2-D DOA Estimation of Multiple Signals Based on Sparse L-shaped Array

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***Abstract*—This paper proposes a novel method for two-dimensional (2-D) direction-of-arrival (DOA) estimation of multiple signals employing a sparse L-shaped array structured by a sparse linear array (SLA), a sparse uniform linear array (SULA) and an auxiliary sensor. In the proposed method, the elevation angles are estimated using the SLA and an improved MUSIC method. The azimuth angles are estimated by two stages. Firstly, the rough azimuth estimates are obtained by a cross-correlation matrix (CCM) of the array received data and the elevation estimates. Secondly, the fine azimuth estimates can be achieved using the SULA and the rough azimuth estimates. The proposed method can achieve automatic pairing of the 2-D DOA estimates. Simulation results show that our approach outperforms the existing methods based on L-shaped array.**

***Index Terms*—Sparse L-shaped array, 2-D DOA estimation, cross-correlation matrix (CCM), automatic pairing.**

1. Introduction

The two-dimensional (2-D) direction-of-arrival (DOA) estimation of multiple signals has received considerable attention in radar, sonar and wireless communications, and other fields. In the past decades, lots of 2-D DOA estimation algorithms have been proposed with L-shaped array since it can provide a larger array aperture. However, the algorithms can not achieve automatic pairing for 2-D DOA estimation. Aiming at the problem, some methods with automatic pairing, e.g., JSVD [1], CCM-ESPRIT-AP [2] and J-2D [3] was proposed. However, the methods [1]–[3] are not very high in estimation accuracy. In this paper, we present a 2-D DOA estimation method based on a sparse L-shaped array. Our approach can achieve higher estimation accuracy and make the 2-D DOA estimates pairing automatically. Simulation results demonstrate the effectiveness of the proposed method.

2. Data Model

Consider an L-shaped array geometry lying in the x-z plane as shown in Fig. 1. It consists of two subarrays: Subarray 1 and Subarray 2. Subarray 1 includes $M+1$ sensor elements and consists of an SULA and an auxiliary sensor, in which the inter-element spacing of the SULA is $d_x = \lambda$ and the distance between the auxiliary sensor and reference sensor is $d = \lambda/2$. Subarray 2 is a SLA with M sensor elements and inter-element spacings $d_i \geq \lambda/2$ $(i = 1, 2, \ldots, M-1)$. In order to avoid phase ambiguity, the minimum inter-sensor spacing of SLA is set to $d_{min} = \lambda/2$. Assume that there are K uncorrelated narrowband signals with wavelength λ impinging on the L-shaped array from distinct directions (θ_k, ϕ_k) $(k = 1, 2, \ldots, K)$, where θ_k and ϕ_k denote, respectively, the elevation and azimuth angles of the kth signal.

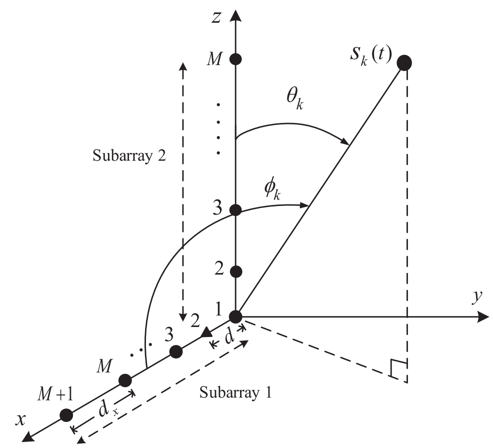

Fig. 1. Sparse L-shaped array for 2-D DOA estimation.

The output vectors of subarrays 1 and 2 can be written as

$$\mathbf{x}(t) = \mathbf{A}_x \mathbf{s}(t) + \mathbf{n}_x(t),\ \mathbf{z}(t) = \mathbf{A}_z \mathbf{s}(t) + \mathbf{n}_z(t) \qquad (1)$$

where $\mathbf{s}(t)$ denotes the signal vector. $\mathbf{n}_x(t)$ and $\mathbf{n}_z(t)$ are the additive noise vectors. $\mathbf{A}_x = [\mathbf{a}_x(\phi_1), \mathbf{a}_x(\phi_2), \cdots, \mathbf{a}_x(\phi_K)]$ is the $(M+1) \times K$ array response matrix of subarray 1, where

$$\mathbf{a}_x(\phi_k) = [1, e^{jd\Phi_k}, e^{jd_x\Phi_k}, \cdots, e^{j(M-1)d_x\Phi_k}]^T \qquad (2)$$

is the steering vector of subarray 1 for the kth signal and $\Phi_k = 2\pi \cos\phi_k/\lambda$. $\mathbf{A}_z = [\mathbf{a}_z(\theta_1), \mathbf{a}_z(\theta_2), \cdots, \mathbf{a}_z(\theta_K)]$ is the $M \times K$ array response matrix of subarray 2, where

$$\mathbf{a}_z(\theta_k) = [1, e^{jd_1\Theta_k}, \cdots, e^{j\sum_{i=1}^{M-1} d_i\Theta_k}]^T \qquad (3)$$

is the steering vector of subarray 2 for the kth signal and $\Theta_k = 2\pi\cos\theta_k/\lambda$.

