Simultaneous Estimation of Azimuth DOA and Angular Spread of Incident Radio Waves by DOA-Matrix Method Using Planar Array

Makoto Jomoto†, Nobuyoshi Kikuma‡, and Kunio Sakakibara‡

†Dept. of Computer Science and Engineering

‡Dept. of Electrical and Mechanical Engineering

Nagoya Institute of Technology, Nagoya 466-8555, Japan

Email: kikuma@m.ieice.org

Abstract **– In estimating DOA of incident waves with high accuracy, we often have to take into consideration the angular spread (AS) of each wave due to reflection, diffraction, and scattering. As a method of estimating DOA and AS simultaneously, DOA-Matrix method was proposed. In this paper, we extend the array configuration from the linear array to the planar array for simultaneous estimation of DOA and AS over whole azimuth angles.**

Index Terms **— DOA, angular spread, DOA-Matrix method, planar array.**

1. Introduction

In order to clarify the radio environments, it is effective to estimate the DOA of individual incident waves at the receiving point. For estimating the DOA, the algorithms such as MUSIC and ESPRIT[1] with array antennas are attractive because of high estimation accuracy and high computational efficiency. However, we often have to take into consideration the angular spread of each wave due to reflection, diffraction, and scattering[2]. As a method of estimating DOA and AS simultaneously, the use of DOA-Matrix method[3] was proposed.

In this paper, we extend the array configuration from the linear array to the planar array for simultaneous estimation of DOA and AS over whole azimuth angles. Then, we propose DOA-Matrix method in which the integrated mode vector [3] is applied to the planar array.

2. Array Antenna and Signal Model

Fig. 1 shows the $K = K_x \times K_y$ element planar array antenna with element spacing of Δx and Δy, which receives L clustered waves with angular spread in azimuth. We assume that the M_l element waves of the l-th clustered wave are in phase and continuously distributed in the angular spread $\Delta\theta_l$ with the center angle θ_l (DOA). Then, the array input vector $\boldsymbol{x}(t)$ can be expressed as follows.

$$\boldsymbol{x}(t) = \sum_{l=1}^{L} s_l(t)\boldsymbol{a}(\theta_l, \Delta\theta_l) + \boldsymbol{n}(t) \tag{1}$$

$$\boldsymbol{a}(\theta_l, \Delta\theta_l) = \left[a_{1,1}(\theta_l, \Delta\theta_l), a_{2,1}(\theta_l, \Delta\theta_l), \cdots, a_{K_x,K_y}(\theta_l, \Delta\theta_l)\right]^T \tag{2}$$

$$a_{k_x,k_y}(\theta, \Delta\theta) = a_{ok_x}(\theta)a_{ok_y}(\theta)\psi_{k_x,k_y}(\theta, \Delta\theta) \tag{3}$$

$$a_{ok_x}(\theta) = e^{-j\frac{2\pi}{\lambda}(k_x-1)\Delta x\cos\theta} \quad (k_x = 1, \cdots, K_x) \tag{4}$$

$$a_{ok_y}(\theta) = e^{-j\frac{2\pi}{\lambda}(k_y-1)\Delta y\sin\theta} \quad (k_y = 1, \cdots, K_y) \tag{5}$$

$$\psi_{k_x,k_y}(\theta, \Delta\theta) = \mathrm{sinc}\left[\frac{\pi}{\lambda}\Delta\theta\{(k_y-1)\Delta y\cos\theta - (k_x-1)\Delta x\sin\theta\}\right] \tag{6}$$

where $s_l(t)$ is the complex amplitude of the l-th clustered wave.

3. AS Estimation by DOA-Matrix Method

If we make the following approximation

$$\psi_{k_x,k_y}(\theta, \Delta\theta) \simeq \psi_{k_x,k_y+1}(\theta, \Delta\theta) \tag{7}$$

then the array input vectors of two subarrays in Fig. 1, $\boldsymbol{x}_1(t)$ and $\boldsymbol{x}_2(t)$, are expressed as follows.

