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AN APPROACH TO SCALAR WAVE SCATTERING FROM A CONDUCTING TARGET SURROUNDED BY RANDOM MEDIA

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1. Introduction

Wave scattering from targets surrounded by random media is an important problem in radar engineering. An electromagnetic wave radiated by the transmitter propagates in a random medium and a part of the wave illuminates a target. Hence, the surface current is induced on the target, the scattered wave is produced and propagates in the random medium. Therefore, the received wave is the sum of both waves propagated in the random medium, one of which is the incident wave and the other the scattered wave. Because the induced surface-current depends on the incident wave propagated through the random medium and on the target shape, it is difficult to obtain the current directly and thus to obtain the scattered wave from the target in the random medium.

In this paper, using the current characteristic independent of the incident wave and using the Green function in random media of which the point source is on the target, we express the wave scattered from the target and propagated in the random medium. In many practical cases, the wave in the neighborhood of the target and the wave in the transmitter region/the receiver region are statistically independent of each other. Therefore, the calculation of the current characteristic can be made independently of the transmitter-target propagation and the target-receiver propagation. Especially, when a spherical wave propagates from an end to the maximum far end of the target replaced by the random medium and the spatial coherence holds at a high degree, the current calculation becomes equivalent to that in a constant medium.

2. Formulation

In order to show an approach to the problem, we deal with the scalar wave for simplicity. A conducting target is surrounded by a random medium (see Fig.1). Let us designate an incident wave by u_{in}, the scattered wave by u_s and the total wave by $u = u_{in} + u_s$. Then u satisfies the following equation in the random medium V except the target V and the transmitting region V_T:

$$[\nabla^2 + k^2(1+\delta\varepsilon(\mathbf{r}))]u(\mathbf{r}) = 0 \qquad \mathbf{r} \notin V_T \cup \overline{V}\ ,\ \mathbf{r} \in V \tag{1}$$

where $\delta\varepsilon(\mathbf{r})$ is a continuous random function of $\mathbf{r}$ for a turbulent medium and

$$\delta\varepsilon(\mathbf{r}) = \sum_{i=0}^{N} \varepsilon_i(\mathbf{r})$$

for a discrete random medium in which, in general, $\varepsilon_i(\mathbf{r})$ is a random function of $\mathbf{r}$, the shape and orientation of the particle.

It is assumed that $\delta\varepsilon(\mathbf{r})$ exists in a finite region and $\delta\varepsilon(\mathbf{r}) \equiv 0$ in the external region. Then u_s is assumed to satisfy the following radiation condition.

$$\lim_{r\to\infty} r\left(\frac{\partial u_s}{\partial r}+jku_s\right)=0 \tag{2}$$

where the time factor $\exp(-j\omega t)$ is assumed. The Green function G in all regions $V+V_T+V$ satisfies the radiation condition (2) and the equation

$$[\nabla^2+k^2(1+\delta\varepsilon(\mathbf{r}))]G(\mathbf{r}|\mathbf{r}')=-\delta(\mathbf{r}-\mathbf{r}') \tag{3}$$

Using the Green theorem, we get the integral representation

$$u(\mathbf{r})=u_{in}(\mathbf{r})+\int_S G(\mathbf{r}|\mathbf{r}_0)\frac{\partial}{\partial n}u(\mathbf{r}_0)d\mathbf{r}_0 \qquad \mathbf{r}\in V \tag{4}$$

where, the boundary condition for u on the target surface S:

$$u(\mathbf{r}_0)=0 \qquad \mathbf{r}_0 \ on\ S \tag{5}$$

was used and $\partial/\partial n$ denotes the normal derivative on S, directed into the target.

It is assumed that $|\delta\varepsilon(\mathbf{r})|<\infty$. Then it can be proved that the singularities of

$$\lim_{\mathbf{r}\to\mathbf{r}_0} G(\mathbf{r}|\mathbf{r}_0) \qquad and \qquad \lim_{\mathbf{r}\to\mathbf{r}_0}\frac{\partial}{\partial n}G(\mathbf{r}|\mathbf{r}_0)$$

are the same as those in homogeneous media. This fact leads to the Fredholm integral equation of the second kind on the surface.

$$\frac{1}{2}\frac{\partial}{\partial n}u(\mathbf{r})+\int_S\frac{\partial}{\partial n}G(\mathbf{r}|\mathbf{r}_0)\frac{\partial}{\partial n}u(\mathbf{r}_0)d\mathbf{r}_0=-\frac{\partial}{\partial n}u_{in}(\mathbf{r}) \tag{6}$$

where $(\partial/\partial n)G$ is the normal derivative at $\mathbf{r}$ on S, and u_{in} is given by

$$u_{in}(\mathbf{r})=\int_{S_T} G(\mathbf{r}_0|\mathbf{r}_t)f(\mathbf{r}_t)d\mathbf{r}_t \tag{7}$$

Substituting the solution $(\partial/\partial n)u$ of (6) into (4), we get the received total wave. However, $(\partial/\partial n)G$ and $(\partial/\partial n)u$ in (6) are statistically coupled, so that it is difficult to obtain the $(\partial/\partial n)u$. This means that the standard approach by which (6) is directly analyzed is not applicable.

