On Direction-of-Arrival Estimation with Khatri-Rao Transform Virtual-Array by Using Sparse Signal Reconstruction

Suguru OHASHI, Hiroyoshi YAMADA, Yoshio YAMAGUCHI
Graduate School of Science & Technology, Niigata University Ikarashi 2-8050, Nishi-ku, Niigata, 950-2181 Japan

***Abstract* – Khatri-Rao (KR) product increases the number of elements virtually and enables to estimate more incoming waves in direction-of-arrival (DOA) estimation using array antenna. The DOA estimation with the KR scheme, however, makes the estimation difficult because the scheme makes the data coherent. The sparse signal reconstruction is a method realizing the DOA estimation regardless of the signal correlation. In this paper, we study the DOA estimation by the sparse signal reconstruction. In this study, Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) is applied to the virtual array data by the KR product. We show that the FISTA has a performance equivalent to the MUSIC algorithm without estimating the number of incident waves and signal decorrelation preprocessing such as Spatial Smoothing Preprocessing (SSP).**

***Index Terms* — DOA estimation, Khatri-Rao product, Sparse signal reconstruction, Minimum Redundancy Array**

1. Introduction

The research on direction-of-arrival (DOA) estimation of incident waves with an array has been actively studied for many years. Increasing the number of receiving elements improves degree of freedom (DOF) of array as well as increasing the array aperture, and can estimate many incoming waves. In realization of large array, we should consider the costs and size-limitation depending on the applications. Another solution is the virtual array technique. The Khatri-Rao (KR) product [1] is one of the virtual array techniques, which enables us to increase the number of elements virtually. By using the KR product, we can increase the array elements, but all of the incident waves become coherent. Hence, algorithms which require a priori information on the number of incident waves can be hardly employed. In addition, the estimation accuracy deteriorates in the case of correlated wave incidence. This is because the KR transform assumes the uncorrelated wave incidence. In this study, we evaluate performance of the sparse signal reconstruction technique for the KR-virtual array. The sparse signal reconstruction can estimate DOA regardless of the wave correlation. Furthermore, the technique can resolve waves without pre-estimation of the number of incident waves. These features are different from superresolution techniques such as multiple signal classification (MUSIC) [2]. In this paper, we adopt Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) [3] which classified $L1$ reconstruction by computer simulation.

2. Data Model and KR Transform Virtual-Array

Let us assume that K plane waves impinge on a uniform linear array (ULA) consisting of M elements. In the DOA estimation by the sparse signal reconstruction, we first divide the estimation angular region into N sections ($N >> K$). Then, the M dimensional received data vector $\boldsymbol{y}$ can be written by

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n}, \qquad (1)$$

where $\boldsymbol{A}$ is the $M \times N$ measurement matrix consisted of mode vectors corresponding to N-divided angles, $\boldsymbol{s}$ is the N dimensional sparse vector whose components has only K non-zero values ($K << N$) and zeros for the others. $\boldsymbol{n}$ is the M dimenstional additive Gaussian noise vector. In this paper, we apply the KR-product transform to the vector $\boldsymbol{y}$ and obtain the virtual array data. This virtual array technique forms larger aperture of array virtually by rearranging the data correlation matrix. In particular, the technique is more effective for non-uniform array such as a Minimum Redundancy Array (MRA) [4] than the ULA. Note that uncorrelated wave incidence is assumed in the KR transform and the transformed data becomes coherent even when all of the waves are uncorrelated. In addition, plenty of snapshots often required to make accurate transformation.

3. Sparse Signal Reconstruction

When the dimension of received data vector, M, is smaller than the dimension N, the problem becomes ill-posed problem, then we cannot obtain the solution uniquely in general. The sparse signal reconstruction estimates the solution by using the sparsity of the N dimensional vector $\boldsymbol{s}$. In this study, we use the algorithm called FISTA based on the $L1$-norm minimization for DOA estimation. The FISTA is the algorithm that solves the following problem

$$\hat{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2^2 + \alpha\|\boldsymbol{x}\|_1, \qquad (2)$$

where $\|\boldsymbol{x}\|_p$ is the Lp norm ($p \geq 1$) and α is the control parameter.

