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Effect of Spatial Correlation with Directional Antenna on MIMO Capacity

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1. Introduction

G. J. Foschini and M. J. Gan have been shown that the use of multiple antennas in both transmitter and receiver, called multiple input multiple output system (MIMO), linearly increases the channel capacity with the number of antennas [1]. The effect of antenna pattern to the capacity has been studied in [2]. The channel model in [2] is based on the angle of arrival (AOA). This model, however, does allow including the radiation pattern into the capacity calculation. The effect of fading correlation without antenna pattern on the MIMO capacity has been studied in [3]. It has been shown that the capacity is reduced when the fading correlation exists [3]. In this paper, we generalize the channel model so that it can take both fading correlation and antenna pattern effect on the capacity into consideration. Having the channel model, we then investigate the effect of antenna pattern to the capacity with fading correlation.

The paper is organized as follows. In the next section, the proposed channel model with antenna radiation pattern is discussed for both single and multiple antenna systems. Simulations of the MIMO capacity in various propagation environments are presented in section 3. Section 4 concludes the paper.

2. Proposed MIMO Model

In this section, a brief review of spatial correlation based on "one-ring" channel model is given. Then, the proposed model of the MIMO system with the effect of antenna pattern is discussed. We conclude this section with a technique for MIMO capacity calculation with effect of both spatial correlation and antenna pattern.

Let $\mathbf{x}$ be a vector of the transmitted signals with $\boldsymbol{n_T}$ transmit antennas and $\mathbf{y}$ be a vector of the received signals with $\boldsymbol{n_R}$ receive antennas. Then the MIMO system model is given by

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n} \tag{1}$$

where $\mathbf{n}$ is an $(\boldsymbol{n_R} \times 1)$ noise vector and $\mathbf{H}$ is an $(\boldsymbol{n_R} \times \boldsymbol{n_T})$ channel matrix.

The "one-ring" channel model is used for fading correlation computation [3]. The channel matrix can be written as $\mathbf{H} = \left(\boldsymbol{h}_1, \boldsymbol{h}_2, \mathbf{L}, \boldsymbol{h}_{\boldsymbol{n_T}}\right)$. Let $vec(H) = (h_1^{'}, h_2^{'}, \mathbf{L}, h_{n_T}^{'})^{'}$ denote a vector formed by stacking the columns of the channel matrix $\mathbf{H}$. The covariance matrix of the vector is given by $\operatorname{cov}(vec(H)) = E[vec(H) \cdot vec(H)^*]$ where $E[\cdot]$ denotes the expectation operator. In a uplink of the "one-ring" model with two transmit antenna elements aligned on the y-axis, the covariance between two elements with *i*-th and *j*-th element spacing $\boldsymbol{d_{i,j}}$ and angle spread Δ is approximated by $y_{i,j} = J_0\left(\Delta 2p\, d_{i,j} / l\right)$ where $J_0(.)$ is the Bessel function of the first kind of the zeroth order [3]. The receive antenna element in the uplink is assumed to be uncorrelated. The channel covariance matrix is then given by $\operatorname{cov}(vec(H)) = \Psi \otimes I_{n_R}$ where Ψ is a $\boldsymbol{n_T} \times \boldsymbol{n_T}$ matrix whose elements are $y_{i,j}$. The realizations of the channel matrix $\mathbf{H}$ can be generated by [3]

$$vec(\mathbf{H}) = (\operatorname{cov}(vec(H)))^{1/2}\, vec(\mathbf{H}_w) \tag{2}$$

where $\mathbf{H}_w$ contains i.i.d. $\overset{\sim}{\boldsymbol{N}}(0,1)$ entries.

Let x and y be a transmitted and received signal respectively. The system model of a narrowband wireless system with single antenna at the both ends can be written as

$$\boldsymbol{y} = \boldsymbol{hx} + \boldsymbol{n} \tag{3}$$

where *h* and *n* are the channel impulse response and additive noise respectively. The radiation pattern of the receive antenna will change the magnitude of the channel impulse response for each angle of arrival (AOA) as shown in Fig. 1.

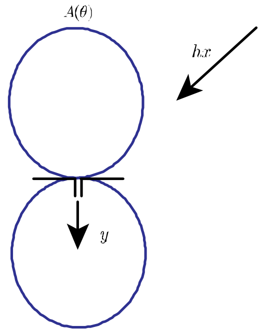


Figure 1: Schematic of the antenna pattern, incoming signal and received signal

The modified channel impulse response with the effect of the radiation pattern can be then written as

