LIGHT SCATTERING OF PARTIALLY COHERENT BEAM PULSES BY RANDOMLY FLUCTUATED COMPOSITE MEDIA

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1. Introduction

Electromagnetic scatterings of Gaussian optical beam pulses with partially coherent characteristics by randomly fluctuated microheterogeneities of composite media are discussed using statistical field method. Light scatterings of laser beam pulses are essential to optical probe method concerned with optical communication and object identification with refractive index distribution measurements in material science and technology(1),(2). Light scatterings due to microinhomogeneities, which are very important factors of optical propagation losses and indispensable for investigation of optical material spectroscopy, have been discussed(3). Spot dancing characteristics by index fluctuation with long correlation length are studied for optical propagation. Most of these scattering analysis are discussed only for forward scatterings. Stationary backward scattering by microheterogeneities was studied by the author, using perturbation theory.

Inverse scattering problems of identification of distance dependencies of random inhomogeneities have been shown for measurement of transverse refractive index distribution of optical fibers. These stationary scattering method can not be applied to inverse problems for long distance inhomogeneities in the propagation direction. Particularly, pulse beam reflection and inverse methods of laser radar are powerful to identification of inhomogeneities distribution in the propagation direction. However, exact electromagnetic scattering and reflection theory of optical beam pulses by inhomogeneous index distribution has not been shown. In this paper, based on the previous statistical stationary theory shown by the author, statistical transient or non-stationary scattering and reflection theories of partially coherent beam pulses are shown for inhomogeneous index distribution with randomly microheterogeneities. Inverse scattering properties of beam pulses are discussed.

2. Electromagnetic Field

The dielectric constants of inhomogeneous media with temporal and spatial refractive index variations can been shown in a general form as follows,

$$\varepsilon(\mathbf{r}) = \varepsilon + \varepsilon \sum_i \Delta \eta_i(\mathbf{r}, t) \tag{1}$$

Equation (1) can be applied to inhomogeneous composite media with lossy and microheterogeneities such as random atmosphere, aerosols, glasses, polymers and liquids. Each terms of $\Delta \eta_i(\mathbf{r}, t)$ shows the refractive index fluctuations caused by different chemical and physical factors. Electromagnetic fields $\mathbf{E}(r,t)$ can be written as Fourier transform of spectrum $\hat{\mathbf{E}}(\mathbf{r}, \omega)$,

$$\hat{\mathbf{E}}(\mathbf{r}, \omega) = \int_{-\infty}^{\infty} \mathbf{E}(\mathbf{r}, t) e^{-j\omega t} dt \tag{2}$$

where,

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2\pi} \int_0^{\infty} \hat{\mathbf{E}}(\mathbf{r}, \omega) e^{-j\omega t} d\omega \tag{3}$$

Spectrum components of fluctuations are

$$\Delta \hat{\eta}_i(\mathbf{r}, \omega) = \int_{-\infty}^{\infty} \eta_i(\mathbf{r}, t) e^{-j\omega t} dt \tag{4}$$

Temporal fluctuations are more slower than variations of optical waves, and spectrum of fluctuations are extremely smaller than those of optical waves. From Maxwell's equation, we have the following fundamental field equation for temporal and spatial fluctuated media,

$$\nabla \times \nabla \times \hat{\mathbf{E}}(\mathbf{r}, \omega) - \mu \varepsilon \omega^2 \hat{\mathbf{E}}(\mathbf{r}, \omega) =$$

$$-\mu \varepsilon \Big[\frac{1}{2\pi} \int (-\omega'^2) \sum_i \Delta \hat{\eta}_i(\omega') \hat{\mathbf{E}}(\omega - \omega') d\omega'$$

$$+ \frac{1}{2\pi} \int (-2\omega'(\omega - \omega')) \sum_i \Delta \hat{\eta}_i \hat{\mathbb{E}}(\omega - \omega') d\omega'$$

$$+ \frac{1}{2\pi} \int (-(\omega - \omega')^2) \Delta \hat{\eta}_i(\omega') \hat{\mathbf{E}}(\omega - \omega') d\omega' \Big] 2\pi \tag{5}$$

In normal case, such as temporal fluctuations which are not so high speedy, compared with the optical frequency, the right hand side of eq. (5) can be approximately written only by the third term.