4. Computer Simulation

Incoming waves often have high correlations for applications such as radar. In this paper, we consider the DOA estimation problem whose number of incoming waves is larger than that of array elements. In addition, all waves are assumed to be coherent. Simulation parameters are listed in TABLE I. In this study, we use the 4-element MRA as shown in Fig. 1 where d_r is the minimum spacing of the elements. By applying the KR product to this array data, we can obtain the virtual ULA having 13 elements. The separable number of incoming waves becomes 6 waves maximally because the KR transformed data are coherent. In this study, we employ the FISTA whose stop condition is defined by

$$\left\|\boldsymbol{x}_i - \boldsymbol{x}_{i-1}\right\|_2^2 \Big/ \left\|\boldsymbol{x}_i\right\|_2^2 < \varepsilon , \qquad (3)$$

where ε is the threshold defined in the TABLE I. Fig. 2 and Fig. 3 show the DOA spectrum estimated by the FISTA and MUSIC algorithm, respectively, where the signal to noise ratio (SNR) is 10 dB and the MUSIC uses 7 subarrays for signal decorrelation. Fig. 4 depicts the Root Mean Squared Error (RMSE) of the estimated DOAs in each SNR. The parameters used here are the same of TABLE I and the RMSE were evaluated by 100 trials. This result shows that the estimation performance of the FISTA has almost the same as that of the MUSIC when SNR is larger than 5 dB.

TABLE I
Simulation Parameters

Array configuration	4-element MRA
Frequency	5.25 GHz
Minimum element spacing	Half wave length versus 5.25 GHz
Incoming wave	6 waves : –60, –30, 0, 20, 40, 60 degree
Correlation coefficient	1
Number of snapshots	1000
Threshold (ε)	10^{-10}
Control parameter (α)	40

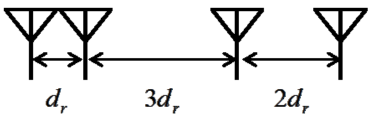

Fig. 1. The 4-element MRA.

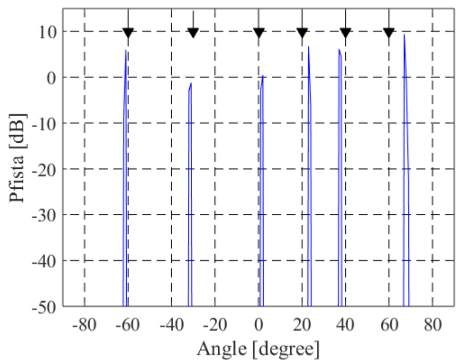

Fig. 2. Estimated FISTA spectrum.

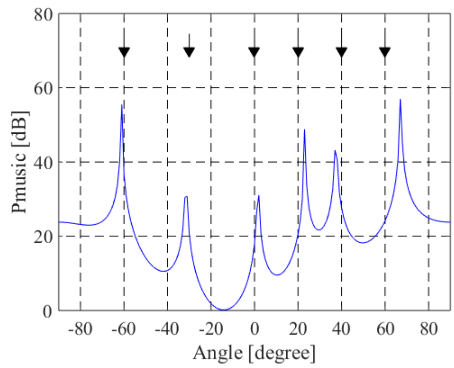

Fig. 3. Estimated MUSIC spectrum.

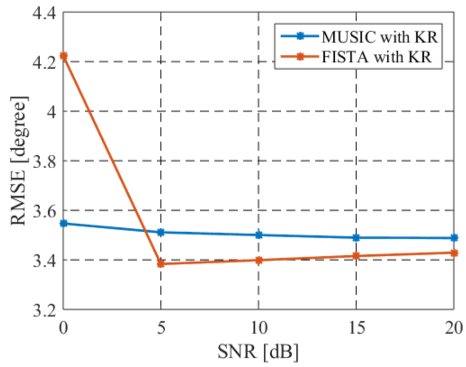

Fig. 4. RMSE vs. each SNR.

5. Conclusion

In this paper, we have shown the validity of the DOA estimation with KR-transformed virtual array by one of the sparse signal reconstructions called FISTA. The simulation results show that FISTA has stable performance even for coherent waves when the number of incoming waves exceeds the number of array elements. Unfortunately, the DOA estimation results have bias. This is because we violate the assumption of the KR transform which requires uncorrelated wave incidence. However, it is a good news that the coherent waves can be resolved. We will study how to decrease the DOA estimation bias by some modifications for the sparse signal reconstruction in the near future.

Acknowledgment

This work was partly supported by JSPS KAKENHI Grant Number 26420346, and research grant by SCAT, Japan.

References

[1] W. K. Ma, T. H. Hsieh, and C. Y. Chi, "DOA estimation of quasi-stationary signals with less sensors than sources and unknown spatial noise covariance: A Khatri-Rao subspace approach," IEEE Trans. Signal Processing, vol.58, no.4, pp.2168-2180, Apr. 2010.
[2] R. O. Schmidt, "Multiple Emitter Location and Signal Parameter Estimation," IEEE Trans. Antennas and Propagat., vol.AP-34, no.3, pp.276-280, Mar. 1986.
[3] A. Beck and M. Teboulle, "A fast iterative shrinkage-thresholding algorithm for linear inverse problem," SIAM Journal on Image Science, vol.2, no. 1, pp. 183-202, Mar. 2009.
[4] A. T. Moffet, "Minimum-Redundancy Linear Arrays," IEEE Trans. Antennas and Propagation, vol.AP-16, no.2, pp.172-175, Mar. 1968.