$$\boldsymbol{h_a} = \boldsymbol{A}(\theta)\boldsymbol{h} \tag{4}$$

where $\boldsymbol{A}(\theta)$ is the antenna pattern as a function of angle. It can be seen that the modified channel impulse response consists of two random variables, i.e. θ and $\boldsymbol{h_c}$. The antenna radiation pattern, $\boldsymbol{A}(\theta)$, will transform the angle random variable, θ, to a new random variable. Hence, the modified channel impulse response is a multiplication of two random variables. The main advantage of the proposed model is that the channel impulse response, AOA, and antenna pattern can be treated separately. The AOA and channel impulse response can be obtained from either a wireless channel measurement [4] or an analytic model [5] whereas either a full wave electromagnetic simulation or antenna radiation pattern measurement can be used to obtain radiation pattern. By using (4) for the modified channel impulse responses with antenna pattern, each row of the channel matrix is given by

$$\mathbf{h}_{\boldsymbol{i}}^{\boldsymbol{a}} = \left[\boldsymbol{A_i}(\theta_1)\boldsymbol{h}_{i1} \quad \boldsymbol{A_i}(\theta_2)\boldsymbol{h}_{i2} \quad \cdots \quad \boldsymbol{A_i}(\theta_{\boldsymbol{n_T}})\boldsymbol{h}_{\boldsymbol{in_T}}\right] \tag{5}$$

where $\boldsymbol{A_i}(\theta_{\boldsymbol{k}})$ is the radiation pattern of i-th receive antenna and $\theta_{\boldsymbol{k}}$ is the AOA of the signal from k-th transmit antenna to i-th receive antenna. Hence, the channel matrix with the effect of receive antenna radiation patterns can be written as

$$\mathbf{H}_{\boldsymbol{a}} = \left[\mathbf{h}_1^{\boldsymbol{a}} \quad \mathbf{h}_2^{\boldsymbol{a}} \quad \cdots \quad \mathbf{h}_{\boldsymbol{n_T}}^{\boldsymbol{a}}\right]^{\boldsymbol{T}} \tag{6}$$

The MIMO system model in (1) with the effect received antenna radiation pattern can be written as

$$\mathbf{y} = \mathbf{H}_{\boldsymbol{a}}\mathbf{x} + \mathbf{n} \tag{7}$$

The narrowband MIMO capacity is a function of the channel matrix and given by

$$\boldsymbol{C} = \log\det\left(\boldsymbol{I}_{\boldsymbol{n_T}} + \frac{\boldsymbol{P}}{\boldsymbol{n}}\mathbf{H}_{\boldsymbol{a}}\mathbf{H}_{\boldsymbol{a}}{}^{\boldsymbol{T}}\right) \tag{8}$$

where $\boldsymbol{P}$ is the signal power and $\boldsymbol{n}$ is the noise power. To calculate the capacity with effect of both spatial correlation and antenna pattern, random matrices with spatial correlations are generated by using (2). Having correlated random matrices, each realization of the channel matrix is obtained by using (5) and (6) where the AOA for each receive antenna is randomly generated.

3. Simulations

The MIMO capacity in (8) is evaluated using Monte Carlo simulations. In the simulations, we use 4 transmit and 4 receive antennas at the both ends. The signal to noise ratio ($\boldsymbol{P} / \boldsymbol{n}$) is 20 dB. The antenna spacing in this simulations are 0.5λ and 4λ for representing correlated and uncorrelated scenarios respectively. The AOAs are generated using Laplacian distribution [3]. Three types of antenna are used in the simulations. An isotropic antenna is used for simulating an ideal scenario. A horizontal polarized dipole and a 5-element Yagi-Uda are used to investigate the effect of antenna radiation pattern and spatial correlation to the channel capacity. The radiation patterns of all antennas are normalized, [3], so that

$$\int_{-\pi}^{\pi} |\boldsymbol{A}(\theta)|^2 \, \boldsymbol{d}\theta = 2\pi \tag{9}$$

The radiation patterns of dipole and Yagi-Uda antennas using in the simulations are obtained from [6]. Only horizontal radiation pattern is considered in the simulations. Fig. 2 shows the radiation pattern of dipole and Yagi-Uda antennas using in the simulations