Let us express the solution of (6) as

$$\frac{\partial}{\partial n}u(\mathbf{r}_0)=\int_S S(\mathbf{r}_0|\mathbf{r}')\frac{\partial}{\partial n}u_{in}(\mathbf{r}')d\mathbf{r}' \tag{8}$$

where the operator $S(\mathbf{r}_0|\mathbf{r}')$ satisfies

$$S(\mathbf{r}_0|\mathbf{r}')=-2\delta(\mathbf{r}_0-\mathbf{r}')-2\int_S\frac{\partial}{\partial n}G(\mathbf{r}_0|\mathbf{r}_1)S(\mathbf{r}_1|\mathbf{r}')d\mathbf{r}_1 \tag{9}$$

We write $S(\mathbf{r}|\mathbf{r}_0)$ as

$$S(\mathbf{r}|\mathbf{r}_0)=-2\delta(\mathbf{r}-\mathbf{r}_0)S(\mathbf{r})+S_e(\mathbf{r},\mathbf{r}_0) \tag{10}$$

where $S_e(\mathbf{r},\mathbf{r}_0)$ is composed of a number of the eigen functions in the homogeneous equation (6) with $(\partial/\partial n)u_{in}\equiv 0$, and satisfies

$$\int_S S_e(\mathbf{r},\mathbf{r}_0)d\mathbf{r}_0=\int_S S_e(\mathbf{r},\mathbf{r}_0)d\mathbf{r}=0 \tag{11}$$

Because the eigen values are the internal resonance frequencies in the target, the number of the eigen functions depends on the target size normalized to the wavelength in free space. For simplicity, in this paper, we deal with a frequency band except the resonance and consider only the $S(\mathbf{r})$ in (10). In this case, it follows from (8) and (9) that

$$\frac{\partial}{\partial n}u(\mathbf{r}_0)=-2S(\mathbf{r}_0)\frac{\partial}{\partial n}u_{in}(\mathbf{r}_0) \tag{12}$$

$$S(\mathbf{r}_0)=1-2\int_S\frac{\partial}{\partial n}G(\mathbf{r}_0|\mathbf{r}_1)S(\mathbf{r}_1)d\mathbf{r}_1 \tag{13}$$

where $S(\mathbf{r}_0)$ shows the current characteristic normalized to the current based on the physical optic approximation.

From (4) and (12), we can express the scattered wave as follows:

$$\begin{aligned}u_s(r)&=-2\int_S G(\mathbf{r}|\mathbf{r}_0)S(\mathbf{r}_0)\frac{\partial}{\partial n}u_{in}(\mathbf{r}_0)d\mathbf{r}_0\\&=-2\int_S d\mathbf{r}_0\int_{S_T}d\mathbf{r}_T G(\mathbf{r}|\mathbf{r}_0)\frac{\partial}{\partial n}G(\mathbf{r}_0|\mathbf{r}_T)S(\mathbf{r}_0)f(\mathbf{r}_T)\end{aligned} \tag{14}$$

It should be noted that $[G(\mathbf{r}|\mathbf{r}_0)\partial/\partial n G(\mathbf{r}_0|\mathbf{r}_T)]$ and $S(\mathbf{r}_0)$ in (14) are statistically independent of each other in many practical cases because $|\mathbf{r}-\mathbf{r}_0|$ and $|\mathbf{r}_0-\mathbf{r}_T|$ are much greater than the size of target. Moreover, $G(\mathbf{r}_0|\mathbf{r}_1)$ in (13) may practically be replaced by the Green function in free space or in a constant medium because $|\mathbf{r}_1-\mathbf{r}_0|$ is very short optical-distance in the random medium. In this case, $S(\mathbf{r}_0)$ is obtained by analyzing the non-random problem.