$$\boldsymbol{x}_1(t) = \boldsymbol{A}_1\boldsymbol{s}(t) + \boldsymbol{n}_1(t) \tag{8}$$

$$\boldsymbol{x}_2(t) = \boldsymbol{A}_2\boldsymbol{s}(t) + \boldsymbol{n}_2(t) \tag{9}$$

$$\boldsymbol{A}_1 = [\boldsymbol{a}_1(\theta_1, \Delta\theta_1), \cdots, \boldsymbol{a}_1(\theta_L, \Delta\theta_L)] \tag{10}$$

$$\boldsymbol{a}_1(\theta_l, \Delta\theta_l) = \left[a_{1,1}(\theta_l, \Delta\theta_l), \cdots, a_{K_x,K_y-1}(\theta_l, \Delta\theta_l)\right]^T \tag{11}$$

$$\boldsymbol{A}_2 = [\boldsymbol{a}_2(\theta_1, \Delta\theta_1), \cdots, \boldsymbol{a}_2(\theta_L, \Delta\theta_L)] \tag{12}$$

$$\boldsymbol{a}_2(\theta_l, \Delta\theta_l) = \left[a_{1,2}(\theta_l, \Delta\theta_l), \cdots, a_{K_x,K_y}(\theta_l, \Delta\theta_l)\right]^T \tag{13}$$

$$\boldsymbol{A}_2 \simeq \boldsymbol{A}_1\boldsymbol{\Phi} \tag{14}$$

$$\boldsymbol{s}(t) = [s_1(t), s_2(t), \cdots, s_L(t)]^T \tag{15}$$

$$\boldsymbol{\Phi} = \mathrm{diag}(\phi_1, \cdots, \phi_L) \quad (\phi_l = e^{-j\frac{2\pi}{\lambda}\Delta y\sin\theta_l}) \tag{16}$$

where $\boldsymbol{n}_1(t)$ and $\boldsymbol{n}_2(t)$ are noise vectors of subarray 1 and subarray 2, respectively.

From the auto-correlation matrix $\boldsymbol{R}_{11} = E[\boldsymbol{x}_1(t)\boldsymbol{x}_1^H(t)]$ and the cross-correlation matrix $\boldsymbol{R}_{21} = E[\boldsymbol{x}_2(t)\boldsymbol{x}_1^H(t)]$, we make $\boldsymbol{R} = \boldsymbol{R}_{21}{\boldsymbol{R}_{11}}^{-1}$. Since $\boldsymbol{R}$ has the relation $\boldsymbol{R}\boldsymbol{A}_1 = \boldsymbol{A}_1\boldsymbol{\Phi}$, we can obtain the $\boldsymbol{A}_1$ and $\boldsymbol{\Phi}$ from the eigendecomposition of $\boldsymbol{R}$.

We derive the DOA and AS estimates over the whole azimuth angles from $\boldsymbol{A}_1$ and $\boldsymbol{\Phi}$ by rearranging the mode vector $\boldsymbol{a}_1$ according to the DOA estimates. Specifically, in the case of $45° \leq |\theta| < 135°$, we obtain the AS estimates from comparison between $\psi_{k_x,k_y}(\theta, \Delta\theta)$ and $\psi_{k_x+1,k_y}(\theta, \Delta\theta)$. On the other hand, in the case of $0° \leq |\theta| < 45°$ and $135° \leq |\theta| < 180°$, we obtain the AS estimates from comparison between $\psi_{k_x,k_y}(\theta, \Delta\theta)$ and $\psi_{k_x,k_y+1}(\theta, \Delta\theta)$.

4. Computer Simulation

Computer simulation is carried out under the conditions described in Table I. Fig. 2 shows the validity of approximation of (7) or (14) by spatial correlation between $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$. Figs. 3 and 4 show the estimation accuracy as a function of DOA. In both figures, two methods i.e. the proposed method and DOA-Matrix method without the rearrangement of mode vector are compared.

It is found from Fig. 2 that the value of correlation coefficient between $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ is greater than 0.9992 within AS of 10°. Therefore, it is demonstrated that the approximation of (7) or (14) is valid enough, and also that DOA-Matrix method is applicable. It is found from Fig. 3 that both methods provide high DOA estimation accuracy. Moreover, Fig. 4 shows that the proposed method provides more accurate AS estimates with error of 17% or less for all azimuth angles.

5. Conclusion

Through computer simulation, the effectiveness of the proposed method for DOA and AS estimation in whole azimuth angles has been demonstrated. Particularly, it is confirmed that the AS estimation is improved by rearrangement of the mode vector according to the DOA estimates.