Moreover, define $\mathbf{x}'(t) = [x_1(t), x_3(t), \cdots, x_{M+1}(t)]^T$ as the received vector of the SULA in subarray 1. Correspondingly, $\mathbf{A}_{x'} = [\mathbf{a}_{x'}(\phi_1), \cdots, \mathbf{a}_{x'}(\phi_K)]$ is the $M \times K$ array response matrix of the SULA, where

$$\mathbf{a}_{x'}(\phi_k) = [1, e^{j\Psi_k}, \cdots, e^{j(M-1)\Psi_k}]^T \qquad (4)$$

and $\Psi_k = (2\pi d_x/\lambda)\cos\phi_k = 2\pi\cos\phi_k$ denotes the phase difference between two adjacent sensors in the SULA.

3. Proposed Method

The covariance matrix of $\mathbf{z}(t)$ can be expressed as

$$\mathbf{R}_{zz} = E\{\mathbf{z}(t)\mathbf{z}^H(t)\} = \mathbf{A}_z\mathbf{R}_s\mathbf{A}_z^H + \sigma^2\mathbf{I}_M \qquad (5)$$

In practice, the covariance matrix $\mathbf{R}_{zz}$ is estimated by

$$\hat{\mathbf{R}}_{zz} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{z}(t) * \mathbf{z}^H(t) \tag{6}$$

where N denotes the number of snapshots.

The eigenvalue decomposition (EVD) of $\hat{\mathbf{R}}_{zz}$ yields

$$\hat{\mathbf{R}}_{zz} = \mathbf{U}_{zs}\mathbf{D}_s\mathbf{U}_{zs}^H + \mathbf{U}_{zn}\mathbf{D}_n\mathbf{U}_{zn}^H \tag{7}$$

where $\mathbf{U}_{zn}$ and $\mathbf{U}_{zs}$ are the noise and signal subspace.

Furthermore, we can construct the MUSIC spectrum

$$P_{MUSIC} = \frac{1}{\mathbf{a}_z^H(\theta)\mathbf{U}_{zn}\mathbf{U}_{zn}^H\mathbf{a}_z(\theta)}. \tag{8}$$

Using (8), we can estimate the elevation angles by two steps: 1) Create a rough grid with searching interval l_1 and find peaks to get K rough estimates of the elevation angles. 2) Create a refined grid with searching interval l_2 around the locations of the peaks in the first search and perform search again to obtain more accurate estimates of K elevation angles.

Next, we will estimate azimuth angles. Let $\mathbf{z}'(t) = [z_2(t), z_3(t), \cdots, z_M(t)]^T$, and then we can obtain a cross-correlation matrix between $\mathbf{x}(t)$ and $\mathbf{z}'(t)$

$$\mathbf{R}_{xz'} = E\{\mathbf{x}(t)\mathbf{z}'(t)^H\} = \mathbf{A}_x\mathbf{R}_s\mathbf{A}_{z'}^H \tag{9}$$

where $\mathbf{A}_{z'} = \mathbf{A}_z(2:M,:)$.

Substituting the elevation estimates $\hat{\theta}$ into $\mathbf{A}_z$, we can obtain $\hat{\mathbf{A}}_{z'} = \hat{\mathbf{A}}_z(2:M,:)$. Under the AGWN environment, we can estimate $\mathbf{A}_x\mathbf{R}_s$ by solving the following least square (LS) problem

$$\widetilde{\mathbf{A}_x\mathbf{R}_s} = \arg\min_{\mathbf{A}_x\mathbf{R}_s}\left\|\mathbf{R}_{xz'} - \mathbf{A}_x\mathbf{R}_s\hat{\mathbf{A}}_{z'}^H\right\|_F^2. \tag{10}$$

The solution of (10) is $\widetilde{\mathbf{A}_x\mathbf{R}_s} = \mathbf{R}_{xz'}(\hat{\mathbf{A}}_{z'}^H)^{\dagger}$.

According to (5) and (7), we have $\mathbf{A}_z\mathbf{R}_s\mathbf{A}_z{}^H \approx \mathbf{U}_{zs}\mathbf{D}_s\mathbf{U}_{zs}^H$, then the signal covariance matrix can be estimated by

$$\hat{\mathbf{R}}_s = \hat{\mathbf{A}}_z^{\dagger}\mathbf{U}_{zs}\mathbf{D}_s\mathbf{U}_{zs}^H(\hat{\mathbf{A}}_z^H)^{\dagger}. \tag{11}$$

Therefore, we can obtain the estimation of $\mathbf{A}_x$

$$\hat{\mathbf{A}}_x = \mathbf{R}_{xz'}(\hat{\mathbf{A}}_{z'}^H)^{\dagger}\left(\hat{\mathbf{A}}_z^{\dagger}\mathbf{U}_{zs}\mathbf{D}_s\mathbf{U}_{zs}^H(\hat{\mathbf{A}}_z^H)^{\dagger}\right)^{-1}. \tag{12}$$

Furthermore, we can obtain $\hat{\mathbf{A}}_{x'}$ through $\hat{\mathbf{A}}_x$.

