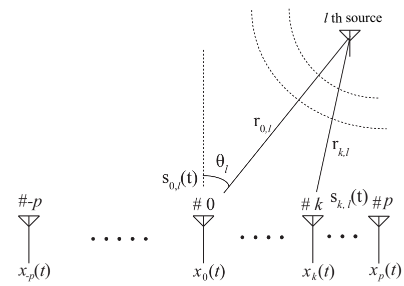

Fig. 1. K-element ULA and received signals

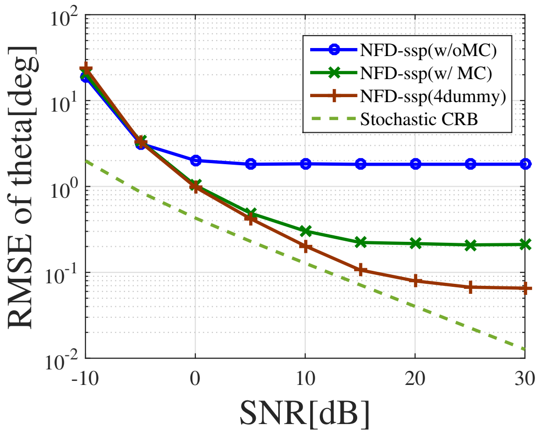

Fig. 2. RMSE of DOA estimates vs. SNR

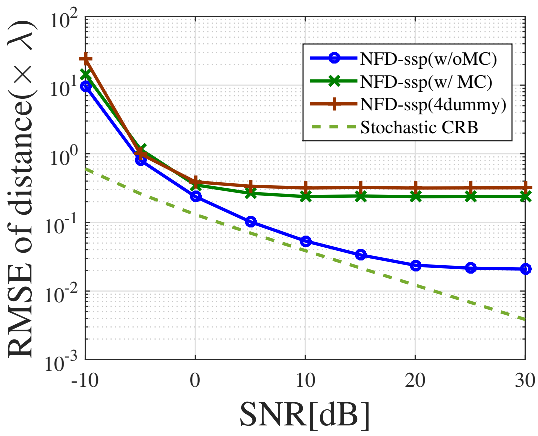

Fig. 3. RMSE of distance estimates vs. SNR