In the future work, we will examine the way of removing the estimation errors due to the overlapping subarrays by using SLS (Structured Least Squares) algorithm[4].

References

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TABLE I
Simulation Conditions

Number of elements	49(7 × 7)
Element spacing	0.45λ
Number of clustered waves	1
DOA of clustered waves	$-180° \sim 180°$
AS of clustered waves	$1° \sim 10°$
Number of element waves in one clustered wave	30
Input SNR	20 [dB]
Number of snapshots	30
Number of trials	100

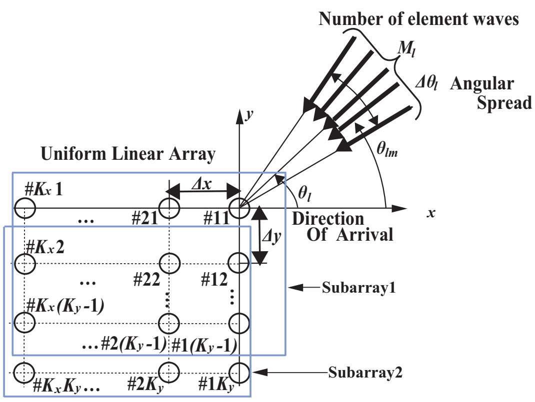

Fig. 1. Planar rectangular array antenna and incident waves with angular spread

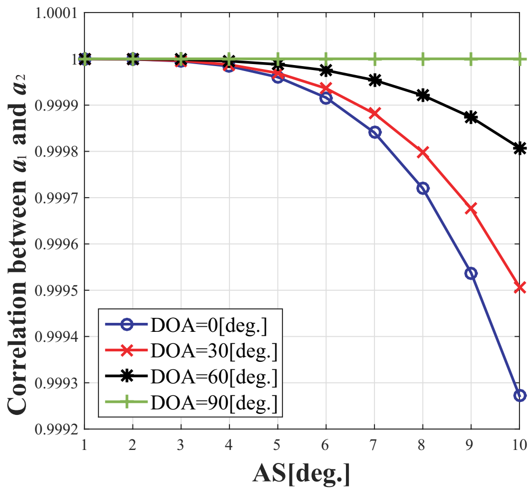

Fig. 2. Spatial correlation between $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ vs. AS

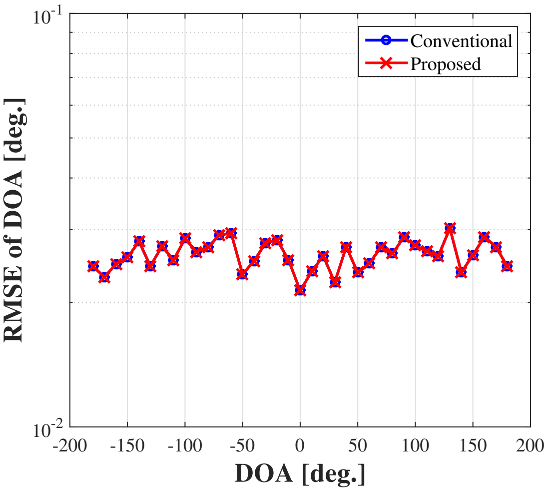

Fig. 3. RMSE of DOA estimates vs. DOA (AS = 6°)

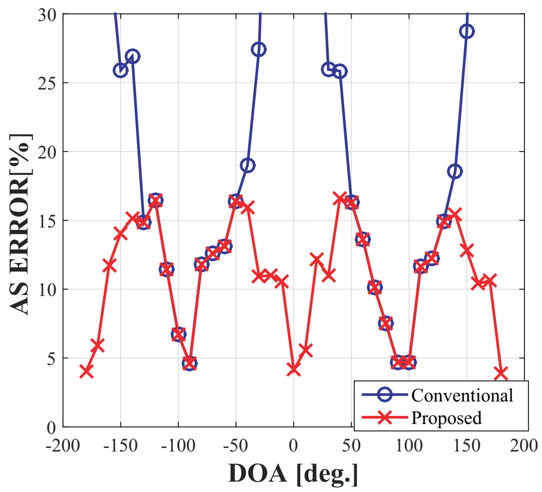

Fig. 4. Error of AS estimates vs. DOA (AS = 6°)