If the incident beam pulse is $\mathbb{E}_0$, total and scattered field, $\hat{\mathbb{E}}$ and $\hat{\mathbf{E}}_{scatt}$, can be shown as

$$\hat{\mathbf{E}} = \hat{\mathbf{E}}_0 + \hat{\mathbf{E}}_{scatt} \tag{6}$$

where,

$$\hat{\mathbf{E}}_{scatt} = \int \hat{\mathbf{\Gamma}} \cdot \hat{\mathbf{F}}(\hat{\mathbf{E}}) dv' \tag{7}$$

Here, $\hat{\mathbf{\Gamma}}$ is the Green's dyadic function satisfying

$$\nabla \times \nabla \times \hat{\mathbf{\Gamma}}(\mathbf{r}, \mathbf{r}') - k^2 \hat{\mathbf{\Gamma}}(\mathbf{r}, \mathbf{r}') = \mathbb{I} \delta(\mathbf{r} - \mathbf{r}') \tag{8}$$

and $\hat{\mathbf{F}}$ is the functional as, if $k^2 = \omega^2 \varepsilon \mu$ and $\omega' \ll \omega$

$$\hat{\mathbf{F}}(\hat{\mathbf{E}}) \cong \mu \varepsilon \sum_i \int \omega^2 \Delta \hat{\eta}_i(\omega') \hat{\mathbf{E}}(\omega - \omega') d\omega' \tag{9}$$

Equations (7) and (9) can be symbolically expressed as

$$\hat{\mathcal{E}}_{sj} = \sum_k \Gamma_{jk} \hat{\mathcal{F}}_k (\Delta \eta_i \underset{\omega}{\otimes} \hat{\mathcal{E}}_k) \tag{10}$$

Cross correlations of scattered fields are

$$\langle \mathbf{E}_{si}(\mathbf{r}_1, t_1) E_{sj}^*(\mathbf{r}_2, t_2) \rangle = \frac{1}{(2\pi)^2} \int_0^{\infty} d\omega_1 \int_0^{\infty} d\omega_2 e^{j\omega_1 t_1 - j\omega_2 t_2}$$

$$\langle \sum_k \Gamma_{jk} \hat{\mathcal{F}}_k(\omega_1) \sum_l \Gamma_{jl}^* \hat{\mathcal{F}}_k(\omega_2) \rangle_e \tag{11}$$

Γ_{jk} denotes the jk tensor component of the Green's dyadic, and $\mathcal{F}_k$ is the k component of perturbations shown in eq.(9)

3. Partially Coherent Pulses

When incident wave is a pulse with the partially coherent wave, the field $\mathbf{E}_0$ is written as follows, using spatial field function $\mathbf{E}_{a0}(r)$

$$\mathbf{E}_0(\mathbf{r}, t) = \mathbf{E}_{a0}(\mathbf{r}) v(t) \text{ and } \hat{\mathbf{E}}_0(\mathbf{r}, \omega) = \mathbf{E}_{a0}(\mathbf{r}) \hat{V}(\omega) \tag{12}$$

Here, $\hat{V}(\omega)$ is fourier component of the pulse packet $v(t)$. The time function can be written as

$$v(t) = S(t) h(t) \tag{13}$$

Where, the carrier wave $S(t)$ of light source is modulated by a pulse wave $h(t)$. Correlations of light sources are, in the stationary case of optical carriers.

$$R(t_1 - t_2) = \langle S(t_1) S(t_2) \rangle = R_0 e^{j\omega_0 \tau - b^2 \tau^2} \tag{14}$$

where $\tau = t_1 - t_2$, ω_0 is center frequency of optical waves and 1/b is the coherence time. Spectrum properties of incident pulse waves are given by center frequency ω_0, coherence factor b and pulse wave h(t)

