Direction-of-Arrival Estimation with Lüneburg Lens and Metamaterial Absorber

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Abstract - **In this article, we introduce a direction-of-arrival (DOA) estimation by combining a Lüneburg lens and a frequency tunable metamaterial absorber. The Lüneburg lens is used to physically separate the multiple incident waves with different DOAs and focus them into the different focal points near the lens surface. The metamaterial absorber is set on the focal plane as a sensor array. Therefore, the DOA information of an incident wave can be obtained based on the position of the sensor element having detected it. The resonance (absorption) frequency of the metamaterial absorber can be changed from 2 to 3 GHz using variable capacitors on the absorber. The error in estimated DOAs obtained by using a correlation algorithm is less than 2 degrees at 2 to 3 GHz, which is confirmed by prototype evaluations.**

Index Terms — **Metamaterial absorber, Lüneburg lens, Direction-of-arrival estimation.**

1. Introduction

Localization of electromagnetic wave sources from electronic and industrial equipment is an important issue in electromagnetic comparability (EMC) field. One of the popular technique for a small electronic device is near-field scanning. However, for a large object, it is too difficult to scan the whole space of interest because of the longer measurement times required. Direction-of-arrival (DOA) estimation is a good choice for a large object and many techniques have been developed. Most of these techniques are based on antenna arrays with a sophisticated algorithm like multiple signal classification (MUSIC) or estimation of signal parameters via rotational invariance technique (ESPRIT). To achieve high resolution in the incident angle, the complicated signal processing algorithms and high performance hardware are required.

A Lüneburg lens [1] with photonic sensors or power detectors [2] is another DOA technique without any sophisticated algorithms. The lens can separate the multiple incident waves with different DOAs. A sensor array is used to detect the incident wave which was concentrated at the focal point. This paper proposes a DOA estimation with Lüneburg lens and a frequency tunable metamaterial absorber [3] with a correlation algorithm.

2. DOA estimation using Lüneburg lens and frequency tunable metamaterial absorber

The proposed technique is mainly composed of a Lüneburg lens and a frequency tunable metamaterial absorber. The absorber is employed as a compact and high-density 2D sensor array whose elements are located at the focal points near the lens surface for the incident waves with different DOAs.

(1) Lüneburg lens

The Luneburg lens is a spherical structure with gradually changing the relative permittivity. The multiple incident waves with different DOAs are physically distinguished by the lens and focused to the different focal points. The relative permittivity of the lens can be described by the following formula.

$$\varepsilon_r(r) = 2 - \left(\frac{r}{R}\right)^2 \quad (1)$$

R: Radius of the lens, r: Distance from center of the lens, ε_r: Relative permittivity

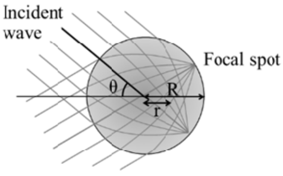


Fig. 1. Lüneburg Lens

In this paper, we used a Lüneburg lens with a diameter of 410 mm (Lun'tech SMR16).

(2) Frequency tunable metamaterial absorber

A compact 2D sensor was designed based on the mushroom-type metamaterial absorber. Fig. 2 shows an example of the mushroom type metamaterial structure with variable capacitor for controlling the resonance frequency.

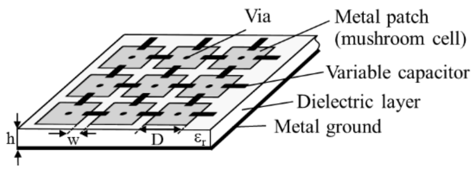


Fig. 2. Structure of a mushroom-type metamaterial absorber

The parasitic capacitance C and parasitic inductance L of the structure can be described approximately by the following formulas (2), (3) [4] and the resonance frequency is given by a simple equation (4) with variable capacitance C_{var}. The power of the incident wave can be measured by connecting a detecting circuit to the variable capacitor.

$$C = \frac{D\varepsilon_0(\varepsilon_r + 1)}{\pi}\cosh^{-1}\left(\frac{D+w}{w}\right) \quad (D<<\lambda,\ w<<D) \quad (2)$$

$$L = \mu_0 h \quad (h<<D) \quad (3)$$

$$f = \frac{1}{2\pi\sqrt{L(C+C_{var})}} \quad (4)$$

TABLE I. shows the parameters of the metamaterial absorber designed for 2 to 3 GHz.