3. Moments

Using (13) and (14), we can obtain the moments of u_s and u in radar problems. For backscattering, we receive u_s of which the average intensity is given by

$$\begin{aligned}<|u_s|^2>=4\int_S d\mathbf{r}_1\int_S d\mathbf{r}_2\int_{S_T}d\mathbf{r}_1^T\int_{S_T}d\mathbf{r}_2^T\Big[&<G(\mathbf{r}|\mathbf{r}_1)G^*(\mathbf{r}|\mathbf{r}_2)\frac{\partial}{\partial n}G(\mathbf{r}_1|\mathbf{r}_1^T)\frac{\partial}{\partial n}G^*(\mathbf{r}_2|\mathbf{r}_2^T)>\\&\cdot<S(\mathbf{r}_1)><S^*(\mathbf{r}_2)>f(\mathbf{r}_1^T)f^*(\mathbf{r}_2^T)\Big]\end{aligned} \tag{15}$$

In the case of forwardscattering, $u = u_{in} + u_s$ is received and the average intensity is written as

$$<|u|^2>=<|u_{in}|^2>+2Re[<u_{in}u_s^*>]+<|u_s|^2> \tag{16}$$

where

$$<|u_{in}|^2>=\int_{S_T}d\mathbf{r}_1^T\int_{S_T}d\mathbf{r}_2^T<G(\mathbf{r}|\mathbf{r}_1^T)G^*(\mathbf{r}|\mathbf{r}_2^T)>f(\mathbf{r}_1^T)f^*(\mathbf{r}_2^T) \tag{17}$$

$$\begin{aligned}<u_{in}u_s^*>=-2\int_{S_T}d\mathbf{r}_1^T\int_{S_T}d\mathbf{r}_2^T\int_S d\mathbf{r}_0\Big[&<G^*(\mathbf{r}|\mathbf{r}_0)G(\mathbf{r}|\mathbf{r}_1^T)><\frac{\partial}{\partial n}G^*(\mathbf{r}_0|\mathbf{r}_2^T)><S^*(\mathbf{r}_0)>\\&\cdot f(\mathbf{r}_1^T)f^*(\mathbf{r}_2^T)\Big]\end{aligned} \tag{18}$$

$$\begin{aligned}<|u_s|^2>=4\int_S d\mathbf{r}_1\int_S d\mathbf{r}_2\int_{S_T}d\mathbf{r}_1^T\int_{S_T}d\mathbf{r}_2^T\Big[&<G(\mathbf{r}|\mathbf{r}_1)G^*(\mathbf{r}|\mathbf{r}_2)>\\&\cdot<\frac{\partial}{\partial n}G(\mathbf{r}_1|\mathbf{r}_1^T)\frac{\partial}{\partial n}G(\mathbf{r}_2|\mathbf{r}_2^T)><S(\mathbf{r}_1)><S^*(\mathbf{r}_2)>f(\mathbf{r}_1^T)f^*(\mathbf{r}_2^T)\Big]\end{aligned} \tag{19}$$

In general, the n-th moment of u_s is expressed in terms of $2n$-th moment of the Green function in the random medium in the backscattering case. On the other hand, in the forwardscattering case, u_{in} and u_s are statistically coupled but $G(\mathbf{r}|\mathbf{r}_0)$ and $(\partial/\partial n)G(\mathbf{r}_0|\mathbf{r}_T)$ in u_s becomes uncoupled; therefore, the $m(m \leqq n)$-th moment of u is expressed in terms of the n-th moment of the Green function. In the other case, we receive $u = u_{in} + u_s$ where u_{in} and u_s are statistically independent of each other, and $G(\mathbf{r}|\mathbf{r}_0)$ and $(\partial/\partial n)G(\mathbf{r}_0|\mathbf{r}_T)$ in u_s also are independent.

4. Conclusion

We present a general approach to the problem of wave scattering from a conducting target surrounded by random media. The scattered wave by the target is expressed through two independent calculations of the current characteristic on the target and of the Green function in the transmitter-target and target-receiver propagations. It seems that in many practical cases, this expression for the scattered wave is useful for analyzing the statistics of received waves. This approach will be extended to vector fields.

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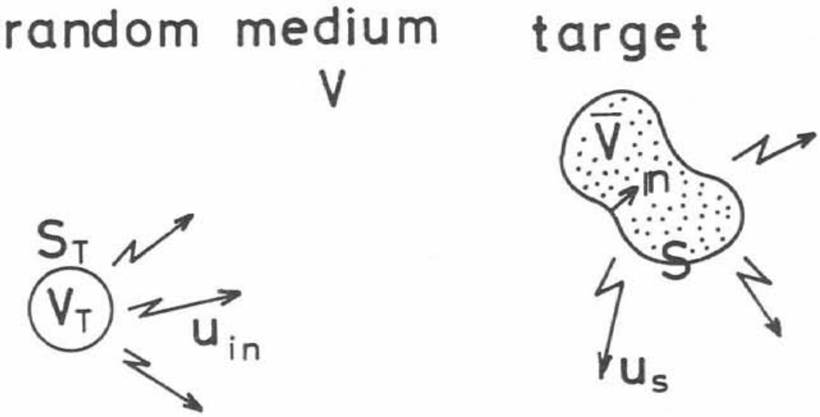

Fig. 1. Geometry of the problem.