4. Scattered Field and Fluctuations

From eq.(11), cross correlations of scattered fields are derived using temporal and spatial correlation function of material fluctuations, and partial coherence functions of incident beam pulse. Scattered field intensities are evaluated by

$$\langle \mathbf{E}(\mathbf{r},t) \times \mathbf{H}^*(\mathbf{r},t) \rangle_e = \frac{1}{(2\pi)^2} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 e^{j\omega_1 t_1 - j\omega_2 t_2} \langle V(\omega_1) V^*(\omega_2) \rangle$$

$$\int dv' \int dv'' \mathbf{K}(\omega_1, \omega_2, \mathbf{r}', \mathbf{r}'', \mathbf{r}) \qquad (15)$$

where field scattering function $\mathbf{K}$ are calculated using correlation functions of fluctuations and Green's dyadic $\mathbf{\Gamma}$, as

$$\langle \Delta \hat{\eta}_i(\mathbf{r}', \omega_1) \Delta \hat{\eta}_j^*(\mathbf{r}'', \omega_2) \rangle = \begin{cases} B^+_{\eta_i}((\mathbf{r}'+\mathbf{r}'')/2) B^-_{\eta_i}(\mathbf{r}'-\mathbf{r}'') B_{\omega_i}(\omega_1 - \omega_2) & (i=j) \\ 0 & (i \neq j) \end{cases} \qquad (16)$$

Field scattering function $\mathbf{K}$ can be expressed as the factorization form when the fluctuation expressed by eq.(16),

$$\mathbf{K}(\omega_1, \omega_2, \mathbf{r}', \mathbf{r}'', \mathbf{r}) = \mathbf{K}_s^-(\mathbf{r}', \mathbf{r}'', \mathbf{r}) \mathbf{K}_s^+ K_t(\omega_1, \omega_2) \qquad (17)$$

In case of a incident beam pulse of the TEM_{00} mode with the spot size r_0, beam waist $z=-z_0$, and coherent pulse packet of $V(t) = V_0 e^{-t^2/\tau_0^2 + j\omega_0 t}$ and $1/b \gg \tau_0$, the scattered and reflected fields are obtained by eq.(15). Backward pulse characteristics $R_s(\mathbf{r})R_0(t)$ are shown, using non-stationary correlation function $B^+_{\eta_i} = B_{ab}^+(z)$ for inhomogeneous region $z_a<z<z_b$, as

$$R_s(\mathbf{r})R_0(t) = \mathbf{K}_s^- C \int_0^\infty \int_0^\infty e^{-j(k_1'-k_2'')(z-z_0)} d\omega_1 d\omega_2$$

$$\int_{z_a}^{z_b} e^{j(k_1'-k_2'')z'} B_{ab}^+(z')dz' \, e^{j\omega_1 t - j\omega_2 t}$$

$$e^{-\tau_0^2(\omega_1-\omega_0)^2/4 - \tau_0^2(\omega_2-\omega_0)^2/4} \qquad (18)$$

where $\mathbf{K}_s^- C$ are spatial scattered factors shown in Fig.2 as numerical examples for local spatial correlation $B^-_{\eta_i} = \overline{\Delta \eta_{0i}}^2 e^{-\rho^2/\rho_{0i}^2}$.

Inverse scattering properties of identification of inhomogeneous index distributions are evaluated from eq.(18).

5. Conclusion

Scattered and reflected fields of beam pulses of partial coherent optical waves due to non-stationary random fluctuations in time and space are exactly evaluated by using statistical transient electromagnetic field theory. Inverse scattering formulations are discussed for inhomogeneous index identification of fluctuations.