TABLE I Parameters of the metamaterial absorber

Patch size D	8.0 mm
Patch distance w	0.5 mm
Thickness h	1.4 mm
Additional capacitance C_{var}	0.5~1.5 pF
Dielectric permittivity ε_r	4.5
Number of patches	32×32

(3) Correlation algorithm

A correlation algorithm was used for increasing the accuracy of DOA estimation. The absorber obtains a diffraction image of the lens. The spot size of the focal point due to the diffraction limit was determined by the diameter of the lens and the frequency of the incident wave. A theoretical diffraction image was calculated by the following formulas [5].

$$U(x_2, y_2, z_2) \propto \frac{\exp(ikz_{12})}{i\lambda z_{12}} \iint \exp\left\{\frac{ik}{2z_{12}}\left[(x_2-x_1)^2+(y_2-y_1)^2\right]\right\} \times U(x_1, y_1, z_1)_l \, dx_1 dy_1 \quad (5)$$

$$U(x_1, y_1, z_1)_l = \exp\left[-i\frac{k}{2f}\left(x_1^2+y_1^2\right)\right] \quad (6)$$

$U(x_1, y_1, z_1)_l$: Lens transfer function, $U(x_2,y_2,z_2)$: Amplitude of the output plane, k: Wave number, λ: Wave length, z_{12}: Distance from the input to the output plane

Theoretical images U_T were calculated by different incident angles from 0 to 20 degrees by every 0.25 degree. The correlation coefficients between the measured distribution U_E and theoretical images U_T were computed by the following formula. The estimated direction of the incident wave is the angle which has the largest correlation coefficient.

$$Corr = \frac{\sum_m \sum_n \left(U_{Tmn} - \overline{U_T}\right)\left(U_{Emn} - \overline{U_E}\right)}{\left(\sum_m \sum_n \left(U_{Tmn} - \overline{U_T}\right)^2\right)\left(\sum_m \sum_n \left(U_{Emn} - \overline{U_E}\right)^2\right)} \quad (7)$$

m, n : Numbers of observation points, Ū: Average of matrix elements

3. Experimental Results

Fig. 3 shows the experimental setup in an anechoic chamber. The focal points of the Lüneburg lens were on a virtual spherical surface 40 mm behind the lens surface. The center of the sensor was located on the 0 degree focal point of the Lüneburg lens. A horn antenna was set 3 meters away from the sensor as an electromagnetic wave source. The angle of incident wave is changed by controlling the antenna height.

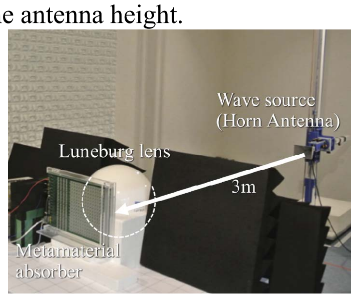
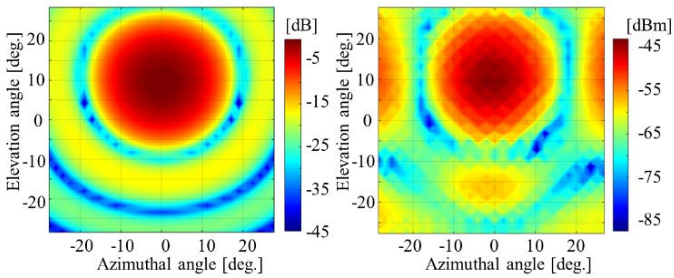


Fig. 3. Experimental setup in an anechoic chamber

Fig. 4 shows the theoretical diffraction image and the measured distribution at an incident angle of 10 degrees at 2.5GHz. The angle of incident wave was obtained according to the vertical position of the maximum power level in Fig 4.



(a) Theoretical image (b) Measured distribution

Fig. 4. Diffraction images at an incident angle of 10 degrees

The incident angle was estimated by the correlation algorithm. The estimation error observed for various angles is shown in Fig. 5. It shows that it is possible to estimate DOAs by this technique from 2 to 3 GHz, with the estimation error less than ± 2.0 degrees. The accuracy was possibly limited by the gain variation among the sensor elements and reflection at the edge of the absorber, which could be improved by calibration.

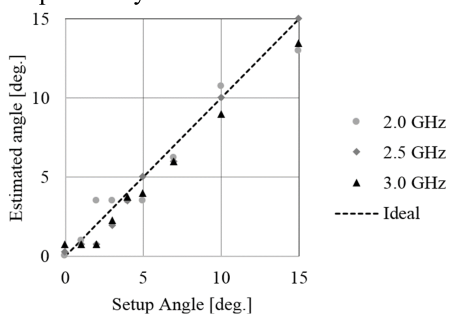


Fig. 5. Estimated angles of incident waves

4. Conclusion

A DOA estimation by combining a Lüneburg lens and a frequency tunable metamaterial absorber with correlation algorithm was studied. It was shown that the proposed method can estimate the angle of incident wave without any complicated signal processing algorithms. The DOA estimation error was less than 2.0 degrees in the frequency range of 2 to 3 GHz

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