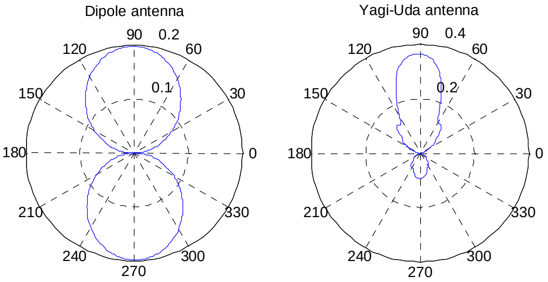
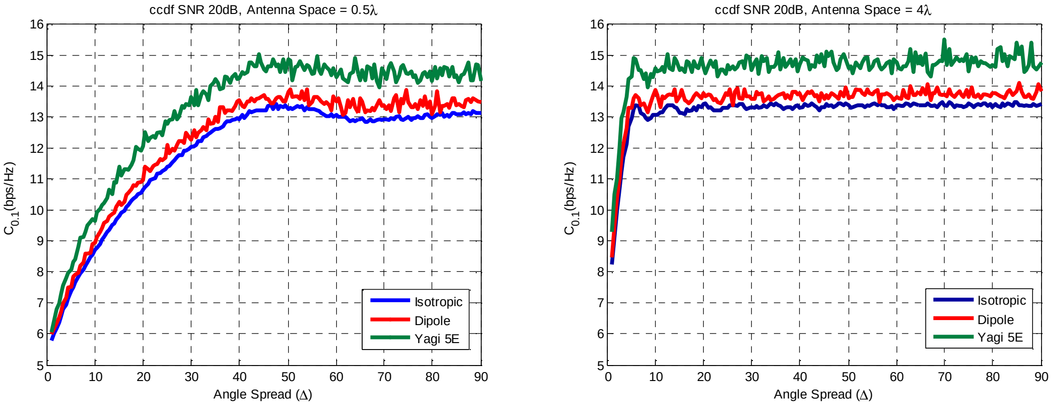


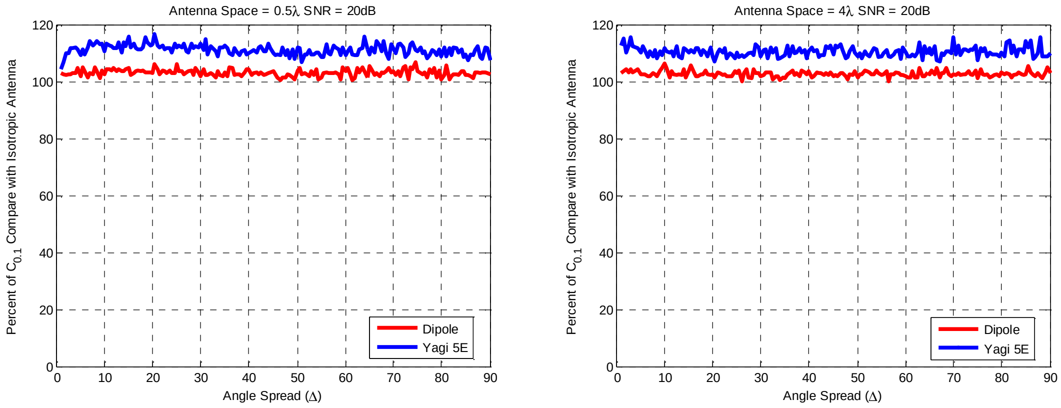
Figure 2: The radiation pattern in linear scale of two types of directional antennas for simulations.



(a) (b)

Figure 3: effect of antenna pattern and antenna spacing with Laplacian distribution on
a) correlated scenario b) uncorrelated scenario

The 10% outage channel capacities, $\boldsymbol{C}_{0.1}$, for correlated and uncorrelated scenarios with various types of antenna are shown in figure 3. It can be seen that the directional antennas improve the MIMO capacity in both correlated and uncorrelated scenarios. The capacities for correlated and uncorrelated scenario approach the same capacity when the angle spreads are about 80 and 20 degrees respectively as shown in figure 3. The $C_{0.1}$ capacities in uncorrelated region for isotropic, dipole and Yagi-5E are 13.4 bps/Hz , 13.8 bps/Hz and 14.8 bps/Hz respectively.



(a) (b)

Figure 4: Percent improvement of channel capacity for directional antenna compare with omni-directional antenna in a) correlated scenario b) uncorrelated scenario

We now compare the capacity improvement with directional antennas to the isotropic case. In correlated case, the percentage of capacity improvement increases as the angle spread and it is steady when the angle spread is about 10 degrees as shown in figure 4(a). The dipole and Yagi-5E antennas give 5% and 10% capacity improvement with respect to isotropic case when the angle spread is greater than 10 degrees. Figure 4(b) shows that the capacities using the dipole and Yagi-5E antennas are about 5% and 10% more than the isotropic capacity respectively in uncorrelated scenario. It can be also seen that the percentage of capacity improvement does not depend to the angle spread in uncorrelated case.

4. Conclusion

We have proposed the channel model for MIMO capacity calculation. The proposed model can be used to calculate the MIMO capacity with the presence of both spatial correlation and antenna radiation pattern. The proposed model allows us to model the spatial correlation, AOA and antenna pattern separately. The “one-ring” channel model is used for capacity calculation. The elements spacing are 0.5λ and 4λ for correlated and uncorrelated cases respectively. We have shown that the directional antennas can improve the MIMO capacity in both correlated and uncorrelated scenarios. The percentage of the capacity improvement with respect to the isotropic capacity is invariant to the angle spread in uncorrelated scenario. In correlated scenario, the capacity improvement is steady when the angle spread is greater than a particular value.

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