References
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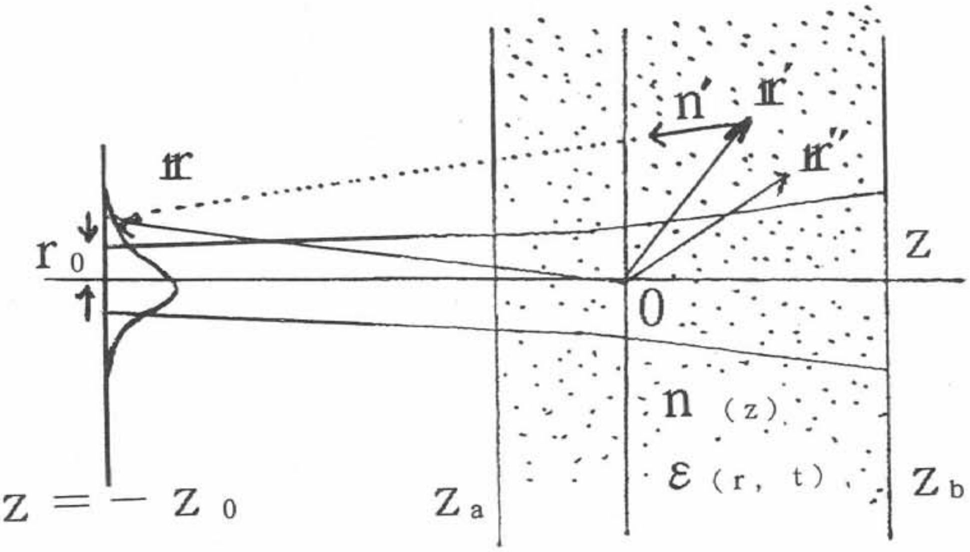
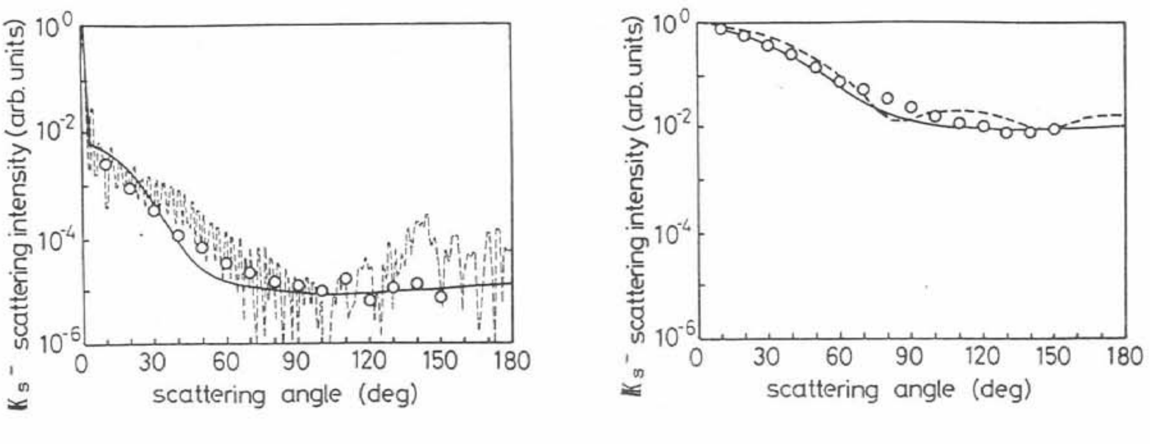


Fig.1 Scatterings and reflections of partially coherent beam pulses in randomly fluctuated media



(a)

(b)

Fig.2 Scattering patterns. The solid line is the calculation using K_s of eq.(17), the dashed line is the Mie scattering theory, and points indicate measured values. (a) Scattering pattern for water droplet clouds, extinction factor $\sigma=0.4\mathrm{m}^{-1}$, $\rho_{\theta i}=14,\ 1.6,\ 0.1\ \mu\mathrm{m}$, $\Delta\overline{\eta_{\theta i}}^2=0.0005,\ 0.0004,\ 0.0004$. In Mie theory, diameter of a particle D=14um, refractive index u=1.33. (b) Scattering pattern for smoke, $\sigma=0.4\mathrm{m}^{-1}$, $\rho_{\theta i}=0.3,\ 0.1\mu\mathrm{m}$, $\Delta\overline{\eta_{\theta i}}^2=0.0004,\ 0.0004$. In Mie theory, D=0.5um, u=1.55.