To solve ambiguity, we firstly obtain K rough estimates of the azimuth angles by

$$\phi_k' = \cos^{-1}\left[\arg\left(\frac{\hat{\mathbf{a}}_{x'',k_1}^H\hat{\mathbf{a}}_{x'',k_2}}{\|\hat{\mathbf{a}}_{x'',k_1}\|\|\hat{\mathbf{a}}_{x'',k_2}\|}\right) / \left(\frac{2\pi d}{\lambda}\right)\right], \tag{13}$$

where $\hat{\mathbf{a}}_{x'',k_1} = [1 \;\; e^{j2\pi d\cos\phi_k/\lambda}]^T$ and $\hat{\mathbf{a}}_{x'',k_2} = [e^{j2\pi d\cos\phi_k/\lambda} \;\; e^{j2\pi d_x\cos\phi_k/\lambda}]^T$.

According to the rough azimuth estimates above, determine the range where each azimuth angle may be, then K fine estimates of the azimuth angles can be obtained by

$$\hat{\phi}_k = \begin{cases} \cos^{-1}[\frac{\hat{\Psi}_k+2\pi}{2\pi d_x/\lambda}], & \phi_k' \in [0, 60^o] \\ \cos^{-1}[\frac{\hat{\Psi}_k}{2\pi d_x/\lambda}], & \phi_k' \in (60^o, 120^o) \\ \cos^{-1}[\frac{\hat{\Psi}_k-2\pi}{2\pi d_x/\lambda}], & \phi_k' \in [120^o, 180^o] \end{cases} \tag{14}$$

where

$$\hat{\Psi}_k = \arg\left(\frac{\hat{\mathbf{a}}_{x',k_1}^H\hat{\mathbf{a}}_{x',k_2}}{\|\hat{\mathbf{a}}_{x',k_1}\|\|\hat{\mathbf{a}}_{x',k_2}\|}\right), \quad k = 1, \cdots, K \tag{15}$$

where $\hat{\mathbf{a}}_{x',k_1}$ and $\hat{\mathbf{a}}_{x',k_2}$ denote the first and last $M-1$ elements of $\hat{\mathbf{a}}_{x',k}$ that denotes the kth column of $\hat{\mathbf{A}}_{x'}$.

4. Simulation Results

We consider an L-shaped array as shown in Fig. 1, and assume that $M = 6$, the inter-sensor spacings of Subarray 2 are $d_1 = \lambda/2, d_2 = 5\lambda/2, d_3 = 3\lambda/2, d_4 = \lambda, d_5 = \lambda$. For J-2D, the inter-element spacings of the SLA along the x-axis are $d_1 = \lambda/2, d_2 = \lambda, d_3 = 3\lambda/2, d_4 = \lambda, d_5 = \lambda$. The searching range of the elevation angle is $[0°, 180°]$ with searching intervals $l_1 = 1°$ and $l_2 = 0.1°$. Two uncorrelated sources with 2-D DOAs $(50°, 55°)$ and $(80°, 65°)$ are considered. The snapshots number is set to $N = 100$, and 1000 independent independent trials are performed. Figs. 2 indicates that the proposed method achieves better performance than the JSVD [1], CCM-ESPRIT-AP [2] and J-2D [3] methods, especially at low SNR.

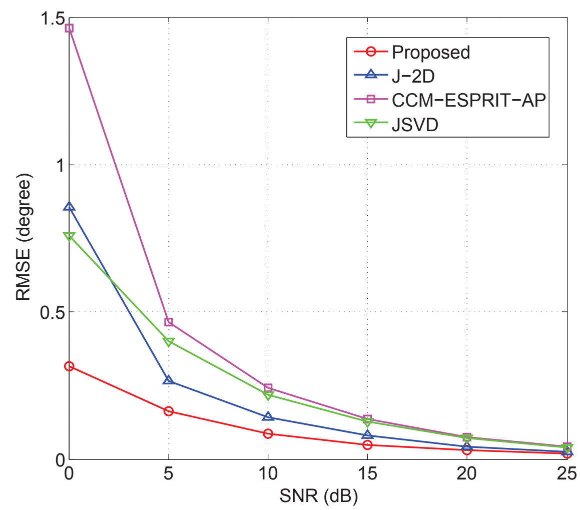

Fig. 2. RMSE of 2-D DOA estimates versus the SNR.

5. Conclusions

A 2-D DOA estimation method based on sparse L-shaped array has been proposed in this paper. The proposed method does not require an extra pairing process and can achieve automatic pairing for 2-D DOA estimation. Simulation results show that our approach exhibit better performance than some existing methods with automatic pairing based on L-shaped array, such as JSVD, CCM-ESPRIT-AP and J-2D.

References

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